

# Graphical Tool to Estimate the Partial Solution to Linear Optimization Problems.



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**Resumen.** This work shows the development of an application with graphical interface that allows to determine approximately the solution to linear optimization problems. This application is based on the descendant gradient method to solve functions of several variables, in addition to implementing a program that looks for local minimums from the "Karush-Kuhn-Tucker" method, which is a generalization of Lagrange multipliers. In both cases tests were performed with different functions to perform the validation of the operation of each application respectively.

**Palabras Clave:** Numerical Method, Optimization, Gradient, Hessian, Minima, Maximum.

## I. INTRODUCTION

In order to solve engineering problems, mathematicians and scientists that have a high degree of complexity or are not solvable by conventional methods, some computational methods arise, including numerical methods that allow approaching a solution in a practical and simple way, since that rely on computers and software developing tools designed for that purpose [1]. This work shows how functions can be solved using the descendant gradient method [2], which consists in moving a point in a plane in the opposite direction to the gradient in that point, also the Karush Kuhn Tucker (KKT) method was implemented [3], which allows solving non-linear programming models because it takes into account conditions of inequality and equality. The latter term is known as a restriction and is the one that limits the space for searching for values for this function. Equation solving tools like this has been applied in several fields, one of them is telecommunications [4], where you can see in the design optimization of transmission networks and relay nodes from the concept of KKT combined with the convex optimization. Although these methods show that they are capable of solving a large number of functions, they have a disadvantage and that is that they depend on the initial point and with this convergence cannot be guaranteed at a global minimum, because if the initial point is not enough Well maybe the scheduled iterations or the termination condition does not have enough time or the necessary step to find the global minimum.

## II. STATE OF THE ART

There are different applications in which numerical methods have been applied for the solution of problems,

within this group are engineering problems, in which is the calibration of detection systems in gamma radiation spectrometry [5], which consists of in the characterization of samples of radioactive content in a quantitative and qualitative way. This technique allows the analysis of several elements, where the balance of the samples is evaluated, but the monetary cost of these experiments is quite high, so it was decided to perform a function that would allow the same analysis to be carried out at the lowest possible cost. Within the engineering line you can also find applications for the evaluation of structures used by civil engineers [6], these can be for elastic and inelastic dynamic analysis of flat and spatial gantries formed by bars, with some characteristics of rigidity, damping, deformation and mass, subjected to one or several force components in the base. This particular software makes use of the Newton Raphson method for the simulation of the mechanical effects of deformation on structures. Also in the optimal planning of electric power generation [7], there is a particular application of KKT, in the methodology for the design of optimal planning models of Distributed Generation in electricity distribution networks, with which it is intended find the best possible electrical distribution model with the restrictions placed by the line engineer in charge. Although you can find applications dedicated to the area of basic sciences [8], as are the interior point methods that apply variants of Newton's method to some equalities and modifying the directions of search and step size, they look for the restriction to be strictly satisfied in each iteration. Finally, variants of Newton's method can be found in fields of social impact, such as in the oil sector [9], through which the following problem is to be solved; There is a problem of two-phase flow (water and oil) through a porous medium, the problem is to displace the oil, which in an initial state completely occupies the pore volume, by injecting water at a constant flow rate Through a section of the boundary, the displaced oil exits through an extraction well or a section of the boundary until the pore volume is replaced by the water phase. The model consists of two coupled equations; The equation of pressure and the saturation equation, derived from some equations, the unknowns to solve in each iteration are; the oil pressure and water saturation. In which it is verified that with linear mobilities there is an optimal performance of Newton's method and the calculation of the variables has an error that is within the tolerance allowed by the industry.

## III. PRESENTATION OF THE PROPOSAL

The methodology that was used for the development of the applications that solve problems using the strategies of Newton Raphson and KKT was the following:

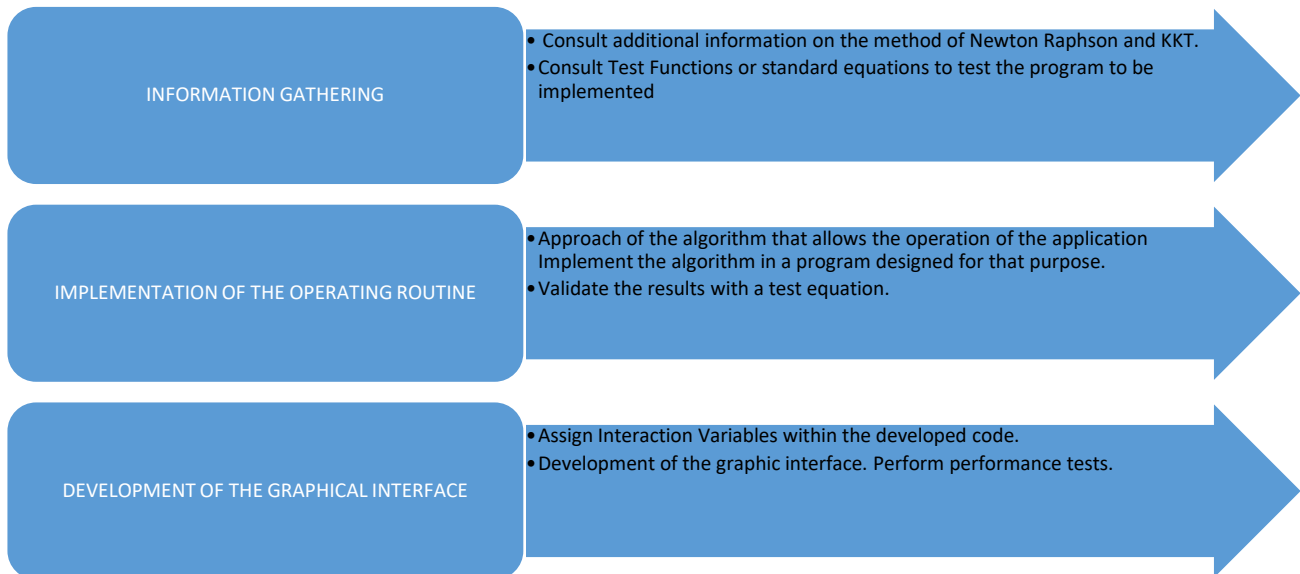
Revised Manuscript Received on 30 July 2019.

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**Fig. 1. Methodology used.**

For the development of the application, the definition of equation (1), which is Newton's method for root search, was taken into account, to which was added a variant shown in equation (2) to simulate the descending gradient method and finally in the application two was taken into account (3) for the implementation of the KKT method. Although it is worth mentioning that the calculation of the matrix was used to implement Newton's Hessian<sup>1</sup> method of the particular equation that is being worked on.

$$X(n+1) = x(n) - \left( \frac{\alpha * f'(x(n))}{f''(x(n))} \right)$$

$$X(n+1) = x(n) - \left( \frac{\alpha * f'(x(n))}{f''(x(n))} \right) + \mu * \Delta f'(x(n) - 1) \quad (2)$$

$$f(x) = f(x) + \sum_{i=1}^m P_i * g_i(x) + \sum_{j=1}^k Q_j * h^2_j(x) \quad \text{con } h^2_j(x) \leq 0 \quad (3)$$

Each of the programs was implemented following the structures shown in algorithms 1 and 2.

**Algorithm 1.** Descendant Gradient Function, This is the way by which the roots of the equations were calculated using this method.

```

Program Gradiente_Desc()
Define Initial Variables;
Read Equation;
Read Number of Variables;
Read Amount of Iterations;
Calculate Gradient;
Calculate Hessian;
while != Stop Condition
Evaluate Hessian
    
```

```

If Hessian > 0
    X(n+1) := X(n) -
    (Alfa * Gradient / Hessian) * Momentum;
    Momentum := f(x(n)) - f(x(n-1));
Else
    break;
    Evaluate Function;
End While
Show results.
End
    
```

**Algorithm 2.** Function of KKT, Using this strategy a test function was reduced using the Karush-Kuhn-Tucker method.

```

Program KKT()
Define Initial Variables;
Form Lagrangian function
While != Stop Condition
Evaluate restrictions;
Evaluate the restrictions in the form of a matrix;
End While
Show results.
End
    
```

## IV. EXPERIMENTATION AND RESULTS

This section is divided into two items which show the results obtained during the development of each application;

### 1.1 Descending Gradient

Fig. 2 shows the graphical interface developed for the interaction with this application which is divided into two; the first part graphs the result of the search when the function is of a single variable and the second only shows the minimum possible, but not graphical since some spaces are not graphically representable.

<sup>1</sup>A Hessian Matrix is the matrix of second partial derivatives of a function.

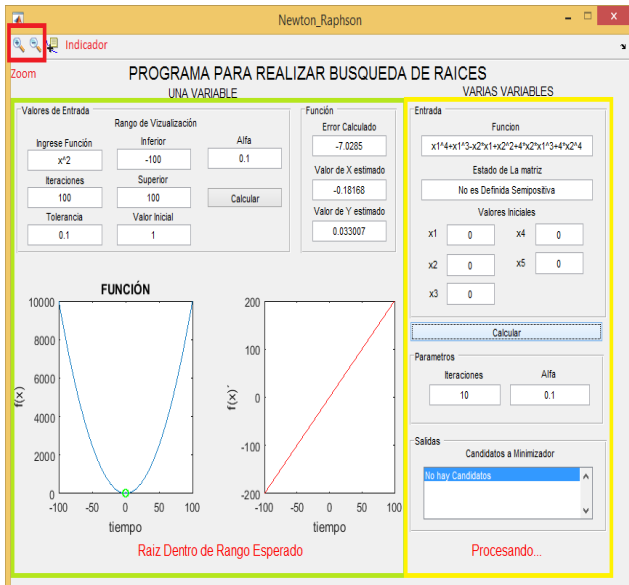


Fig. 2. Graphic Interface Gradient Gradient

In the evaluations of the function  $x^2$  it can be seen that this method corrects its error and can thus reach a local minimum after a certain number of iterations;

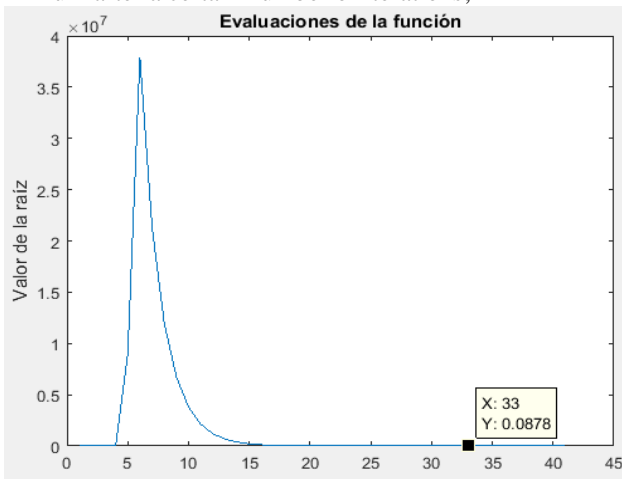


Fig. 3. Evaluación de la función de prueba

Once the method for a single variable has been tested, the same is done with a function of two form variables (4) whose results can be found in graph number 4;

$$x1^4 + x1^3 - x2 * x1 + x2^2 + 4 * x2 * x1^3 + 4 * x2^4 \quad (4)$$

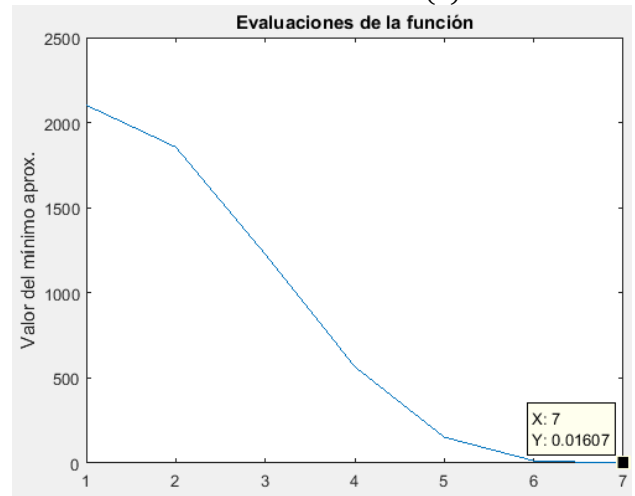


Fig. 4. Evaluation of the test function II

### 1.2 KKT

Like the previous program, a graphical interface was used to enter the function and the expanded summation as shown in figure 5, and figure 6 shows the results obtained by optimizing the test function (5) for this Program.

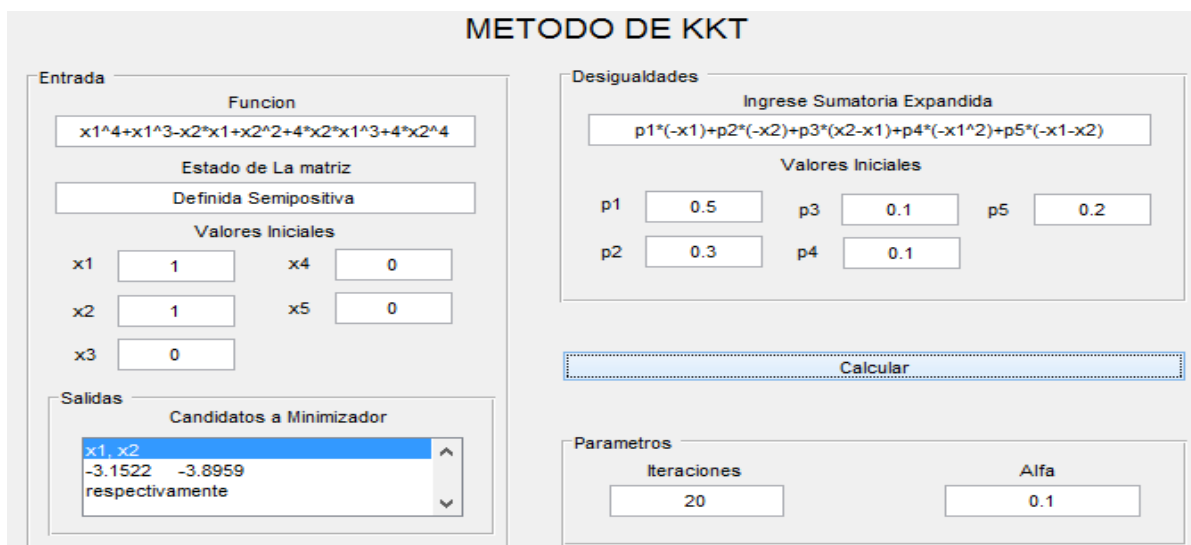


Fig. 5.KKT interface

## Graphical Tool to Estimate the Partial Solution to Linear Optimization Problems.

$$x_1^4 + x_1^3 - x_2 * x_1 + x_2^2 + 4 * x_2 * x_1^3 + 4 * x_2^4 \quad (5)$$

Hold it:

$$x_1 < 0, x_2 < 0, x_2 - x_1 > 0, x_1^2 < 0, x_1 + x_2 < 0$$

And the result obtained is reported in the text box at the bottom or by a graph as shown below;

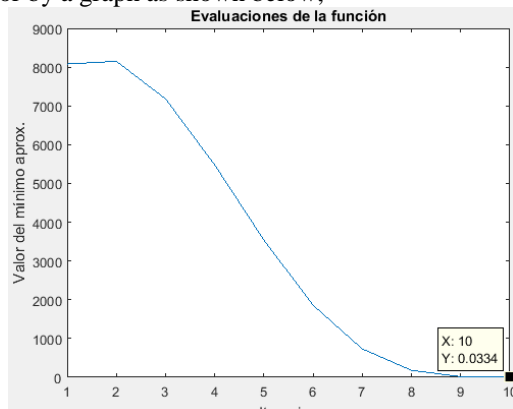


Fig. 6. Optimized function

The result of the functions is shown in an orderly manner in section 4.3, where they specify the conditions in which the tests were performed.

### 1.3 Results of the Experiments

Table 1. Results of the different Experiments

	Eq	Coefficients					Results			Iterations	
		Alfa	P1	P2	P3	P4	P5	X1	X2		F(X)
1	X <sup>2</sup>	0.5	-	-	-	-	-	0.25	--	0.063	40
2	(4)	0.2	-	-	-	-	-	1.99	1.8	0.016	10
3	(5)	0.15	0.3	0.1	0.1	0.1	0.2	3.15	3.8	0.33	20

These results show which is the local minimum of the test functions shown in the previous literals, where function 1 and 2 were evaluated without restriction.

## V. CONCLUSIONS

- The methods discussed in this document do not ensure convergence because they depend on the initial conditions to function during their operation, although in the tests carried out, they always found a local minimum.
- Sometimes, when too many iterations are programmed, the point does not remain in the local minimum, but goes to other zones, but returns to the same point thanks to the gradient indicating the route to follow to converge again.
- Restrictions make the search process of a local minimum somewhat more complex, since satisfying them all is sometimes impossible, but thanks to the allocation of weights made, solutions can be found close to an ideal solution.

## VI. FUTURE WORK

- You can implement search methods for functions that your Hessian is equal to zero, similarly look for numerical methods that allow convergence in a fast way and that guarantee that when the system finds a global minimum or maximum it automatically stops.
- These methods can be used in engineering to determine critical parameters of stability, with which it is possible to carry out better development of structural projects, design, among others. These parameters are important because they can influence the cost or development time of a project.

## REFERENCIAS

1. F. Vazquez, « INTRODUCCIÓN A LOS MÉTODOS NUMÉRICOS,» [En línea]. Available: <http://www.monografias.com/trabajos-pdf5/metodos-numericos-unidad-i/metodos-numericos-unidad-i.shtml>. [Último acceso: 23 11 2015].
2. DLEGORRETA, «Optimizacion,» [En línea]. Available: <https://dlegorreta.wordpress.com/tag/gradiente-descendente/>. [Último acceso: 23 11 2015].
3. H. W. Kuhn, «Nonlinear Programming. Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,» University of California Press, Berkeley.
4. E. M. F. Velasco, Herramientas de optimización Convexa, Leganes, 2011.
5. N. A. C. Díaz, DESARROLLO DE UN CÓDIGO DE SIMULACIÓN POR MONTE CARLO PARA LA CALIBRACIÓN DE SISTEMAS DE DETECCIÓN EN ESPECTROMETRÍA DE RADIACIÓN GAMMA, Badajoz, 2009.
6. M. P. D. L. S. Ing, Programa Didáctico a Código Abierto de Análisis Dinámico de Estructuras UNDIN 1.0, Bogota, 2013.
7. C. P. Corral, PLANIFICACIÓN ÓPTIMA DE LA GENERACIÓN DISTRIBUIDA EN REDES DE DISTRIBUCIÓN DE ENERGÍA ELÉCTRICA, Zaragoza, España., 2010.
8. A. K. Cruz, «MATRICES ESCALONADAS Y METODOS PRIMAL DUAL DE PUNTO INTERIOR,» REVISTA INVESTIGACION OPERACIONAL, vol. 23, n° 3, pp. 100-113, 2002.
9. E. Z. SOSA, EL METODO DE NEWTON RAPHSON EN LA RECUPERACION MEJORADA DEL PETROLEO, UNIVERSITARIA, 2013.