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Intuitionistic Nanogonal Fuzzy Numbers and its Application in FDE

C. Jesuraj, A. Rajkumar

***Abstract:** In this paper, we have introduced a new way to solve an ordinary differential equation by using Nanogonal intuitionistic fuzzy numbers. A new version of INFN addressed here. Here we formed different types of NIFN System and we discussed the properties said fuzzy numbers. Demonstration of Nanogonal intuitionistic fuzzy solution in FDE is carried out with INFN.Finally one examples problem also solved with pictorial representation of said method.*

***Keywords:** Fuzzy Differential Equation, Fuzzy Numbers, INFN.*

I. INTRODUCTION

Prof. L. A. Zadeh was introduced fuzzy set theory to the world in 1965. Using fuzzy set theory Dubois developed a new method to determine fuzzy differential equation in 1978 later in 1999 K.Atanassov etal,. Introduced intuitionistic concepts in fuzzy environment which deals with both membership and non–membership functions [1]. Kaleva,O solved Cauchy problems by using fuzzy differential concepts [2,3]. J.J. Buckley, T. Feuring solved one first order differential equation by initial conditions as fuzzy numbers to get best solution[5,6].Kumar and A.Sadeghi apply to analyse time dependent intuitionistic FDE for some reliability of the industrial system. C.Duraisamy, B. Usha, used modified Euler method to find the solution of FDE [7]. Sankar Prasad Mondal used generalised triangular fuzzy numbers to find FDE solution, and he used Laplace transform to solve FDE[8,9,10,11] finally Sankar Prasad Mondal and Manimohan Mandal used pentagonal fuzzy numbers , its application in fuzzy equation [12].arithmetic operation of different fuzzy numbers were discussed by Bongju lee and Yong Sik Yun [13].Intuitionistic pentagonal fuzzy numbers introduce by Ponnivalavan and Pathinathan[14]. A.Felix, S.Christopher, A.Victor Devadoss used nanogonal fuzzy numbers for real life problems [15].A.Rajkumar and C.Jesuraj used INFN to solve FDE and applied modelled real life to FDE problem[16]. So many research work done related to FDE buy all are used linear membership only taken in symmetry concept but what will happen if its asymmetry so, in this paper we discussed the properties of INFN and its application in FDE with the help of numerical examples. In second section having the basic concepts and definition related to this work and third section explains the properties of said fuzzy numbers with their membership function and non-membership function with graphical representation using alpha beta cut. Fourth section shows the solution method for FDE using INFNs. Fifth one the application of real life problems and final section has the conclusion of this paper.

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II. PRELIMINARIES

Definition 2.1
Fuzzy subset of a universe X is a set A described by its membership function μA: X→ [0, 1] The standard membership degrees are represented by one and zero.

Definition 2.2
The membership function obtain by any component in the fuzzy set and it's given by $h(\tilde{U}) = \sup \mu_{\tilde{U}}(x)$

Definition 2.3

Let $\tilde{U} = \{x, \mu_{\tilde{U}}(x) / x \in X\}$ is called to be convex if and only if $\forall\ x_1, x_2 \in X$, the function of U satisfies the following

$\mu_{\tilde{U}}\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_{\tilde{U}}(x_1), \mu_{\tilde{U}}(x_2)\}, \lambda \in [0,1]$

Definition 2.4
Let a set A is unchanged. An intuitionistic fuzzy set U in A is an object of the following form:
$U = \{x, \mu_U(x), \vartheta_U(x) / x \in X\}$, $v_U(x) = 1 - \mu_U(x)$,$\forall x \in A$

Definition 2.5
An Intuitionistic Fuzzy set of real line is called an IFN's if the following are satisfied then $\tilde{A}^i$

i) is normal.
ii) is convex, i.e, the membership function of μ is fuzzy convex and its non-membership function of ϑ is fuzzy concave.
iii) $\mu_{\tilde{A}^i}, \vartheta_{\tilde{A}^i}$ are the major and minor semi continuous respectively.
iv) Supp $U = \{x\, belongs\, to\, X / \vartheta_{\tilde{U}^i}(x) < 1\}$ bounded

III. INTUITIONISTIC NANOGONAL FUZZY NUMBER

Properties of INFS are linear and non-linear with symmetry and linear and non-linear asymmetry cases are discussed below with their membership and non-membership function are defined and graphical representation also given.

3.1. Linear Nonogonal Intuitionistic Fuzzy Number with Symmetry (LNIFNS). A Linear Nonogonal Intuitionistic Fuzzy Number with Symmetry is written as

$$\tilde{A}_{LINFNS} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9; b_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, b_9 p, q, r\}$$

Membership and non-membership functions are given below

$$\mu_{\tilde{A}}(x) = \begin{cases} p\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ p+(q-p)\left(\dfrac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ q+(r-q)\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ r+(1-r)\left(\dfrac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ 1+(r-1)\left(\dfrac{x-a_5}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ r+(q-r)\left(\dfrac{x-a_6}{a_7-a_6}\right), & a_6 \le x \le a_7 \\ q+(p-q)\left(\dfrac{x-a_7}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ p-p\left(\dfrac{x-a_8}{a_9-a_8}\right), & a_8 \le x < a_9 \\ 0, & x < a_1, a_9 \le x \end{cases}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} 1+(r-1)\left(\dfrac{x-b_1}{a_2-b_1}\right), & b_1 \le x \le a_2 \\ r+(q-r)\left(\dfrac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ q+(p-q)\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ p-p\left(\dfrac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ p\left(\dfrac{x-a_5}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ p+(q-p)\left(\dfrac{x-a_6}{a_7-a_6}\right), & a_6 \le x \le a_7 \\ q+(r-q)\left(\dfrac{x-a_7}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ r+(1-r)\left(\dfrac{x-a_8}{b_9-a_8}\right), & a_8 \le x < b_9 \\ 1, & x < b_1, \; b_9 \le x \end{cases}$$

Graphical representation of Linear Nonogonal Intuitionistic Fuzzy Number with Symmetry

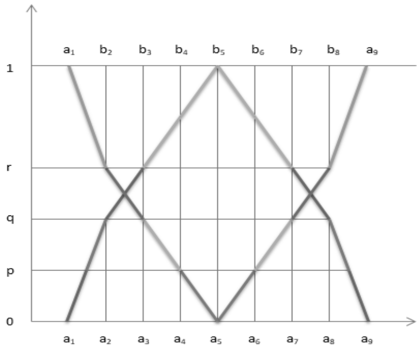

Figure 1: LINFNS

Alpha and beta cut or parametric form of LINFNS In this LINFNS the membership function is increasing from 0 to 1 and decreasing from 1 to 0. Non-membership is decreasing from 1 to 0 and increasing from 0 to 1.

$$A(\alpha) = \begin{cases} a_1+\left(\dfrac{\alpha}{p}\right)(a_2-a_1), & [0,p] \\ a_2+(a_3-a_2)\left(\dfrac{\alpha-p}{q-p}\right), & [p,q] \\ a_3+(a_4-a_3)\left(\dfrac{\alpha-q}{r-q}\right), & [q,r] \\ a_4+(a_5-a_4)\left(\dfrac{\alpha-r}{1-r}\right), & [r,1] \\ a_5+(a_6-a_5)\left(\dfrac{\alpha-1}{r-1}\right), & [1,r] \\ a_6+(a_7-a_6)\left(\dfrac{\alpha-r}{q-r}\right), & [r,q] \\ a_7+(a_8-a_7)\left(\dfrac{\alpha-q}{p-q}\right), & [q,p] \\ a_8-\dfrac{(\alpha-p)}{p}(a_9-a_8), & [p,0] \end{cases}$$

$$A(\beta) = \begin{cases} b_1+\left(\dfrac{\beta-1}{r-1}\right)(a_2-b_1), & [1,r] \\ a_2+(a_3-a_2)\left(\dfrac{\beta-r}{q-r}\right), & [r,q] \\ a_3+(a_4-a_3)\left(\dfrac{\beta-q}{p-q}\right), & [q,p] \\ a_4+(a_5-a_4)\left(\dfrac{\beta-p}{-p}\right), & [p,0] \\ a_5+(a_6-a_5)\left(\dfrac{\beta}{p}\right), & [0,p] \\ a_6+(a_7-a_6)\left(\dfrac{\beta-p}{q-p}\right), & [p,q] \\ a_7+(a_8-a_7)\left(\dfrac{\beta-q}{r-q}\right), & [q,r] \\ a_8-\dfrac{(\beta-r)}{(1-r)}(b_9-a_8), & [r,1] \end{cases}$$

3.2. Linear Nonogonal Intuitionistic Fuzzy Number with Asymmetry

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A Linear Nonogonal Intuitionistic Fuzzy Number with Asymmetry is written as

$$\bar{A}_{LINFNAS}=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9;b_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,b_9;p,q,r,s\}$$

Membership and non-membership functions are given below

$$\mu_{\tilde{A}}(x)=\begin{cases} q\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1\le x\le a_2\\ q+(r-q)\left(\dfrac{x-a_2}{a_3-a_2}\right), & a_2\le x\le a_3\\ r+(s-r)\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3\le x\le a_4\\ s+(1-s)\left(\dfrac{x-a_4}{a_5-a_4}\right), & a_4\le x\le a_5\\ 1+(t-1)\left(\dfrac{x-a_5}{a_6-a_5}\right), & a_5\le x\le a_6\\ t+(r-t)\left(\dfrac{x-a_6}{a_7-a_6}\right), & a_6\le x\le a_7\\ r+(p-r)\left(\dfrac{x-a_7}{a_8-a_7}\right), & a_7\le x\le a_8\\ p-p\left(\dfrac{x-a_8}{a_9-a_8}\right), & a_8\le x< a_9\\ 0, & x<a_1, a_9\le x \end{cases}$$

$$\mu_{\tilde{B}}(x)=\begin{cases} 1+(s-1)\left(\dfrac{x-b_1}{a_2-b_1}\right), & b_1\le x\le a_2\\ s+(q-s)\left(\dfrac{x-a_2}{a_3-a_2}\right), & a_2\le x\le a_3\\ q+(p-q)\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3\le x\le a_4\\ p-p\left(\dfrac{x-a_4}{a_5-a_4}\right), & a_4\le x\le a_5\\ p\left(\dfrac{x-a_5}{a_6-a_5}\right), & a_5\le x\le a_6\\ p+(q-p)\left(\dfrac{x-a_6}{a_7-a_6}\right), & a_6\le x\le a_7\\ q+(r-q)\left(\dfrac{x-a_7}{a_8-a_7}\right), & a_7\le x\le a_8\\ r+(1-r)\left(\dfrac{x-a_8}{b_9-a_8}\right), & a_8\le x< b_9\\ 1, & x<b_1,\ b_9\le x \end{cases}$$

Graphical representation of Linear Nonogonal Intuitionistic Fuzzy Number with Asymmetry

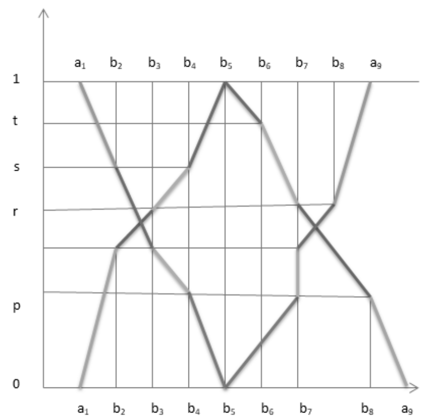

Figure 2: LINFNAS

Alpha and beta cut or parametric form of LINFNAS

$$A(\alpha)=\begin{cases} a_1+\left(\dfrac{\alpha}{q}\right)(a_2-a_1), & [0,q]\\ a_2+(a_3-a_2)\left(\dfrac{\alpha-q}{r-q}\right), & [q,r]\\ a_3+(a_4-a_3)\left(\dfrac{\alpha-r}{s-r}\right), & [r,s]\\ a_4+(a_5-a_4)\left(\dfrac{\alpha-s}{1-s}\right), & [s,1]\\ a_5+(a_6-a_5)\left(\dfrac{\alpha-1}{t-1}\right), & [1,t]\\ a_6+(a_7-a_6)\left(\dfrac{\alpha-t}{r-t}\right), & [t,r]\\ a_7+(a_8-a_7)\left(\dfrac{\alpha-r}{p-r}\right), & [r,p]\\ a_8-\dfrac{(\alpha-p)}{p}(a_9-a_8), & [p,0] \end{cases}$$

$$A(\beta)=\begin{cases} b_1+\left(\dfrac{\beta-1}{s-1}\right)(a_2-b_1), & [1,s]\\ a_2+(a_3-a_2)\left(\dfrac{\beta-s}{q-s}\right), & [s,q]\\ a_3+(a_4-a_3)\left(\dfrac{\beta-q}{p-q}\right), & [q,p]\\ a_4+(a_5-a_4)\left(\dfrac{\beta-p}{-p}\right), & [p,0]\\ a_5+(a_6-a_5)\left(\dfrac{\beta}{p}\right), & [0,p]\\ a_6+(a_7-a_6)\left(\dfrac{\beta-p}{q-p}\right), & [p,q]\\ a_7+(a_8-a_7)\left(\dfrac{\beta-q}{r-q}\right), & [q,r]\\ a_8-\dfrac{(\beta-r)}{(1-r)}(b_9-a_8), & [r,1] \end{cases}$$

Where the membership function is increasing from 0 to 1 and decreasing from 1 to 0. Nonmembership is decreasing from 1 to 0 and increasing from 0 to1

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3.3. Non- Linear Nonogonal Intuitionistic Fuzzy Number with Symmetry

A non-Linear Nonogonal Intuitionistic Fuzzy Number with symmetry is written as

$$\tilde{A}_{LINFNAS} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9; b_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, b_9, p, q, r\}$$

Membership and non-membership functions are given below

$$\mu_{\tilde{A}}(x) = \begin{cases} p\left(\frac{x-a_1}{a_2-a_1}\right)^{n_1}, & a_1 \le x \le a_2 \\ p+(q-p)\left(\frac{x-a_2}{a_3-a_2}\right)^{n_2}, & a_2 \le x \le a_3 \\ q+(r-q)\left(\frac{x-a_3}{a_4-a_3}\right)^{n_3}, & a_3 \le x \le a_4 \\ r+(1-r)\left(\frac{x-a_4}{a_5-a_4}\right)^{n_4}, & a_4 \le x \le a_5 \\ 1+(r-1)\left(\frac{x-a_5}{a_6-a_5}\right)^{m_1}, & a_5 \le x \le a_6 \\ r+(q-r)\left(\frac{x-a_6}{a_7-a_6}\right)^{m_2}, & a_6 \le x \le a_7 \\ q+(p-q)\left(\frac{x-a_7}{a_8-a_7}\right)^{m_3}, & a_7 \le x \le a_8 \\ p-p\left(\frac{x-a_8}{a_9-a_8}\right)^{m_4}, & a_8 \le x < a_9 \\ 0, & x < a_1, a_9 \le x \end{cases}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} 1+(r-1)\left(\frac{x-b_1}{a_2-b_1}\right)^{n_1}, & b_1 \le x \le a_2 \\ r+(q-r)\left(\frac{x-a_2}{a_3-a_2}\right)^{n_2}, & a_2 \le x \le a_3 \\ q+(p-q)\left(\frac{x-a_3}{a_4-a_3}\right)^{n_3}, & a_3 \le x \le a_4 \\ p-p\left(\frac{x-a_4}{a_5-a_4}\right)^{n_4}, & a_4 \le x \le a_5 \\ p\left(\frac{x-a_5}{a_6-a_5}\right)^{m_1}, & a_5 \le x \le a_6 \\ p+(q-p)\left(\frac{x-a_6}{a_7-a_6}\right)^{m_2}, & a_6 \le x \le a_7 \\ q+(r-q)\left(\frac{x-a_7}{a_8-a_7}\right)^{m_3}, & a_7 \le x \le a_8 \\ r+(1-r)\left(\frac{x-a_8}{b_9-a_8}\right)^{m_4}, & a_8 \le x < b_9 \\ 1, & x < b_1,\ b_9 \le x \end{cases}$$

Graphical representation of non-linear Nonogonal Intuitionistic Fuzzy Number with Symmetry

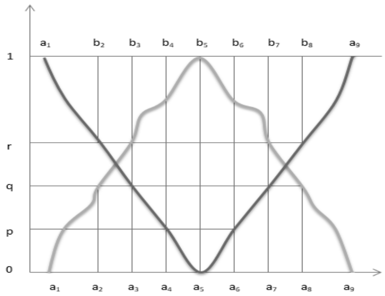

Figure 3: NLINFNS

Alpha and beta cut or parametric form of non-LINFNS:

$$A(\alpha) = \begin{cases} a_1 + \left(\frac{\alpha}{p}\right)^{n_1}(a_2-a_1), & [0,p] \\ a_2 + (a_3-a_2)\left(\frac{\alpha-p}{q-p}\right)^{n_2}, & [p,q] \\ a_3 + (a_4-a_3)\left(\frac{\alpha-q}{r-q}\right)^{n_3}, & [q,r] \\ a_4 + (a_5-a_4)\left(\frac{\alpha-r}{1-r}\right)^{n_4}, & [r,1] \\ a_5 + (a_6-a_5)\left(\frac{\alpha-1}{r-1}\right)^{m_1}, & [1,r] \\ a_6 + (a_7-a_6)\left(\frac{\alpha-r}{q-r}\right)^{m_2}, & [r,q] \\ a_7 + (a_8-a_7)\left(\frac{\alpha-q}{p-q}\right)^{m_3}, & [q,p] \\ a_8 - \left(\frac{\alpha-p}{p}\right)^{m_4}(a_9-a_8), & [p,0] \end{cases}$$

$$A(\beta) = \begin{cases} b_1 + \left(\frac{\beta-1}{r-1}\right)^{n_1}(a_2-b_1), & [1,\mathrm{r}] \\ a_2 + (a_3-a_2)\left(\frac{\beta-r}{q-r}\right)^{n_2}, & [\mathrm{r},q] \\ a_3 + (a_4-a_3)\left(\frac{\beta-q}{p-q}\right)^{n_3}, & [q,\mathrm{p}] \\ a_4 + (a_5-a_4)\left(\frac{\beta-p}{-p}\right)^{n_4}, & [\mathrm{p},0] \\ a_5 + (a_6-a_5)\left(\frac{\beta}{p}\right)^{m_1}, & [0,\mathrm{p}] \\ a_6 + (a_7-a_6)\left(\frac{\beta-p}{q-p}\right)^{m_2}, & [\mathrm{p},q] \\ a_7 + (a_8-a_7)\left(\frac{\beta-q}{r-q}\right)^{m_3}, & [q,\mathrm{r}] \\ a_8 - \left(\frac{\beta-r}{1-r}\right)^{m_4}(b_9-a_8), & [\mathrm{r},1] \end{cases}$$

Where the membership function is increasing from 0 to 1 and decreasing from 1 to 0. Non-membership is decreasing from 1 to 0 and increasing from 0 to 1.

3.4. Non- Linear Nonogonal Intuitionistic Fuzzy Number with Asymmetry

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A non-Linear Nonogonal Intuitionistic Fuzzy Number with Asymmetry is written as

$\tilde{A}_{LINFNAS} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9; b_1, a_2, a_3, a_4, a_5, a_6, a_7, a_{\cdot}$

Whose membership and non-membership functions are given below

$$\mu_{\tilde{A}}(x) = \begin{cases} q\left(\dfrac{x-a_1}{a_2-a_1}\right)^{n_1}, & a_1 \le x \le a_2 \\ q+(r-q)\left(\dfrac{x-a_2}{a_3-a_2}\right)^{n_2}, & a_2 \le x \le a_3 \\ r+(s-r)\left(\dfrac{x-a_3}{a_4-a_3}\right)^{n_3}, & a_3 \le x \le a_4 \\ s+(1-s)\left(\dfrac{x-a_4}{a_5-a_4}\right)^{n_4}, & a_4 \le x \le a_5 \\ 1+(t-1)\left(\dfrac{x-a_5}{a_6-a_5}\right)^{m_1}, & a_5 \le x \le a_6 \\ t+(r-t)\left(\dfrac{x-a_6}{a_7-a_6}\right)^{m_2}, & a_6 \le x \le a_7 \\ r+(p-r)\left(\dfrac{x-a_7}{a_8-a_7}\right)^{m_3}, & a_7 \le x \le a_8 \\ p-p\left(\dfrac{x-a_8}{a_9-a_8}\right)^{m_4}, & a_8 \le x < a_9 \\ 0, & x < a_1, a_9 \le x \end{cases}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} 1+(s-1)\left(\dfrac{x-b_1}{a_2-b_1}\right)^{n_1}, & b_1 \le x \le a_2 \\ s+(q-s)\left(\dfrac{x-a_2}{a_3-a_2}\right)^{n_2}, & a_2 \le x \le a_3 \\ q+(p-q)\left(\dfrac{x-a_3}{a_4-a_3}\right)^{n_3}, & a_3 \le x \le a_4 \\ p-p\left(\dfrac{x-a_4}{a_5-a_4}\right)^{n_4}, & a_4 \le x \le a_5 \\ p\left(\dfrac{x-a_5}{a_6-a_5}\right)^{m_1}, & a_5 \le x \le a_6 \\ p+(q-p)\left(\dfrac{x-a_6}{a_7-a_6}\right)^{m_2}, & a_6 \le x \le a_7 \\ q+(r-q)\left(\dfrac{x-a_7}{a_8-a_7}\right)^{m_3}, & a_7 \le x \le a_8 \\ r+(1-r)\left(\dfrac{x-a_8}{b_9-a_8}\right)^{m_4}, & a_8 \le x < b_9 \\ 1, & x < b_1,\ b_9 \le x \end{cases}$$

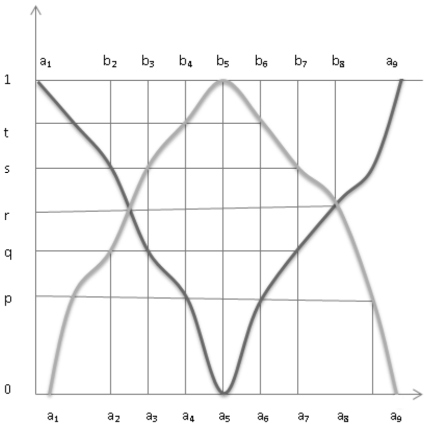

Figure 4: NLINFNAS

Alpha and beta cut or parametric form of non-linear intuitionistic nanogonal fuzzy number with asymmetry is represented by the formula

$$A(\alpha) = \begin{cases} a_1 + \left(\dfrac{\alpha}{q}\right)^{n_1}(a_2-a_1), & [0,q] \\ a_2 + (a_3-a_2)\left(\dfrac{\alpha-q}{r-q}\right)^{n_2}, & [q,r] \\ a_3 + (a_4-a_3)\left(\dfrac{\alpha-r}{s-r}\right)^{n_3}, & [r,s] \\ a_4 + (a_5-a_4)\left(\dfrac{\alpha-s}{1-s}\right)^{n_4}, & [s,1] \\ a_5 + (a_6-a_5)\left(\dfrac{\alpha-1}{t-1}\right)^{m_1}, & [1,t] \\ a_6 + (a_7-a_6)\left(\dfrac{\alpha-t}{r-t}\right)^{m_2}, & [t,r] \\ a_7 + (a_8-a_7)\left(\dfrac{\alpha-r}{p-r}\right)^{m_3}, & [r,p] \\ a_8 - \left(\dfrac{\alpha-p}{p}\right)^{m_4}(a_9-a_8), & [p,0] \end{cases}$$

$$A(\beta) = \begin{cases} b_1 + \left(\dfrac{\beta-1}{s-1}\right)^{n_1}(a_2-b_1), & [1,s] \\ a_2 + (a_3-a_2)\left(\dfrac{\beta-s}{q-s}\right)^{n_2}, & [s,q] \\ a_3 + (a_4-a_3)\left(\dfrac{\beta-q}{p-q}\right)^{n_3}, & [q,p] \\ a_4 + (a_5-a_4)\left(\dfrac{\beta-p}{-p}\right)^{n_4}, & [p,0] \\ a_5 + (a_6-a_5)\left(\dfrac{\beta}{p}\right)^{m_1}, & [0,p] \\ a_6 + (a_7-a_6)\left(\dfrac{\beta-p}{q-p}\right)^{m_2}, & [p,q] \\ a_7 + (a_8-a_7)\left(\dfrac{\beta-q}{r-q}\right)^{m_3}, & [q,r] \\ a_8 - \left(\dfrac{\beta-r}{1-r}\right)^{m_4}(b_9-a_8), & [r,1] \end{cases}$$

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Where the membership function is increasing from 0 to 1 and decreasing from 1 to 0. Non-membership is decreasing from 1 to 0 and increasing from 0 to 1.

3.5 Finding **S**olution of first order fuzzy differential equation (Strong and weak).

Consider an intuitionistic fuzzy ordinary differential equation $\frac{dx}{dt}=mx, x(t_o)=x_o$ with x_o as IFNs. The result of the given differential equation can be $\tilde{X}(t)$ and it's $(\alpha,\beta)-$cuts are given by

$(i)\frac{dx_1(\alpha)}{d\alpha}>0, \frac{dx_8(\alpha)}{d\alpha}<0, \forall\alpha\in[0,0.25]$,

$x_1(t,0.25)\le x_8(t,0.25)$

$(ii)\frac{dx_2(\alpha)}{d\alpha}>0, \frac{dx_7(\alpha)}{d\alpha}<0, \forall\alpha\in[0.25,0.5]$,

$x_2(t,0.5)\le x_7(t,0.5)$

$(iii)\frac{dx_3(\alpha)}{d\alpha}>0, \frac{dx_6(\alpha)}{d\alpha}<0, \forall\alpha\in[0.5,0.75]$,

$x_3(t,0.75)\le x_6(t,0.75)$

$(iv)\frac{dx_4(\alpha)}{d\alpha}>0, \frac{dx_5(\alpha)}{d\alpha}<0, \forall\alpha\in[0.75,1]$,

$x_4(t,1)\le x_5(t,1)$

$(v)\frac{dx_1'(\beta)}{d\beta}<0, \frac{dx_8'(\beta)}{d\beta}>0, \forall\beta\in[1,0.75]$,

$x_1'(t,0.75)\le x_8'(t,0.75)$

$(vi)\frac{dx_2'(\beta)}{d\beta}<0, \frac{dx_7'(\beta)}{d\beta}>0, \forall\beta\in[0.75,0.5]$,

$x'_2(t,0.5)\le x'_7(t,0.5)$

$(vii)\frac{dx_3'(\beta)}{d\beta}<0, \frac{dx_4'(\beta)}{d\beta}>0, \forall\beta\in[0.5,0.25]$,

$x'_3(t,0.25)\le x'_4(t,0.25)$

$(viii)\frac{dx_4'(\beta)}{d\beta}<0, \frac{dx_5'(\beta)}{d\beta}>0, \forall\beta\in[0.25,0]$,

$x'_4(t,0)\le x'_5(t,0)$

Otherwise the solution is a very weak solution.

IV. ODE'S WITH INITIAL VALUE AS INFNS

The ordinary fuzzy differential equations initial condition as INFNS

$$y'=mx, x(0)=\tilde{A}^i=\{(a_1,\dots\dots,a_9);(b_1,a_2,\dots\dots,a_8,b_9)\}$$

Case (i): if m> 0, Let $m=r$, Taking $(\alpha,\beta)-$cuts of the above equation we get

$$\frac{d}{dt}\left[\left(x_1(t,\alpha),\dots\dots x_8(t,\alpha)\right);\left(x_1'(t,\beta),\dots\dots x_8'(t,\beta)\right)\right]$$

$$=r\left[\left(x_1(t,\alpha),\dots\dots x_8(t,\alpha)\right);\left(x_1'(t,\beta),\dots\dots,x_8'(t,\beta)\right)\right]$$

The initial condition is given by

$$x(t_o;\alpha,\beta)=\left[\left(a_1(\alpha),\dots\dots,a_8(\alpha)\right);\left(a_1'(\beta),\dots\dots,a_8'(\beta)\right)\right]$$

$$\frac{d}{dt}x_1(t,\alpha)=rx_1(t,\alpha),\dots\dots\dots\frac{d}{dt}x_8(t,\alpha)=rx_8(t,\alpha),$$

Similarly for non-membership

$$\frac{d}{dt}x_1'(t,\beta)=rx_1(t,\beta)\dots\dots,\frac{d}{dt}x_8'(t,\beta)=rx_8(t,\beta),$$

With the initial condition as NIFN's

$$\{x_1(t_o,\alpha)=a_1(\alpha),\dots\dots\dots\dots x_8(t_o,\alpha)=a_8(\alpha)\}$$

$$\{x_1'(t_o,\beta)=a_1'(\beta),\dots\dots\dots x_8'(t_o,\beta)=a_8'(\beta)\}$$

The solution of the given first order differential equation is given by

$$x_1(t,\alpha)=a_1(\alpha)e^{r(t-t_o)}, x_2(t,\alpha)=a_2(\alpha)e^{r(t-t_o)},$$

$$x_3(t,\alpha)=a_3(\alpha)e^{r(t-t_o)}, x_4(t,\alpha)=a_4(\alpha)e^{r(t-t_o)},$$

$$x_5(t,\alpha)=a_5(\alpha)e^{r(t-t_o)}, x_6(t,\alpha)=a_6(\alpha)e^{r(t-t_o)},$$

$$x_7(t,\alpha)=a_7(\alpha)e^{r(t-t_o)}, x_8(t,\alpha)=a_8(\alpha)e^{r(t-t_o)},$$

$$x_1'(t,\beta)=a_1'(\beta)e^{r(t-t_o)}, x_2'(t,\beta)=a_2'(\beta)e^{r(t-t_o)},$$

$$x_3'(t,\beta)=a_3(\beta)e^{r(t-t_o)}, x_4'(t,\beta)=a_4'(\beta)e^{r(t-t_o)},$$

$$x_5'(t,\beta)=a_5'(\beta)e^{r(t-t_o)}, x_6'(t,\beta)=a_6'(\beta)e^{r(t-t_o)},$$

$$x_7'(t,\beta)=a_7(\beta)e^{r(t-t_o)}, x_8'(t,\beta)=a_8'(\beta)e^{r(t-t_o)}$$

Case (ii).Let $m=-r$, Taking $(\alpha,\beta)-$ cuts of the above equation we get

$$\frac{d}{dt}\left[\left(x_1(t,\alpha),\dots\dots,x_8(t,\alpha)\right);\left(x_1'(t,\beta),\dots\dots,x_8'(t,\beta)\right)\right]$$

$$=-r\left[\left(x_1(t,\alpha),\dots\dots,x_8(t,\alpha)\right);\left(x_1'(t,\beta),\dots\dots,x_8'(t,\beta)\right)\right]$$

The initial condition is given by

$$x(t_o;\alpha,\beta)=\left[\left(a_1(\alpha),),\dots\dots,a_8(\alpha)\right);\left(a_1'(\beta),\dots\dots\dots,a_8'(\beta)\right)\right]$$

$$\frac{d}{dt}x_1(t,\alpha)=-rx_8(t,\alpha),\dots\dots\dots\dots\dots\frac{d}{dt}x_8(t,\alpha)=-rx_1(t,\alpha)$$

$$\frac{d}{dt}x_1'(t,\beta)=-rx_8(t,\beta),\dots\dots\dots\dots\frac{d}{dt}x_8'(t,\beta)=-rx_1(t,\beta)$$

With the initial condition

$$x_1(t_o,\alpha)=a_1(\alpha),\dots\dots\dots\dots\dots x_8(t_o,\alpha)=a_8(\alpha)$$

$$x_1'(t_o,\beta)=a_1'(\beta),\dots\dots\dots\dots\dots\dots x_8'(t_o,\beta)=a_8'(\beta)$$

The solution of the above given first order differential equation is given by

$$x_1(t,\alpha)=\left(\frac{a_1(\alpha)+a_8(\alpha)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_1(\alpha)-a_8(\alpha)}{4}\right)e^{r(t-t_o)}$$

$$x_2(t,\alpha)=\left(\frac{a_2(\alpha)+a_7(\alpha)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_2(\alpha)-a_7(\alpha)}{4}\right)e^{r(t-t_o)}$$

$$x_3(t,\alpha)=\left(\frac{a_3(\alpha)+a_6(\alpha)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_3(\alpha)-a_6(\alpha)}{4}\right)e^{r(t-t_o)}$$

$$x_4(t,\alpha)=\left(\frac{a_4(\alpha)+a_5(\alpha)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_4(\alpha)-a_5(\alpha)}{4}\right)e^{r(t-t_o)}$$
$$x_5(t,\alpha)=\left(\frac{a_5(\alpha)+a_4(\alpha)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_5(\alpha)-a_4(\alpha)}{4}\right)e^{r(t-t_o)}$$
$$x_6(t,\alpha)=\left(\frac{a_6(\alpha)+a_3(\alpha)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_6(\alpha)-a_3(\alpha)}{4}\right)e^{r(t-t_o)}$$
$$x_7(t,\alpha)=\left(\frac{a_7(\alpha)+a_2(\alpha)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_7(\alpha)-a_2(\alpha)}{4}\right)e^{r(t-t_o)}$$
$$x_8(t,\alpha)=\left(\frac{a_8(\alpha)+a_1(\alpha)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_8(\alpha)-a_1(\alpha)}{4}\right)e^{r(t-t_o)}$$
$$x_1'(t,\beta)=\left(\frac{a_1'(\beta)+a_8'(\beta)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_1'(\beta)-a_8'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_2'(t,\beta)=\left(\frac{a_2'(\beta)+a_7'(\beta)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_2'(\beta)-a_7'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_3'(t,\beta)=\left(\frac{a_3'(\beta)+a_6'(\beta)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_3'(\beta)-a_6'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_4'(t,\beta)=\left(\frac{a_4'(\beta)+a_5'(\beta)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_4'(\beta)-a_5'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_5'(t,\beta)=\left(\frac{a_5'(\beta)+a_4'(\beta)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_5'(\beta)-a_4'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_6'(t,\beta)=\left(\frac{a_6'(\beta)+a_3'(\beta)}{4}\right)e^{-r(t-t_o)}+\left(\frac{a_6'(\beta)-a_3'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_7'(t,\beta)=\left(\frac{a_7'(\beta)+a_2'(\beta)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_7'(\beta)-a_2'(\beta)}{4}\right)e^{r(t-t_o)}$$
$$x_8'(t,\beta)=\left(\frac{a_8'(\beta)+a_1'(\beta)}{4}\right)e^{-r(t-t_o)}-\left(\frac{a_8'(\beta)-a_1'(\beta)}{4}\right)e^{r(t-t_o)}$$

V. APPLICATION

The rate of gradual increase in sale of petrol in Tamilnadu from Jan-2017 to Feb-2018 was assume to be comparative y itself , what will be the sale of petrol in Tamilnadu after ten years in the initial sales are {65.46,67.71,69.93,71.78,73.36,71.65,71.74,72.53,75.77;64.47,67.71,69.93,71.78,73.36,71.65,71.74,72.53,76.10} considering the proportionality as 0.10.

Solution : Let $\frac{dy}{dt}=mx$, where $m=0.10$ and

$$y(0)=\{(65.46,67.71,69.93,71.78,73.36,71.65,71.74,72.53,75.77);\ (64.47,67.71,69.93,71.78,73.36,71.65,71.74,72.53,76.10)\}$$

$y_1(t,\alpha)=(9\alpha+65.46)e^{0.10t}$;

$y_8(t,\alpha)=(-12.96\alpha+75.77)e^{0.10t}$, $\alpha\in[0,0.25]$

$y_2(t,\alpha)=(8.88\alpha+65.49)e^{0.10t}$;

$y_7(t,\alpha)=(-3.16\alpha+73.32)e^{0.10t}$, $\alpha\in[0.25,0.5]$

$y_3(t,\alpha)=(7.4\alpha+66.23)e^{0.10t}$;

$y_6(t,\alpha)=(0.08\alpha+71.70)e^{0.10t}$, $\alpha\in[0.5,0.75]$

$y_4(t,\alpha)=(6.32\alpha+67.04)e^{0.10t}$;

$y_5(t,\alpha)=(6.4\alpha+66.96)e^{0.10t}$, $\alpha\in[0.75,1]$

$y_1'(t,\beta)=(12.96\beta+57.99)e^{0.10t}$;

$y_8'(t,\beta)=(14.24\beta+61.85)e^{0.10t}$, $\beta\in[0,0.25]$

$y_2'(t,\beta)=(8.88\beta+65.49)e^{0.10t}$;

$y_7'(t,\beta)=(3.16\beta+70.16)e^{0.10t}$, $\beta\in[0.25,0.5]$

$y_3'(t,\beta)=(7.4\beta+69.93)e^{0.10t}$;

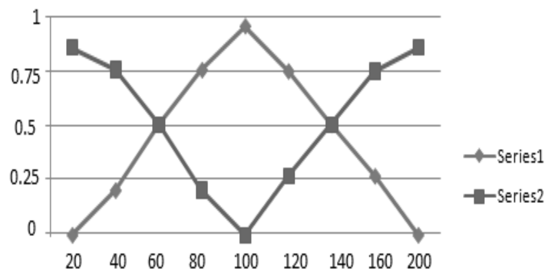
$y_6'(t,\beta)=(0.36\beta+71.56)e^{0.10t}$, $\beta\in[0.5,0.75]$

$y_4'(t,\beta)=(-6.32\beta+73.36)e^{0.10t}$;

$y_5'(t,\beta)=(-6.84\beta+73.36)e^{0.10t}$, $\beta\in[0.75,1]$

The solution of given problems is nanogonal intuitionist fuzzy numbers can be expressed as

$$\tilde{A}'=\{[65.46e^{0.10t},67.71e^{0.10t},69.93e^{0.10t},71.78e^{0.10t},73.36e^{0.10t},71.65e^{0.10t},71.74e^{0.10t},72.53e^{0.10t},75.77e^{0.10t}];$$
$$[64.47e^{0.10t},67.71e^{0.10t},69.93e^{0.10t},71.78e^{0.10t},73.36e^{0.10t},71.65e^{0.10t},71.74e^{0.10t},72.53e^{0.10t},76.10e^{0.10t}]\}$$



Graphical representation and the solution of the above problems at t=10

VI. CONCLUSION

In this paper, we derived the different properties of INFN with four cases and we have arrived solution to a first ODE,s with initial condition as Nanogonal IFN Solution procedure are given to solve any real life problems in fuzzy environment using intuitionistic fuzzy numbers. This work can be used for higher order fuzzy differential equation, partial differential equation and fractional differential equation of higher order.

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