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Estimation of Parameters in Erlang Distribution using Prior Information

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Abstract: *This paper focuses on the estimation of parameters of an Erlang density using known Coefficients of variation and Kurtosis of the population basing on past experience and a simple random sample of size n from the population. Estimators using Searle D.T approach are proposed and their bias $B(T)$ and mean squared error $M(T)$ are calculated. The relative efficiency of T over conventional estimator $\bar{x}$ is also tabulated for various sample sizes and various C.V values, Kurtosis values. The proposed estimators are observed to be more efficient than $\bar{x}$ under the conditions established.*

Subject Classification: AMS 62J13

Key Words: *Erlang Density, Coefficient of Variation, Coefficient of Kurtosis, Relative Efficiency, Bias, Mean squared error*

I. INTRODUCTION TO ERLANG DISTRIBUTION

The Erlang distribution is a two-parameter family continuous probability distribution with probability density function (pdf) given by

$$f(x:\lambda,k) = \frac{\lambda^K x^{K-1}}{(K-1)!}e^{-\lambda x}:\ K \in N, \lambda > 0, x \in (0,\propto)$$

Here K denotes shape parameter and λ denotes scaling parameter. Mean and variance of the distribution are respectively $\frac{K}{\lambda}$ and $\frac{K}{\lambda^2}$. This distribution is a special case of Gamma distribution. For $K = 1$ Erlang distribution reduces to exponential distribution.

This distribution is used to examine the number of telephone calls which might be made at the same time to the operators of different switching stations. This work on telephone traffic engineering has been expanded to consider waiting times in Queuing systems in general. Erlang distribution has wide applications in the field of stochastic processes and Bio-mathematics.

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II. SEARLE'S ESTIMATOR

D.T. Searle(1964) suggested the use of known coefficient of variation (C.V) $\sqrt{C}$ of the population to obtain an estimator for population mean $\bar{X}$, which has less mean squared error(MSE) than the conventional estimator $\bar{x}$(sample mean). If $(x_1, x_2, \dots\dots, x_n)$ is a random sample of size n, then Searle suggested an estimator T as $T = w\sum_{i=1}^{n} x_i$, where w is a scalar to be chosen to minimize the $MSE(T)$.

$$MSE(T) = nw^2\sigma^2 + \bar{X}^2(1-nw)^2$$

σ^2 being population variance (unknown)

$$\frac{\partial MSE(T)}{\partial w} = 0 \implies w = \frac{1}{n+c}\ \&\ T = \frac{1}{n+c}\sum_{i=1}^{n} x_i$$

from the basic calculus.

The bias and MMSE of proposed estimator are respectively given by

$$B(T) = E(T) - \bar{X} = \frac{-n}{n+c}\bar{X}$$

$$M(T) = V(T) + (B(T))^2 = \frac{c}{n+c}\bar{X}^2$$

$REF(T,\bar{x}) = \frac{V(\bar{x})}{M(T)} = 1 + \frac{c}{n}$ is the relative efficiency of T over conventional estimator $\bar{x}$.

In the similar manner a Searle's estimator $T_1$is proposed for the population variance as

$$T_1 = w\sum_{i=1}^{n}(x_i - \bar{x})^2$$

where $\bar{x} = \frac{1}{n}\sum x_i$ is sample mean.

The bias and minimum measured error of $T_1$are respectively obtained below.

$$B(T_1) = E(T_1) - \sigma^2 = (w_1(n-1) - 1)\,\sigma^2$$

$$MSE(T_1) = w_1^2(n-1)^2E(S^4) + \sigma^4(1 - 2w_1(n-1))$$

Minimizing $MSE(T_1)$ using calculus

$$\frac{\partial MSE(T_1)}{\partial w_1} = 0 \implies w_1 = \frac{\sigma^4}{(n-1)E(S^4)}$$

where $S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is sample mean square unbiased for σ^2.

Now $E(S^4) = V(S^2) + \left(E(S^2)\right)^2 = \frac{n^2}{(n-1)^2}V(s^2) + \sigma^2$

where $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$ is the sample variance. Using relations between sample moments m_r and population moments μ_r (r^{th} central moments) (Adv. Theo. Stats. Kendall &Stuart Vol. I) to the order $O(n^{-\frac{1}{2}})$

$$V(m_r) = \frac{1}{n}(\mu_{2r} - \mu_r^2 + r^2\mu_2\mu_{r-1}^2 - 2r\mu_{r-1}\mu_{r+1})$$

$$\Rightarrow V(m_2) = V(s^2) = \frac{1}{n}(\mu_4 - \mu_2^2)$$

$$E(S^4) = \frac{\sigma^4}{(n-1)^2}(n(\beta_2 - 1) + (n-1)^2)$$

$$w_1 = \frac{(n-1)}{n(\beta_2 - 1) + (n-1)^2}$$

where $\beta_2 = \frac{\mu_4}{\mu_2^2}$ is the coefficient of Kurtosis which is assumed known.

We shall now obtain Bias & MMSE of T_1 as follows.

$$T_1 = \frac{(n-1)}{n(\beta_2 - 1) + (n-1)^2}\sum_{i=0}^{n}(x_i - \overline{x})^2$$

$$B(T_1) = \frac{-n\sigma^2(\beta_2 - 1)}{n(\beta_2 - 1) + (n-1)^2}$$

$$M(T_1) = \frac{n(\beta_2 - 1)}{n(\beta_2 - 1) + (n-1)^2}\sigma^4$$

$$REF(T_1, S^2) = \frac{n(\beta_2 - 1) + (n-1)^2}{(n-1)^2} \geq 1 \text{ always}$$

2.1 Proposed Estimators For Population Mean And Variance In Erlang Distribution & Results

Using Searle's approach we shall now propose two estimators for population mean and variance respectively for Erlang density as follows.

$$t = \frac{1}{n+c}\sum_{i=1}^{n} x_i$$

$$t_1 = \frac{n-1}{n(\beta_2 - 1) + (n-1)^2}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

where $\sqrt{c}$ is known C.V. and β_2 is known coefficient of Kurtosis of Erlang population from certain past experiences.

Respective Biases and Minimum Mean Squared Errors of proposed Estimators are obtained as below

$$B(t) = \left(\frac{-n}{n+c}\right)\frac{K}{\lambda}$$

$$M(t) = \left(\frac{c}{n+c}\right)\frac{K^2}{\lambda^2}$$

$$REF(t, \overline{x}) = 1 + \frac{c}{n}$$

$$B(t_1) = \left(\frac{-n(\beta_2 - 1)}{n(\beta_2 - 1) + (n-1)^2}\right)\frac{K}{\lambda^2}$$

$$M(t_1) = \left(\frac{n(\beta_2 - 1)}{n(\beta_2 - 1) + (n-1)^2}\right)\frac{K^2}{\lambda^4}$$

$$REF(t_1, S^2) = \frac{n(\beta_2 - 1) + (n-1)^2}{(n-1)^2} \geq 1 \text{ always .}$$

The $REF(t, \overline{x})$ and $REF(t_1, S^2)$ are tabulated below for various sample sizes and values of $\sqrt{c}, \beta_2$.

Table-1

$$\boldsymbol{REF(t, \overline{x}) = 1 + \frac{c}{n} \text{ in } \%}$$

n \ $\sqrt{c}$	0.5	1	1.5	2	2.5
5	105	145	145	180	225
10	102.5	110	122.5	140	162.5
15	101.7	106.7	1115	126.7	141.7
20	101.25	105	111.25	120	131.25
25	101	104	109	116	125
30	100.08	103.3	107.5	113.3	120.83

Table-2

$$\boldsymbol{REF(t_1, S^2) = 1 + \frac{n(\beta_2 - 1)}{(n-1)^2} \text{ in } \%}$$

n \ β_2	1.5	2	2.5	3	3.5
5	115.625	131.25	146.87	162.5	178.125
10	106.173	112.34	118.5	124.69	130.86
15	103.82	107.65	111.48	115.3	119.13
20	102.8	105.54	108.31	111.08	113.85
25	102.17	104.34	106.5	108.7	110.85
30	101.78	103.57	105.35	107.13	108.91

III. CONCLUSIONS

1. t and t_1 are respectively observed to be more efficient than conventional estimators $\overline{x}$ and S^2 for all sample sizes n.
2. With increasing n for a fixed $\sqrt{c}$ value (C.V.) the REF decreases for t over $\overline{x}$ and also for a fixed β_2(Kutosis) value $REF(t_1, S^2)$ also decreases for increasing n.
3. For fixed n and increasing $\sqrt{c}, \beta_2$ values $REF(t, \overline{x}), REF(t_1, S^2)$ respectively increase.
4. The proposed estimators are useful to estimate mean & variance of an Erlang population and hence can be applied in forecasting theory of waiting time in telephone traffic Engineering like departments.

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