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Correlation of CLPT and FSDT Responses in CNT/Polymer Composite Beams Under Deflection and Buckling

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Abstract--- The principal intention of this research paper is to compare the Classical Laminate Plate Theory (CLPT) and First-order Shear Deformation Theory (FSDT) based on the buckling and deflection performance of Carbon Nanotube (CNT) Reinforced Polymer Composite beams. A symmetric, eight layered, and simply supported nanocomposite beam is taken for the comparison of CLPT and FSDT. The Mori-Tanaka micro-mechanical approach is used for the computation of elastic constants as a function of volume fraction of CNT reinforcement for the CNT/Polymer composite material. The elastic constants obtained are extensively utilized to discover the critical buckling loads and deflections by using CLPT and FSDT. The variations are presented in graphical form for the buckling and deflection of beams for various stacking sequences and CNT volume fractions. The effect of shear deformation is studied in both deflection and buckling of nanocomposite beams. The effect of beam thickness on the bending loads and buckling loads in both the theories is also studied from the results.

Keywords— Carbon nanotube, Polymer Composite, Nanocomposite, FSDT,CLPT.

I. INTRODUCTION

To anticipate the failure behaviour of a loaded structural elements made of composites, a few theories of have been proposed depending upon the consideration of the transverse shear deformation effect in to account. Classical Laminate Plate Theory (CLPT) is based on the postulates Love-Kirchhoff's, according to which a straight line perpendicular to the plane of plate before loading remains unchanged even after deformation. First-order shear deformation theory (FSDT) expands the classical plate theory by taking the transverse shear effect into consideration; in this theory the stresses and deformations are invariable across the thickness of the plate. These two theories are extensively compared for the buckling and bending loads by considering a simply supported CNT reinforced polymer composite beam.

Carbon nanotubes are the unbelievable substances that came across to modernize the technological scenario in the recent past. In future, the world will be fashioned by CNT applications, presently as silicon-based equipment dictates the social order nowadays. Space ships; hydrogen vehicle; artificial muscles; Ultra light-weight structures: these are just only some of the scientific wonders that might be made achievable by the advent of carbon nanotubes [1].

Most of the investigations on CNT immediately after its discovery by Ijima in 1991 [2] intended to find their physical and chemical characterization only. An abundant research was conducted for evolution of electrical and mechanical properties of CNTs in addition to CNT-reinforced polymer composites via micromechanical, molecular and investigational based methods [3–6]. In addition, the discovered outstanding physical and mechanical properties of CNTs, for instance, a high Young's modulus of the order of 1 TPa and tensile strength of 200 GPa [7], has made them one of the prominent reinforcing elements in composites. Later on experimental and theoretical studies carried-out on carbon nanotubes have paid attention on the mechanical behavior of reinforced polymer composites to support the progress of the nanocomposites [8-10]. Even though these investigations are quite useful in understanding the mechanical properties of CNTs and CNT reinforced polymer composites, their actual structural behavior is to be determined before put to use in structural elements like beams and plates. The study is also carried out on structural behavior of CNT/Polymer composite beams and plates under different loads, boundary conditions and using deferent theories [11-13].

In the present research, an analytical comparison of CLPT and FSDT on static behavior of Carbon nanotube Reinforced Composite (CNTRC) beams is conceded with a prospect towards appraising the efficacy of these two theories in the development of structural nanocomposites. For the evolution of CNTRC in mechanical behavior, precise property–microstructure associations are mandatory, in the style of micromechanics techniques. In the present research, micromechanics properties of CNTRC are worked out by means of Mori-Tanaka micromechanics method as given in [13-14].

II. MICRO-MECHANICS MODEL

Developing the analytical methods for the polymer composite materials reinforced with nano-scale fibres to approximate their mechanical characteristics has been an issue of great importance in the composite research.

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In general, the mechanical characteristics like stiffness and tensile strength of polymers are assumed to be enhanced with the reinforcement of elastically stiff fillers and can be reduced by adding soft materials. There are numerous micro-mechanics approaches for the computation of the elastic coefficients of nano-composite materials. The micro-mechanics models such as the Halpin–Tsai model, Nielsen method, Mori–Tanaka approach, and Eshelby model, etc., are applied to estimate the stiffness of the nano-composite based on the geometry, elastic constants of the fiber and resin, and inclination of the reinforcement. The Mori-Tanaka micro-mechanics approach used in the current study consists of an isotropic elastic and homogeneous polymer reinforced with straight and unidirectional CNT fibers. The CNT considered is assumed as considerably long, unbroken, and rigid fillers with laterally-isotropic stiffness characteristics and the magnitudes of these elastic constants are absorbed from Popov et.,al [15]. The composite material is assumed to be transversely isotropic and its governing stress strain relations $\sigma = C\,\varepsilon$ can be presented as

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{13} \\ \sigma_{23} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} n & l & l & 0 & 0 & 0 \\ l & k+m & k-m & 0 & 0 & 0 \\ l & k-m & k+m & 0 & 0 & 0 \\ 0 & 0 & 0 & 2p & 0 & 0 \\ 0 & 0 & 0 & 0 & 2m & 0 \\ 0 & 0 & 0 & 0 & 0 & 2p \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{13} \\ \varepsilon_{23} \\ \varepsilon_{12} \end{Bmatrix} \qquad (1)$$

Where m and p are the shear moduli in planes normal and parallel to the fiber direction, k is the plane-strain bulk modulus perpendicular to the fiber orientation, n is the uni-axial tension coefficient in the fiber direction, l is the relative cross modulus, are Hill's elastic coefficients correspondingly. A nano-composite with a fiber phase volume fractionc_r, matrix phase elastic modulus E_m and matrix phase Poisson's ratio υ_m is employed. By means of the Mori_Tanaka methodology, the Hill's elastic moduli are expressed as:

$$k = \frac{E_m\{E_m c_m + 2k_r(1+\nu_m)[1+c_r(1-2\nu_m)]\}}{2(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]}$$

$$l = \frac{E_m\{c_m \nu_m[E_m+2k_r(1+\nu_m)]+2c_r l_r(1-\nu_m^2)\}}{(1+\nu_m)[2c_m k_r(1-\nu_m-2\nu_m^2)+E_m(1+c_r-2\nu_m)]}$$

$$n = \frac{c_m E_m^2(1+c_r-c_m\nu_m)+2c_r c_m(k_r n_r-l_r^2)(1-2\nu_m)(1+\nu_m)^2}{(1+\nu_m)\{2k_r c_m(1-\nu_m-2\nu_m^2)+E_m(1+c_r-2\nu_m)\}} + \frac{E_m[2{c_m}^2 k_r(1-\nu_m)+c_r n_r(1-2\nu_m+c_r)+4c_m c_r l_r \nu_m]}{2k_r c_m(1-\nu_m-2{\nu_m}^2)+E_m(1+c_r-2\nu_m)}$$

$$p = \frac{E_m[E_m c_m+2(1+c_r)p_r(1+\nu_m)]}{2(1+\nu_m)[E_m(1+c_r)+2c_m p_r(1+\nu_m)]}$$

$$m = \frac{E_m[E_m c_m+2m_r(1+\nu_m)(3+c_r-4\nu_m)]}{2(1+\nu_m)\{E_m[c_m+4c_r(1-\nu_m)]+2c_r m_r(3-\nu_m-4{\nu_m}^2)\}} \qquad (2)$$

Where $m_r, p_r, l_r, k_r,$ *and* n_r are the Hill's elastic coefficients for the CNT fiber reinforcing phase. The global elastic constant terms for the CNTRC as functions of the local elastic stiffness constants derived for a unidirectional lamina are as follows:

$$E_L = n - \frac{l^2}{k}, \qquad E_T = \frac{4m(kn-l^2)}{kn-l^2+mn} \qquad G_{LT} = 2p \quad \text{and} \quad \nu_{LT} = \frac{l}{2k} \qquad (3)$$

III. MATERIALS AND METHODOLOGY

The material of beam under this study is consisting of polystyrene as the resin material with the Young's modulus of elasticity and Poisson's ratio of $E_m = 1.9$GPa and $\upsilon_m = 0.3$ correspondingly. The radius of CNT is taken to be 10Å for every case if not mentioned for which the commissioned values of the stiffness coefficients of single-walled carbon nano-tubes (SWCNTs) are: $m_r = 1$ GPa, $p_r = 1$ GPa, $n_r = 450$ GPa, $l_r = 10$ GPa, $k_r = 30$ GPa. The joining at the nanotube-polymer boundary is assumed to be perfect. A methodical study under CLPT & FSDT is accomplished on the deflection and buckling response of CNTRC beams for a range of stacking sequences and side-to-thickness ratios. The effect of volume percentage of CNT is also observed on the bending & buckling resistance of the above said beams.

The bending and buckling behavior of symmetrically laminated carbon nano-tube reinforced polymer beam is to be estimated under CLPT & FSDT and the important equations are reviewed in this section. For the specially-orthotropic laminated composite beams subjected to static loads, the governing equations with standard notations for the bending deflections and buckling loads under CLPT & FSDT are given by Reddy [16] as follows:

The maximum value transverse deflection of a laminated simply supported beam subjected to UDL according to CLPT is given by

$$w_{max} = \frac{5}{384}\left(\frac{q_0 b a^4}{E_{xx}^b I_{yy}}\right) \qquad (4)$$

The maximum value transverse deflection of a laminated simply supported beam subjected to UDL according to FSDT is given by

$$w_{max} = \frac{5}{384}\left(\frac{q_0 ba^4}{E_{xx}^b I_{yy}}\right) + \frac{1}{8}\left(\frac{q_0 ba^4}{KG_{xz}^b bh}\right) \quad 5)$$

The following non-dimensionalization of maximum deflection is used for presenting the results

$$\bar{w} = \frac{w_{max} E_2 h^3 X\ 10^2}{q_0 a^4} \quad (6)$$

The Critical buckling loads of a laminated simply supported beam according to CLPT is given by

$$N_{cr} = \left(\frac{\pi^2}{12}\right)\frac{E_{xx}^b h^3}{a^2} \quad (7)$$

The Critical buckling loads of a laminated simply supported beam according to FSDT is given by

$$bN_{cr} = E_{xx}^b I_{yy}\left(\frac{\pi}{a}\right)^2\left[1 - \frac{E_{xx}^b I_{yy}\left(\frac{\pi}{a}\right)^2}{KG_{xz}^b bh + E_{xx}^b I_{yy}\left(\frac{\pi}{a}\right)^2}\right] \quad (8)$$

The nondimensionalised critical buckling loads presented in the results are given by

$$\bar{N} = N_{cr}\left(\frac{a^2}{E_2 h^3}\right) \quad (9)$$

IV. NUMERICAL RESULTS

A well ordered code using MATLAB is programmed to acquire the elastic stiffness constants of an eight layered symmetrically laminated CNRTC beam and the results are verified with the existing results published in the literature [11-13]. The results achieved by using the developed MATLAB program and the available results in the literature are found to be in exceptional concurrence. Followed by the code is subsequently extended to calculate the deflections and critical buckling loads by means of the mathematical formulation under CLPT & FSDT as presented in equations (4) to (9).

4.1 Comparison of buckling behaviour of simply supported beam under CLPT and FSDT:

An extensive analytical study has been performed on simply-supported beams for a variety of stacking sequences to look into the effect volume fraction of reinforcing CNT on buckling of beams and the results are publicized in Figure 1 & Figure 2.

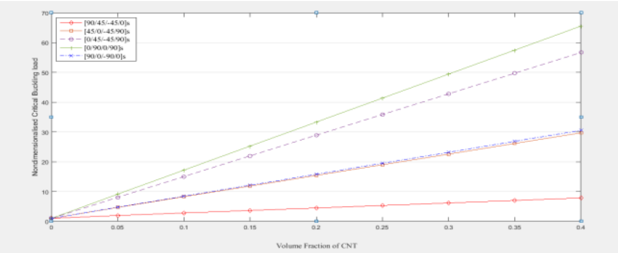

Fig. 1 Non-dimensionalised critical buckling load vs fiber volume fraction for different ply sequences under CLPT.

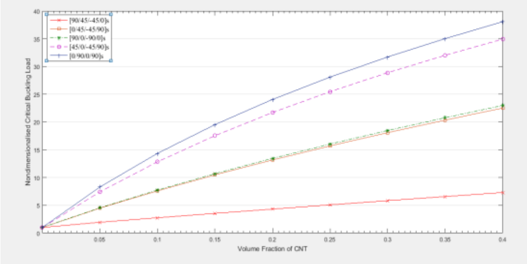

Fig. 2 Non-dimensionalised critical buckling load vs fiber volume fraction for different ply sequences under FSDT.

Form the Fig. 1 & Fig 2, it can be depicted that the lamination sequence [90/45/-45/0]s is defending least critical buckling load while the sequence of lamination [0/90/0/90]s is resisting the maximum in both the theories. Higher resistance to the buckling is desirable in structural elements.

The effect of thickness of the beam on the buckling behavior against different stacking sequences is also studied under FSDT at particular CNT volume fraction 0.3 and the results are presented in fig 3.

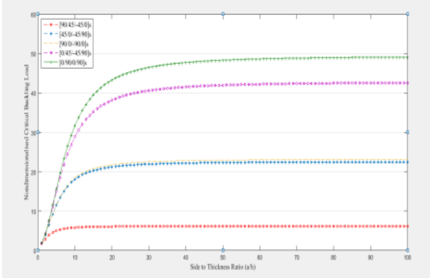

Fig. 3 Non-dimensionalised critical buckling load vs side-to-thickness ratio under FSDT for different stacking orders.

From the above results, it is evident that there is no substantial variation in buckling load as the transverse shear has no significant effect on the critical buckling loads of thin plates i.e., a/h>20. Conversely, the transverse shear influences the critical buckling loads of thick plates (a/h<=20) to a great extent.

The comparison of buckling loads of composite beam under CLPT and FSDT for the composite lamination sequence [0/90/0/90]s at CNT volume fraction 0.3 is shown in fig 4 and fig 5.

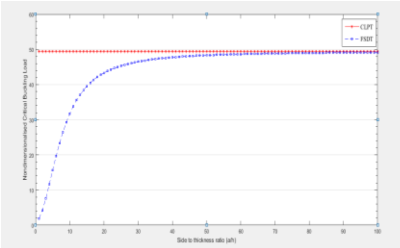

Fig. 4 Comparision of Non-dimensionalised critical buckling loads for beam side-to-thickness ratio (a/h) under CLPT & FSDT.

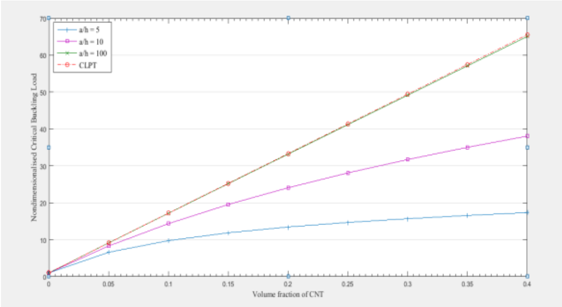

Fig. 5 Comparision of Non-dimensionalised critical buckling loads for CNT volume fractions under CLPT & FSDT.

The above results show that the CLPT is independent of thickness of the beam that is the theory does not consider the beam thickness into consideration where as in FSDT, the effect of thickness of the beam on nondimensionalised minimum buckling loads is clearly observed.

4.2 Comparison of Deflection behaviour of simply supported beam under CLPT and FSDT:

A similar analytical study, like buckling behaviour on simply supported beams for various stacking orders of laminated composite is conducted to explore the effect of thickness of the beam and fiber volume fraction of CNT on deflection behaviour of beams under FSDT and the results are compared with CLPT.

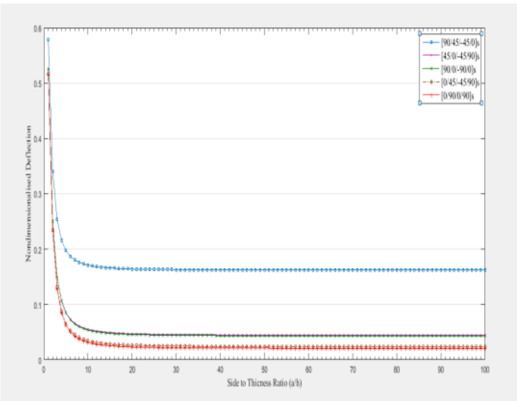

Fig. 6 Non-dimensionalised deflections vs side to thickness ratio for various stacking order under FSDT.

From the results it is apparent that the deflections decrease as the proportion of CNT dispersion raises.

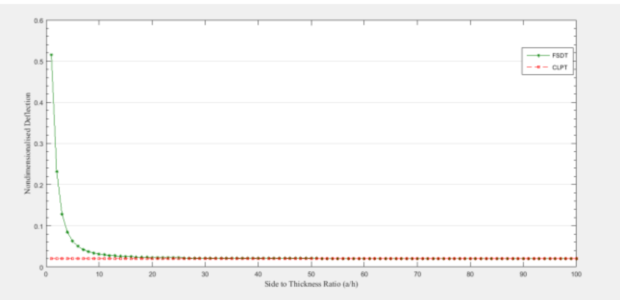

Fig. 7 comparision of Non-dimensionalised deflections vs side to thickness ratio for [0/90/0/90]s under FSDT & CLPT.

A comparison of deflection in beams under CLPT & FSDT is presented in fig 8 & fig 9.

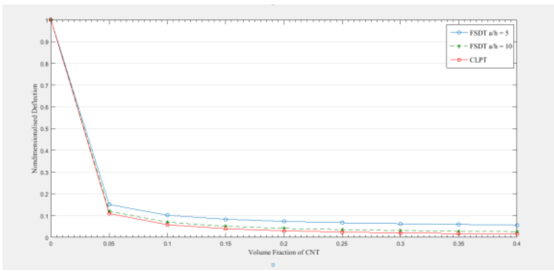

Fig. 8 Comparision of Non-dimensionalised deflections for CNT volume fractions under FSDT & CLPT.

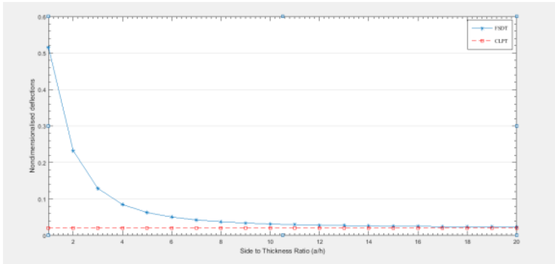

Fig. 9 Comparision of Non-dimensionalised deflections for sidetothickness ratio for thick beams under FSDT & CLPT.

From the above graphs, it can be seen that when transverse shear is taken into account as in FSDT, there is a significant change in the stiffness, with stiffness being higher in case of FSDT compared to CLPT. In case of CLPT, the stiffness remains constant throughout.

V. CONCLUSIONS

Deflections

The response of nano-composite beams for deflections was examined by the application of micromechanics relationships to discover the elastic modulii in terms of carbon nanotube volume fraction. The effect of the stacking order of the laminated composite on the nondimensionalised maximum deflection of CNT/Polymer composite beam has been studied. In case of aligned CNT reinforced polymer nanocomposite beams it has been acknowledged that a minute amount of percentage of addition of carbon nanotube as reinforcement gives a notable gross improvements in the stiffness of the composite material which observed in both cases of CLPT and FSDT, with FSDT resulting in higher stiffness, indicating that the transverse shear has an influence on the stiffness. It was also made known that stacking sequence is yet another essential constraint in both the cases, for reducing the maximum deflection.

Buckling

The nondimensionalised critical buckling loads of a CNT/Polymer nano-composite beam are examined by exercising the micromechanics or property-microstructure

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relationships to verify the elastic modulii in terms of carbon nanotube volume proportion. Also, it can be observed that there is no considerable variation in buckling load of the beams as the transverse shear has any effect on the thin beams. However, the transverse shear considerably affects the buckling loads of thick beams. This shows that the CLPT is independent of thickness of the beam where as in FSDT the effect of thickness of the simply supported beam on nondimensionalised critical buckling loads is clearly observed. In fact there is a substantial raise in the non-dimensionalised critical buckling load.

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