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Control of Beam Vibrations using Viscoelastically Damped Absorber System

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Abstract: The aim of this paper is to study the use of viscoelasically damped vibration absorber systems to optimally control the vibrations of a fixed-fixed beam. In this paper, a main beam with fixed-fixed boundary condition with viscoelstically damped cantilever beam as absorber is taken for the analysis. The paper includes optimum design & analysis of vibration absorber with viscoelastic damping. The equations of motion of the system have been derived to find the vibration response with absorber and used for optimization of parameters. The classical and Den Hartong optimization methods are used to optimize the design parameters and optimum values of design are found out. Theoretical calculations have been done for a fixed-fixed beam with three types of absorber beams (undamped, unconstrained treated damped absorber beam and constrained treated absorber beam). To validate the theoretical calculations, experiments have performed and deviation from theoretical data is discussed.

Index Terms: Vibrations Control, Damped Absorber system, and Constrained Treatments, Viscoelastic Materials, Optimization

I. INTRODUCTION

The subject of vibration control has received considerable attention due to vibration problems occurring wherever there are rotating or moving part in a machine. Apart from the machinery itself, the surrounding structure also faces the vibration hazard because of this vibrating machinery. So, there is a need to reduce unwanted vibrations. Many researchers have done research and given their views to reduce the unwanted vibration. Some of them have investigated the model for optimum control of vibrations. Vibration control has been carried out by several means including reduction of excitations, avoiding of resonance by proper choice of stiffness and mass parameters, use of vibration absorbers and dampers. The dampers are of viscous damping type, coulomb damping, materials or hysteretic damping types using conventional materials. For vibration control over a wide range as in aero-space, automotive and other applications, the above methods have limitations and the use of viscoelstic damping involving use of polymeric or elastomeric materials is finding considerable importance.

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The use of viscoelastic damping is beneficial in situations involving a wide range of excitation frequencies. Sandwich structures with bonded viscoelastic materials are widely used in aerospace, aeronautical, automobile industries due to their performance in reducing structural vibrations. Nakra[5] had studied the vibration control of machines and structures incorporating the viscoelstic materials for wide excitation frequency range and he has suggested the optimal parameters for viscoelastically treated structures. The growing use of these types of treatments has initiated the conduct of many studies in predicting the dynamic behavior of viscoelastically damped structure. Macioc [4], in his article, suggested the behavior of viscoelstic material during loading by combining the behaviors of purely elastic and viscous material. A visoelastic material is characterized by possessing both viscous and elastic behavior. Since the polymeric materials, essentially viscoelstic with high damping, are easily available and so, it is possible to effectively control the vibratory response of a structure. The polymeric material cannot be used on their own due to less strength and rigidity reasons, and hence a composite construction of metal and polymers can achieve both high damping and strength. Many researchers have given the procedure for use of viscoelastic material with metallic plates. There are two types of such a damping treatment, namely, unconstrained and constrained. In unconstrained treatment as shown in figure 1(a), the damping layer is put on side of vibrating plate or panel. The figure indicates both undeformed and deformed configurations during the flexural vibration. The vibrational energy is dissipated due to the extensional deformation of the high-damping viscoelasic layer. In constrained treatment as shown in figure 1(b), the damping layer is sandwiched between the vibrating surface and a stiff constraining layer. In this treatment, most of energy is dissipated due to the shear deformation of the viscoelastic layer. There are some parameters that govern the effectiveness of the two damping treatments. We can express the effectiveness of a damping treatment by the equivalent overall loss factor (η) of the composite structure.

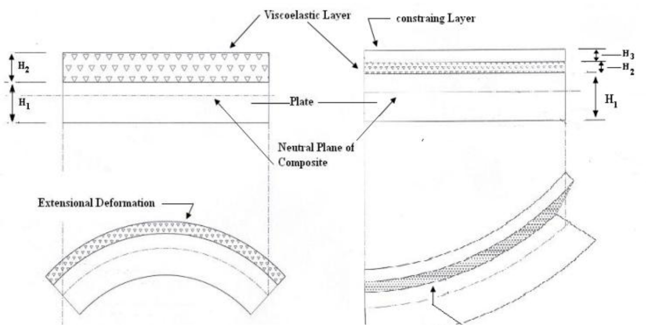

Figure 1(a). Unconstrained treatment of viscoelastic damping. Figure 1(b). Constrained treatment of viscoelastic damping

II. ANALYSIS OF PRIMARY SYSTEM WITH ABSORBER SYSTEM (VISCOELASTIC DAMPING INCLUDED IN ABSORBER SYSTEM)

The system under study is shown in figure (2) below, which consists of a primary mass spring system, whose vibrations has to be reduced, attached to a secondary viscoelastically damped system.The equation of motion for above system (figure 2) in matrix form is-

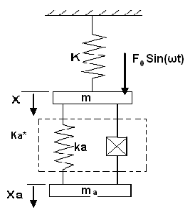

Figure 2. Two DOF System with Viscoelastic Damping in Absorber System

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix}\begin{bmatrix} \ddot{x}(t) \\ \ddot{x}_a(t) \end{bmatrix} + \begin{bmatrix} (k+k_a^*) & -k_a^* \\ -k_a^* & k_a^* \end{bmatrix}\begin{bmatrix} x(t) \\ x_a(t) \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} Sin(\omega t) \quad (1)$$

On solving the equation (1) for response of primary system and the absorber mass, it is found that,

$$X = \frac{\left[(k_a^* - m_a\omega^2)\right]F_0}{k_a^*(k - m\omega^2 - m_a\omega^2) - (km_a\omega^2 + mm_a\omega^4)} \quad (2)$$

$$X_a = \frac{(k_a^*)F_0}{k_a^*(k - m\omega^2 - m_a\omega^2) - (km_a\omega^2 + mm_a\omega^4)} \quad (3)$$

These two equations (2) and (3) express the magnitude of the response of the primary mass and absorber mass. The stiffness of the viscoelastically damped absorber is a complex stiffness and it is represented by

$$k_a^* = k_a(1 + \eta j).$$

$$X = \frac{\left[(k_a - m_a\omega^2)\right]F_0 + (k_a\eta j)F_0}{(k_a + k_a\eta j)(k - m\omega^2 - m_a\omega^2) - (km_a\omega^2 + mm_a\omega^4)} \quad (4)$$

$$X_a = \frac{(k_a + k_a\eta j)F_0}{(k_a + k_a\eta j)(k - m\omega^2 - m_a\omega^2) - (km_a\omega^2 + mm_a\omega^4)} \quad (5)$$

The dimensionless amplitude of vibration of primary system can be found by replacing the terms in the equation (4) as follows

$$\frac{\omega_a}{\omega_p} = \beta, \quad \frac{\omega}{\omega_p} = r \quad \text{and} \quad \frac{m_a}{m} = \mu.$$

The dimensionless amplitude of vibration of primary system is

$$\left|\frac{Xk}{F_0}\right| = \sqrt{\frac{\left[r^2 - \beta^2\right]^2 + (\eta\beta^2)^2}{(\eta\beta^2)^2(\mu r^2 - 1 + r^2)^2 + [\mu r^2\beta^2 - (r^2 - 1)(r^2 - \beta^2)]^2}} \quad (6)$$

It is seen from the equation (6) that the amplitude of the primary system response is determined by four physical parameter values μ, β, r, and η.

These four numbers can be considered as design variables and are chosen to give the smallest possible value of the primary mass's response, X, for a given application.

III. OPTIMIZATION OF DESIGN PARAMETERS

The effectiveness of viscoelastic treatments can be increased by proper choice of material and geometry. This can be achieved by optimizing the design parameters using some optimization methodology. The design parameters of the above system can be optimized to get the smallest possible value of response of primary system. Trindade [8], in his paper, presented the geometrical optimization of the passive damping treatment using finite element model which is applied for laminated composite structure. In this section, two optimization techniques, classical method given by Inman[2] and the technique given by Hartog [1] for viscous system are used to find the optimum design parameters.

A. Classical Technique

In this technique of optimization, the classical method (calculus method) is used to optimize the parameters. To find the optimum values for smallest response of primary system, equation (6) is differentiated partially with respect to variables and equating to zero.

Let, normalized response of primary system is a function of r, β and η.

$$\left|\frac{Xk}{F_0}\right| = f(r,\beta,\eta) = \sqrt{\frac{\left[r^2 - \beta^2\right]^2 + (\eta\beta^2)^2}{(\eta\beta^2)^2(\mu r^2 - 1 + r^2)^2 + [\mu r^2\beta^2 - (r^2 - 1)(r^2 - \beta^2)]^2}}$$

After optimization by calculus method the optimum values of parameters can be found as:

$$r^2 = \frac{1}{1+\mu} \pm \frac{1}{1+\mu}\sqrt{\frac{\mu}{2+\mu}} \quad (7)$$

$$\beta = \frac{1}{1+\mu} \quad (8)$$

$$\eta_{opt} = \sqrt{\frac{3\mu(1+\mu)}{(3\mu+2)}} \quad (9)$$

Now for different values of μ, the optimum values of β, r, η, and the values of normalized response of primary system (Xk/F0) can be calculated. In the above equations, for μ=0.25, the normalized amplitude has a minimum values for large range of r. So the optimum values of β, r and η can be selected corresponding to μ=0.25.

B. Use Of Den Hartog Technique For Absorber With Viscoelastic Damping:

Hartog[1] has given the technique of optimization of parameters of a two degree of freedom vibration system. In this technique, the optimization can be done by making optimally tuned vibration absorber. In this section, the technique of Hortog[1] is used to determine the optimal values of design parameters for maximum reduction of amplitude of vibration of primary system and for widening the range of working frequency.

To make the optimally tuned vibration absorber the amplitude of vibrations at the points from where all the curves passes (the two common points A and B as shown in figure (3)) are equated, and optimal value of η can be found by making the response curve (Xk/F0) as flat as possible at peak A and B.

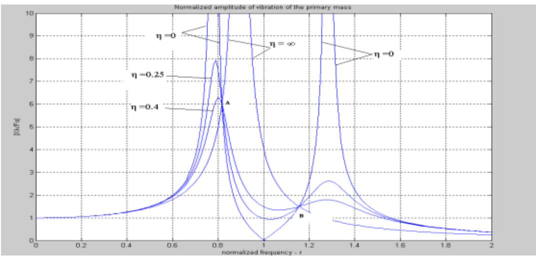

Figure 3.Amplitude of vibration of the primary mass for different values of loss factor as function of the frequency ratio

If equation (6) is plotted against r, for different values of η, and taking μ and β as constant. It is seen from the figure (3) that all curves intersect at point A and B for different values of loss factor η. These point can be located analytically by substituting the extreme cases of η = 0 and η = ∞ into equation (6) and equating them.

For η =0, equation (6) becomes

$$\left(\frac{Xk}{F_0}\right)_0 = \sqrt{\frac{\left[(r^2-\beta^2)^2\right]}{\left[\mu r^2\beta^2-(r^2-1)(r^2-\beta^2)\right]^2}} \qquad (10)$$

And for η = ∞, equation (6) becomes

$$\left(\frac{Xk}{F_0}\right)_\infty = \sqrt{\frac{1}{(r^2-1+\mu r^2)^2}} \qquad (11)$$

So equating Equation (10) and (11), it is found that,

$$\sqrt{\frac{\left[(r^2-\beta^2)^2\right]}{\left[\mu r^2\beta^2-(r^2-1)(r^2-\beta^2)\right]^2}} = \sqrt{\frac{1}{(r^2-1+\mu r^2)^2}}$$

$$\Rightarrow \quad r^4 - \frac{2r^2(1+\beta^2+\mu\beta^2)}{2+\mu} + \frac{2\beta^2}{2+\mu} = 0 \qquad (12)$$

The two roots of equation (12) indicates the values of the frequency ratio rA = ωA/ωp and rB = ωB/ωp, corresponding to the points A and B. the ordinates of A and B can be found by substituting the values of rA and rB, respectively, into equation (6). It has been observed that the most efficient vibration absorber is one for which the ordinates of the points A and B are equal.

So finding ordinates on points A and B, and equating them, it is found that

$$\beta = \frac{1}{1+\mu} \qquad 13)$$

Using the value of β from equation (13) in equation (12), the value of $r^2$is

$$r^2 = \frac{1}{1+\mu} \pm \frac{1}{1+\mu}\sqrt{\frac{\mu}{2+\mu}} \qquad (14)$$

So an absorber satisfying equation (14) can be called the tuned vibration absorber. But the equation (14) does not indicate the optimal values of the damping ratio η and the corresponding value of normalized response of primary system. The optimal value of η can be found by making the response curve (Xk/F0) as flat as possible at peaks A and B in figure 4. This can be achieved by making the curve horizontal at either A or B point. For this, first equation (13) is substituted in to equation (6) to make the resulting equation applicable to the case of optimum tuning. Then modified equation (6) is differentiated with respect to r to find the slope of the curve of (Xk/F0) and then equating to zero. It is found that the optimum value of damping ratio η is

$$\eta_{opt} = \sqrt{\frac{3\mu}{2(\mu+1)^2}\left[1+\sqrt{\left(\frac{\mu}{\mu+2}\right)}\right]} \qquad (15)$$

Now for different values of μ, the optimum values of β, r, η, and the values of normalized response of primary system (Xk/F0) can be calculated.

The equation (6) is plotted against r for different values of μ and corresponding optimal values of β, r, and η as shown in figure (4). It is seen from figure 4 that, for μ=0.25, the normalized amplitude has a minimum values for large range

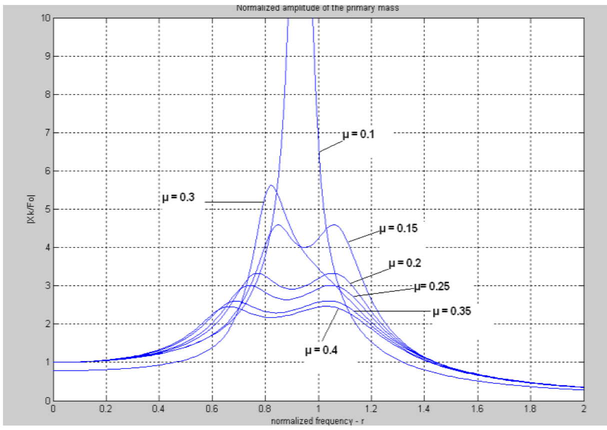

Figure 4.Normalized Amplitude of vibration of the primary mass as function of the frequency ratio for several values of mass ratio

of r. So the optimum values of β, r and η are selected corresponding to μ=0.25. So, in both the above techniques of optimization, the parameters can be selected corresponding to μ=0.25 but if both methods is compared, as given in table 1, then it is seen from table 1 that the optimization by Hartog method is more preferable because of laser value of β.

Table I. Comparison between D.J.Inman method and Den Hartog method of optimization (for viscoelastic system)

	Ratio of absorber mass to the primary mass (μ)	Ratio of decoupled natural frequencies (β)	Ratio of driving frequency to primary natural frequency (r)		Damping ratio (η)	Normalized amplitude of vibration of primary mass (Xk/F_0)
By D.J.Inman method	0.25	0.894427	0.7302	1.03279	0.4330	2.73315
By Den Hartog method	0.25	0.800	0.7302	1.03279	0.5656	3.00

Main beam				Absorber Beam				Viscoelastic Material		
Material: Mild Steel (E= 2×10^{11} N/m^2)				Material: Mild Steel(E= 2×10^{11} N/m^2)				Material: PVC		
Density (ρ)	Length (L)	Width (B)	Thickness (T)	Density (ρ_c)	Length (L_c)	Width (B_c)	Thickness (T_c)	Length (L_v)	Width (B_v)	Thickness (H_2)
7800 kg/m^3	0.88 m	0.05 m	0.0072 m	7800 kg/m^3	0.76 m	0.02 m	0.005 m	0.76 m	0.02 m	0.0058 m

Table II. Specification of Set-Up

To validate the above model, authors have done the theoretical calculations and performed the experiments on a set-up as shown in figure (5)of specifications as given in table 2 (without taking the optimization into consideration), and the experimental values are compared with the theoretical values. In this experiment, a main beam of known dimensions with fixed-fixed boundary condition is taken to reduce the vibrations. A cantilever beam with viscoelastic material is taken as the absorber system in which the absorber masses are attached to vary the natural frequency of absorber system. Two type of treatments (unconstrained and constrained) are made with mild steel beam and the PVC material.

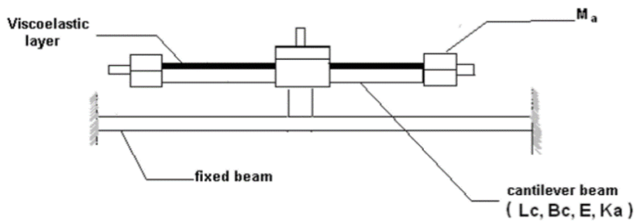

Figure 5.Set-Up of Fixed-fixed Beam with Viscoelstically Damped Vibration Absorber

IV. THEORETICAL CALCULATIONS

Viscoelastic material properties are frequency and temperature dependent. Hence, modeling of the frequency dependence of stiffness and damping properties of viscoelastically damped structures has studies by several research groups.

The model of Nakra [6],[7] which gives very simplified equations for overall loss factors of composite structure, is used for theoretical calculation of overall loss factor of viscoelastically damped absorber system for both unconstrained and constrained treatment at different frequencies for modal number 1.

The expression for the over all loss factor of composite section for figure 8 is given by Nakra [7]

$$\eta = \frac{Y\eta_{g2}}{[(1+e_3h_3^3)(P^2+\eta_{g2}^2)+YP]} \qquad (16)$$

Where,

$(\eta g)_2$= loss factor of viscoelastic layer in shear deformation.

$$Y = (3\psi/h_2)(1+h_3+2h_2)^2(1+\eta_{g2}^2),$$

$$P = (\psi/h_2)[(1+e_3h_3)+1](1+\eta_{g2}^2)+1,$$

Shear parameter $\psi = \dfrac{G_2}{[E_1H_1^2(n\pi/L_c)^2]},$

G_2= storage shear modulus of viscoelastic layer.

$$h_2 = \frac{H_2}{H_1} = \frac{\text{thickness of viscoelastic layer}}{\text{thickness of base (cantilever beam) layer}}$$

$$h_3 = \frac{H_3}{H_1} = \frac{\text{thickness of constraining layer}}{\text{thickness of base (cantilever beam) layer}}$$

n = modal number

$$e_3 = \frac{E_3}{E_1} = \frac{\text{storage Young's modulus of constraining layer}}{\text{storage Young's modulus of base (cantilever beam) layer}}$$

It is also concluded that the relations given below are best suited for viscoelastic material.

$(\eta_E)_2 = (\eta_g)_2$ and E* = 3G*.

Nakra [6], in his paper, has presented the variation of η against ψ . He conclude that η is maximum only at certain values of ψ . Thus a change in modal number n or in G_2 due to temperature or frequency change may change the damping available. It is seen that for a given increase in size or weight, constrained type arrangement gives higher damping effectiveness than an unconstrained one.

Kundra and Nakra [3] have given the variation of shear modulus and loss factor of 5 mm thick PVC with frequency at 320C temperature. These properties are used in this paper to do the theoretical calculations of shear modulus and overall loss factor at different resonance frequencies. For theoretical calculations, the resonance frequencies (ω_1 and ω_2) are found out using the well known equation (17) for resonance of two degree of freedom system.

$$\left(\frac{\omega}{\omega_p}\right)^2 = 1 + \frac{\mu}{2} \pm \left(\mu + \frac{\mu^2}{4}\right)^{1/2} \qquad (17)$$

Table III. Theoretical values of overall loss factors and response of main beam at resonance frequencies for given set-up.

Main Beam	Main Beam with Absorber (with Unconstrained Treatment)				Main Beam with Absorber (with Constrained Treatment)			
	Overall Loss factor of absorber system (η)		Response of Main Beam **(X)**		Overall Loss factor of absorber system (η)		Response of Main Beam **(X)**	
ω_p = 33.2 Hz	At ω_1 = 21.77 Hz	At ω_2 = 46.95	At ω_1 = 21.77 Hz	At ω_2 = 46.95	At ω_1 = 21.16 Hz	At ω_2 = 51.34 Hz	At ω_1 = 21.16 Hz	At ω_2 = 51.34 Hz
	1.0425×10^{-3}	1.361×10^{-3}	1.897 mm	4.2216 mm	0.1419	0.16619	1.3056 mm	1.5821 mm

Using equations (15) and (16), the overall loss factors are calculated at different resonance frequencies and then the response of main beam are calculated on those resonance frequencies. The theoretical calculations are shown in table 3, which shows that the response of main beam, in case of main beam with constrained treated absorber has smaller than the unconstrained treatment at both resonance frequencies.

V. EXPERIMENTAL WORK ON EXPERIMENTAL SET-UPS

To do the experiment on the set-up, the circuit connection has been done. Experiments have been performed on the actual set-up to find the response of main beam corresponding to the excitation frequencies. The experimental values of response of main beam corresponding to the excitation frequencies are shown in the table 4. It is seen from table 4 that the experimental response of main beam, in case of main beam with constrained treated absorber has smaller than the unconstrained treatment at both resonance frequencies, which is like in case of theoretical calculations.

Table IV. Experimental values response of main beam at resonance frequencies for given set-up

	Main Beam with Absorber (without damping)		**Main Beam with Absorber (with Unconstrained Treatment)**		**Main Beam with Absorber (with constrained Treatment)**	
	Natural Frequency	**Displacement of main beam**	**Natural Frequency**	**Displacement of main beam**	**Frequency**	**Displacement of main beam**
First Resonance Point	**20.00 Hz**	**0.382 mm**	**19.05 Hz**	**0.315 mm**	**18.75 Hz**	**0.258 mm**
Lowest Amplitude Point	**31.00 Hz**	**0.001 mm**	**30.50 Hz**	**0.002 mm**	**27.00 Hz**	**0.004 mm**
Second Resonance Point	**34.75 Hz**	**0.464 mm**	**35.00 Hz**	**0.390 mm**	**37.00 Hz**	**0.278 mm**

VI. RESULT DISCUSSION AND CONCLUSION

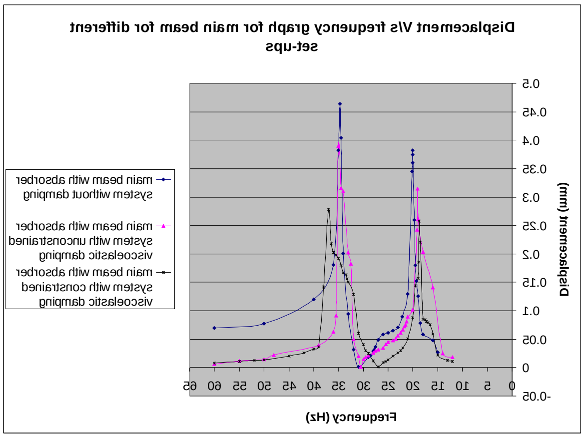

Figure 6. Experimental Displacement v/s Frequency graph for three set-ups (comparative graph)

For constrained damping, at Tuning length of 11.5 cm the two experimental resonant frequencies are 18.75 Hz and 37 Hz, whereas the theoretical resonant frequencies come out to be 23.6 Hz and 46.75 Hz. (Table III). The range of frequency between two resonance points has become wider than the undamped system and unconstrained treated systems. It is seen from the figure 6 that there is greater reduction in response of main beam at peaks in case of constrained viscoelastically damped absorber system attached to main beam than undamped and unconstrained visoelastically damped absorber system attached to main beam. There is also increase in the range of frequency between two resonance points in case of constrained treated absorber system attached to the main system.

REFERENCES

1. Hartog, D., and J.P., "Mechanical Vibrations", MaGraw-Hill. New York, 1956.
2. D. J. Inman, "Engineering Vibration". Prentice Hall, Upper Saddle River, New Jersey. Second Edition, 2001.
3. T. K. Kundra and B. C. Nakra, "Passive and Active Damping Control of Vibration and Noise in Machines and Structure", SERC Research Project Report, 2003.
4. P. Macioc, "Viscoelastic Damping", Rousb Industries, Inc, 2006. Available at.
5. http://www.roush.com/Portals/1/Downloads/Articles/Insight.pdf
6. B. C. Nakra, "Vibration Control in Machines and Structures Using Viscoelastic Damping", Journal of Sound and Vibration, 211(3), 1998, pp. 449-469.
7. B. C. Nakra, "Structural Dynamic Modification Using Additive Damping", Sadhna, India, 2000, pp. 277-289.
8. B. C. Nakra, "Vibration Damping", PINSA, India, , 2001, pp. 461-478.
9. M. A. Trindade, "Optimization of Passive Constrained Layer Damping Treatments Applied to Composite Beams", Latin American Journal of Solids and Structures, 4, 2007, pp. 19-38.

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