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Some Integral Results Associated with Generalized Hypergeometric Function

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Abstract: *In previous papers, it has been introduced a generalized hypergeometric function of two variables. The present paper aims at to derive different types of integral representations for the generalized hypergeometric function. The results derived here are very general in nature and are interesting and can obtain some known and new integrals for various polynomials. Each result is followed by its applications to the classical orthogonal polynomials.*

Index Terms: *Hypergeometric function, Generalized hypergeometric function, modified Jacobi polynomial, Laguerre polynomials.*

I. INTRODUCTION

In applications of mathematics, several special functions can be written in terms of Gauss Hypergeometric function ${}_2F_1\left[a,b;c;x\right]$ (see[12]). Later on many researchers investigated the generalized functions of Gauss hypergeometric function in different ways and applied it in solving the problems regarding mathematical physics. V.S.Bhagavan [14] studied integral representations of Generalized Hypergeometric polynomials and S.Haq et al. [10] studied new class of integrals associated with Geenralized Struve Function. M.Lahiri et al. [4]-[7] introduced generalized hypergeometric function by using a difference operator and derived many useful relations. In the previous paper [1], we introduced generalized hypergeometric function of two discrete variables $B_n^{(\alpha,\beta)}(x,y,w)$ and derived bilateral generating functions of it [2] . The generalized hypergeometric function $B_n^{(\alpha,\beta)}(x,y,w)$ is as follows:

$$B_n^{(\alpha,\beta)}(x,y,w)=\frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{n!}\times\sum_{r=0}^{n}\frac{(-1)^r\,y^{[rw]}\,J_{n-r}^{(\alpha)}(x,w)}{r!\Gamma(n+\alpha-r+1)\Gamma(r+\beta+1)} \quad (1)$$

where $J_n^{(\alpha)}(x,w)$ is modified Jacobi polynomial([3],[4]).

The function $B_n^{(\alpha,\beta)}(x,y,w)$ defined in (1) can be expressed as a double sum by using $J_n^{(\alpha)}(x,w)$ and applying the relations (7), (8) and (9) as follows :

$$B_n^{(\alpha,\beta)}(x,y,w)=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2}\times\sum_{r=0}^{n}\sum_{s=0}^{n-r}\frac{(-n)_{r+s}\left(-\frac{y}{w}\right)_r\left(\frac{x}{w}\right)_s(-w)^r(w)^s}{r!\,s!\,(1+\alpha)_s(1+\beta)_r} \quad (2)$$

$$=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2}F_{-:1;1}^{1:1;1}\left[\begin{matrix}-n:-\frac{y}{w},\ \frac{x}{w};-w,w\\ -:1+\beta;1+\alpha;\end{matrix}\right] \quad (3)$$

where $F_{q:s,v}^{p:r;u}$ is a double hypergeometric function (see [11],[12]).

By considering the limit $w\to 0$; $\beta=0, y=0$ and $w\to 0$; $\alpha=0, x=0$ and $w\to 0$ in (2), it leads to the following special cases

$$L_n^{(\alpha,\beta)}(x,y)=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2}\times\sum_{r=0}^{n}\sum_{s=0}^{n-r}\frac{(-n)_{r+s}(y)^r(x)^s}{r!\,s!\,(1+\alpha)_s(1+\beta)_r} \quad (4)$$

where $L_n^{(\alpha,\beta)}(x,y)$ is Laguerre polynomial of two variables defined by Ragab, S.F. [9].(see also[13]).

By writing $\beta=0, y=0$ and taking $w\to 0$ in (2), it reduces to

$$\underset{w\to 0}{Lt}\, B_n^{(\alpha,0)}(x,0,w)=L_n^{\alpha}(x)$$
(5)

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where $L_n^{\alpha}(x)$ is Laguerre polynomial, Rainville, E.D.[8].

By writing $\alpha = 0, x = 0$ and taking $w \to 0$ in (2), it reduces to

$$\underset{w\to 0}{Lt}\; B_n^{(0,\beta)}(0, y, w) = L_n^{\beta}(y) \quad (6)$$

where $L_n^{\beta}(y)$ is Laguerre polynomial, Rainville, E.D.[8].

The following familiar notations and results has been used

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1 & if\ n = 0 \\ a(a+1)\ldots(a+n-1) & if\ n = 1, 2, \ldots \& a \neq 0 \end{cases} \quad (7)$$

$$(a)_{n-t} = \frac{(-1)^t (a)_n}{(1-a-n)_t} \quad (8)$$

$$(-n)_t = \frac{(-1)^t n!}{(n-t)!} \quad (9)$$

We have the familiar Maclaurin's theorem

$$f(\upsilon) = \sum_{r=0}^{\infty} \frac{f^{(r)}(0)\, \upsilon^r}{r!} \quad (10)$$

We find the coefficients $f^r(0), r = 0, 1, 2, ---$ are given by means of the integrals

$$f^r(0) = \frac{r!}{2\pi i} \int^{(0+)} \frac{f(\upsilon)\, d\upsilon}{\upsilon^{n+1}},\ r = 0, 1, 2, --- \quad (11)$$

$$\int_0^{\pi/2} Cos^m\theta\, Cos(n\theta)\, d\theta = \frac{\Gamma(m+1)\pi}{2^{m+1}\Gamma\left(\frac{m+n+2}{2}\right)\Gamma\left(\frac{m-n+2}{2}\right)} \quad (12)$$

$$\int_0^1 t^{s-1} (\log t)^{x-1} dt = \frac{(-1)^{x-1}\Gamma(x)}{s^x} \quad (13)$$

Where Re(x) > 1 and s > 1.

If Re(x) > 0, Re(y) > 0, Re(λ) > 0, then

$$\iint_A u^{x-1} v^{y-1} (1-u-v)^{\lambda-1} du\, dv = \frac{\Gamma(x)\Gamma(y)\Gamma(\lambda)}{\Gamma(x+y+\lambda)} \quad (14)$$

where the area A is bound by

$u \geq 0,\ v \geq 0\ and\ u + v \leq 1$ (see [12]).

If Re(m) > 0 and Re(n) > 0, then

$$\int_0^1 \int_0^1 \frac{(1-x)^{m-1} y^m (1-y)^{n-1}}{(1-xy)^{m+n-1}} dx\, dy = \beta(m, n) \quad (15)$$

We know that for Re(p) > 0 and Re(q-p) > 0,

$$\frac{(p)_j}{(q)_j} = \frac{\Gamma(q)}{\Gamma(p)\Gamma(q-p)} \int_0^1 \xi^{p+j-1} (1-\xi)^{q-p-1} d\xi \quad (16)$$

$$\Gamma(n) = \int_{-\infty}^{\infty} e^{-t^2}\, t^{2n-1}\, dt \quad (17)$$

II. INTEGRAL REPRESENTATIONS FOR $B_n^{(\alpha,\beta)}(x, y, w)$

A. *A Contour Integral Representation*

From the generating relation of $B_n^{(\alpha,\beta)}(x, y, w)$, (see[1]), we have

$$\sum_{n=0}^{\infty} \frac{n!\, B_n^{(\alpha,\beta)}(x, y, w)}{(1+\alpha)_n (1+\beta)_n} t^n = e^t\, {}_1F_1\left[\begin{matrix} -\frac{y}{w}; \\ 1+\beta; \end{matrix}\ wt\right] {}_1F_1\left[\begin{matrix} \frac{x}{w}; \\ 1+\alpha; \end{matrix}\ -wt\right] \quad (18)$$

Now, by considering

$$f(u) = e^u\, {}_1F_1\left[\begin{matrix} -\frac{y}{w}; \\ 1+\beta; \end{matrix}\ wu\right] {}_1F_1\left[\begin{matrix} \frac{x}{w}; \\ 1+\alpha; \end{matrix}\ -wu\right] \quad (19)$$

and using the Maclaurin's theorem (10) and by using (11), we arrive at the following theorem:

Theorem 1: If

$$e^u\, {}_1F_1\left[\begin{matrix} -\frac{y}{w}; \\ 1+\beta; \end{matrix}\ wu\right] {}_1F_1\left[\begin{matrix} \frac{x}{w}; \\ 1+\alpha; \end{matrix}\ -wu\right] = \sum_{n=0}^{\infty} \frac{n!\, B_n^{(\alpha,\beta)}(x, y, w)}{(1+\alpha)_n (1+\beta)_n} u^n \text{, then}$$

$B_n^{(\alpha,\beta)}(x, y, w)$

$$= \frac{(1+\alpha)_n (1+\beta)_n}{n!\,(2\pi i)} \int^{0+} u^{-n-1} e^{u}$$

$$\times {}_1F_1\left[\begin{matrix} -\frac{y}{w};\ wu \\ 1+\beta; \end{matrix}\right] {}_1F_1\left[\begin{matrix} \frac{x}{w};\ -wu \\ 1+\alpha; \end{matrix}\right] du \qquad (20)$$

where the contour integration encircles the origin of the u-plane in the positive direction.

B. *The Real Integral Representation*

From (20), we obtain (on taking the contour $u = e^{i\theta}$)

$$B_n^{(\alpha,\beta)}(x,y,w)$$

$$= \frac{(1+\alpha)_n (1+\beta)_n}{n!\,(2\pi)} \sum_{r,s,m=0}^{\infty} \frac{\left(-\frac{y}{w}\right)_r \left(\frac{x}{w}\right)_s w^r (-w)^s}{(1+\beta)_r (1+\alpha)_s\, m!\,r!\,s!}$$

$$\times \int_0^{2\pi} Cis(m-n+r+s)\theta\, d\theta$$

Consequently, we arrive at

$$= \frac{(1+\alpha)_n (1+\beta)_n}{n!\,(\pi)} \sum_{r,s,m=0}^{\infty} \frac{\left(-\frac{y}{w}\right)_r \left(\frac{x}{w}\right)_s w^r (-w)^s}{(1+\beta)_r (1+\alpha)_s\, m!\,r!\,s!}$$

$$\times \int_0^{\pi} Cis(m-n+r+s)\theta\, d\theta \qquad (21)$$

C. *Finite Single Integral Representation*

Now, on applying (16),

$$\frac{(1+\alpha)_n (1+\beta)_n}{(n!)^2} F^{1:1;1}_{-:1;1}\left[\begin{matrix} -n: -\frac{y}{w}; \frac{x}{w} \quad ; -w, w \\ -:1+\beta; 1+\alpha; \end{matrix}\right]$$

$$= \frac{(1+\alpha)_n (1+\beta)_n}{(n!)^2} \sum_{r=0}^{n} \sum_{s=0}^{n-r} \frac{(-n)_{r+s}}{r!\,s!\,(1+\alpha)_s}$$

$$\times \frac{\left(-\frac{y}{w}\right)_r \left(\frac{x}{w}\right)_s (-w)^r (w)^s (b)_{r+s} \Gamma(b)}{(1+\beta)_r (a)_{r+s} \Gamma(a)\Gamma(b-a)}$$

$$\times \int_0^1 t^{a+r+s-1} (1-t)^{b-a-1} dt$$

and we come across with the following theorem:

Theorem 2: If $\mathrm{Re}(a) > 0$ and $\mathrm{Re}(b-a) > 0$, then

$$B_n^{(\alpha,\beta)}(x,y,w) = \frac{\Gamma(b)}{\Gamma(a)\,\Gamma(b-a)} \int_0^1 t^{a-1}(1-t)^{b-a-1}$$

$$\times \frac{(1+\alpha)_n (1+\beta)_n}{(n!)^2} F^{2:1;1}_{1:1;1}\left[\begin{matrix} -n, b; -\frac{y}{w}; \frac{x}{w}; -wt, wt \\ a; 1+\beta; 1+\alpha; \end{matrix}\right] dt \qquad (22)$$

Applications:

(i) If a = b, the above theorem reduces into

$$B_n^{(\alpha,\beta)}(x,y,w) = \int_0^1 \frac{t^{a-1}(1+\alpha)_n (1+\beta)_n}{(1-t)\,(n!)^2}$$

$$\times F^{1:1;1}_{-:1;1}\left[\begin{matrix} -n: -\frac{y}{w}; \frac{x}{w}; -wt, wt \\ -:1+\beta; 1+\alpha; \end{matrix}\right] dt$$

$$= \int_0^1 \frac{t^{a-1}}{(1-t)} B_n^{(\alpha,\beta)}(x,y,wt)\, dt$$

(ii) If a=b and $w \to 0$, the above theorem leads to

$$B_n^{(\alpha,\beta)}(x,y,w) = \int_0^1 \frac{t^{a-1}}{(1-t)} L_n^{(\alpha,\beta)}(xt,yt)\, dt$$

where $L_n^{(\alpha,\beta)}(xt,yt)$ is the Laguerre polynomial of two variables by Ragab, S.F.(see [9])

(iii) By considering a=b, β=0, y=0 and $w \to 0$, the above theorem leads to

$$B_n^{(\alpha,\beta)}(x,y,w) = \int_0^1 \frac{t^{a-1}}{(1-t)} L_n^{\alpha}(xt)\, dt$$

where $L_n^{\alpha}(xt)$ is the Laguerre polynomial (see [1],[8]).

iv) By considering a=b, α=0, x=0 and $w \to 0$, the above theorem leads to

$$B_n^{(\alpha,\beta)}(x,y,w) = \int_0^1 \frac{t^{a-1}}{(1-t)} L_n^{\beta}(yt)\, dt$$

where $L_n^{\beta}(yt)$ is the Laguerre polynomial (see [1],[8]).

Theorem 3: If $\mathrm{Re}(a) > -\frac{1}{2}$ *and* $\mathrm{Re}(b) > -\frac{1}{2}$, *then*

$$B_n^{(\alpha,\beta)}(x,y,w) = \frac{\Gamma(a+b)(1+\alpha)_n (1+\beta)_n}{\Gamma(a)\Gamma(b)(n!)^2}$$

$$\times 2\int_0^{\pi/2} (Sint)^{2a-1}(Cost)^{2b-1}$$

$$\times F^{1:3;1}_{-:3;1}\left[\begin{matrix} -n:-\frac{y}{w},\frac{a+b}{2},\frac{a+b}{2}+\frac{1}{2}; & \frac{x}{w}; \\ -\ :\ 1+\beta,\ a,\ b\ \ ; & 1+\alpha; \end{matrix}\ -w\sin^2 t\cos^2 t, w\right]dt \qquad (23)$$

Proof: The hypergeometric representation of $B_n^{(\alpha,\beta)}(x,y,w)$ (see[1]), gives

$$B_n^{(\alpha,\beta)}(x,y,w)=\frac{\Gamma(a+b)(1+\alpha)_n(1+\beta)_n}{\Gamma(a)\Gamma(b)(n!)^2}$$

$$\times\sum_{r=0}^{n}\sum_{s=0}^{n-r}\frac{(-n)_{r+s}(a+b)_r\left(-\frac{y}{w}\right)_r\left(\frac{x}{w}\right)_s}{r!s!(1+\alpha)_s(1+\beta)_r}$$

$$\times\frac{2(-w)^r w^s}{(a)_r(b)_r}\int_0^{\pi/2}(Sint)^{2(a+r)-1}(Cost)^{2(b+r)-1}dt$$

By interchanging summation and integration, we have

$$=\frac{\Gamma(a+b)(1+\alpha)_n(1+\beta)_n}{\Gamma(a)\Gamma(b)(n!)^2}$$

$$\times 2\int_0^{\pi/2}(Sint)^{2a-1}(Cost)^{2b-1}$$

$$\times F^{1:3;1}_{-:3;1}\left[\begin{matrix} -n:-\frac{y}{w},\frac{a+b}{2},\frac{a+b}{2}+\frac{1}{2}; & \frac{x}{w}; \\ -\ :\ 1+\beta,\ a,\ b\ \ ; & 1+\alpha; \end{matrix}\ -w\sin^2 t\cos^2 t, w\right]dt$$

D. *Infinite Single Integral Representation*

Theorem 4 :

$$B_n^{(\alpha,\beta)}(x,y,w)$$

$$=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2\Gamma(a)}\int_{-\infty}^{\infty}e^{-t^2}t^{2a-1}$$

$$\times F^{1:1;1}_{1:1;1}\left[\begin{matrix} -n:-\frac{y}{w}; & \frac{x}{w}; \\ a:1+\beta; & 1+\alpha; \end{matrix}\ -wt^2, wt^2\right]dt \qquad (24)$$

Proof: The hypergeometric representation of $B_n^{(\alpha,\beta)}(x,y,w)$ (see[1]) and on making use of integral formula (17) (see [8]), we get

$$B_n^{(\alpha,\beta)}(x,y,w)$$

$$=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2\Gamma(a)}\sum_{r=0}^{n}\sum_{s=0}^{n-r}\frac{(-n)_{r+s}\left(-\frac{y}{w}\right)_r}{r!\,s!\,(1+\alpha)_s}$$

$$\times\frac{\left(\frac{x}{w}\right)_s(-w)^r(w)^s}{(1+\beta)_r(a)_{r+s}}\int_{-\infty}^{\infty}e^{-t^2}t^{2(a+r+s)-1}dt$$

On changing the order of summation and integration, we arrive at the following

$$B_n^{(\alpha,\beta)}(x,y,w)$$

$$=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2\Gamma(a)}\int_{-\infty}^{\infty}e^{-t^2}t^{2a-1}$$

$$\times F^{1:1;1}_{1:1;1}\left[\begin{matrix} -n:-\frac{y}{w}; & \frac{x}{w}; \\ a:1+\beta; & 1+\alpha; \end{matrix}\ -wt^2, wt^2\right]dt$$

Applications:

(i) If $w\to 0$, the above theorem leads to

$$B_n^{(\alpha,\beta)}(x,y,w)=\frac{(1+\alpha)_n(1+\beta)_n}{(n!)^2\Gamma(a)}\int_{-\infty}^{\infty}e^{-t^2}t^{2a-1}$$

$$\times F^{1:-;-}_{1:1;1}\left[\begin{matrix} -n:-;-; \\ a:1+\beta;1+\alpha; \end{matrix}\ yt, xt\right]dt$$

E. *Double Integral Representation*

Applying (14) on hypergeometric representation of $B_n^{(\alpha,\beta)}(x,y,w)$, we get

$$B_n^{(\alpha,\beta)}(x,y,w)$$

$$=\frac{\Gamma(\lambda)(1+\alpha)_n(1+\beta)_n}{\Gamma(\lambda-a-b)\Gamma(a)\Gamma(b)(n!)^2}$$

$$\times\sum_{r=0}^{n}\sum_{s=0}^{n-r}\frac{(-n)_{r+s}\left(\frac{\lambda}{2}\right)_r\left(\frac{\lambda}{2}+\frac{1}{2}\right)_r\left(-\frac{y}{w}\right)_r}{r!s!(a)_r(b)_s(1+\beta)_r}$$

$$\times\frac{\left(\frac{x}{w}\right)_s(-w)^r w^s}{(1+\alpha)_s}$$

$$\times\iint u^{a+r-1}v^{b+r-1}(1-u-v)^{\lambda-a-b-1}du\,dv$$

Finally, on interchanging the integration and summation, we arrive at the following theorem

Theorem 5 : If Re(a) > 0, Re(b) > 0, Re(λ) > 0, then

$$B_n^{(\alpha,\beta)}(x,y,w) = \frac{\Gamma(\lambda)(1+\alpha)_n(1+\beta)_n}{\Gamma(\lambda-a-b)\Gamma(a)\Gamma(b)(n!)^2} \iint u^{a-1}v^{b-1}(1-u-v)^{\lambda-a-b-1} \times F_{-:3;1}^{1:3;1}\left[\begin{matrix} -n: \frac{\lambda}{2}, \frac{\lambda}{2}+\frac{1}{2}, -\frac{y}{w}; \frac{x}{w}; -uvw, w \\ -: a,\ b,\ 1+\beta\ ;\ 1+\alpha; \end{matrix}\right] du\,dv \quad (25)$$

III. CONCLUSION

In this paper contour integral representation and different type of integral representations has been discussed for generalized hypergeometric function and some applications for it also given. In the same manner, we can derive some other type of double integral representations and also transforms of integrals like Laplace, Euler etc., can be derived which are useful in engineering and mathematical physics.

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