Statistical Hypothesis Test on Industrial Applications through Ranks from Cog of TRFNS

Parthiban. S, Gopinathan. P

Abstract— This paper aims at a procedure for the test of hypothesis of two sample t-test by representing a defuzzification method of Triangular Fuzzy Numbers (TFNs) and Trapezoidal Fuzzy Numbers (TrFNs) based on the Rank obtained from Centre of Gravity (COG) which can be followed in industries and other mathematical applications.

Index Terms— Alpha Cut, COG, TrFNs, Test of Hypothesis, Two Sample t – test.

1. INTRODUCTION

Fuzzy logic is a powerful tool to characterize the uncertainty with multiple values which provides a reliable resource for the problems of decision making. On the other hand, statistical hypothesis test is a very essential device to arrive decisions in real time problems. Generally, in the hypothesis test, the concerned data are assumed to be exact numbers in nature but in real time observations, it is not quite viable to get the data that are precise in nature. In this case, the test statistics will also provide ambiguous number. This article proposes an approach for the test of hypothesis on the basis of the collected data are employed as TrFNs. The test of hypothesis under fuzzy environment has been analysed by many authors based on the concepts of fuzzy set theory established by Zadeh [18].

In the proposed work a new method is attempted for the test of fuzzy hypothesis of two-sample t-test in which the observed samples are in terms of TrFNs. The sample TrFNs can be defuzzfied using the COG of TrFNs and based on the defuzzified data, the hypothesis test is performed.

2. PRELIMINARIES

Fuzzy numbers play a significant role in fuzzy mathematics just like the role played by means of the ordinary numbers in classical mathematics. The general facts can be referred in the subject book [12]. The simplest form of fuzzy number is likely a Triangular Fuzzy Number (TFN).

Definition 2.1. A generalized fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4; \omega)$ is defined as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

i. $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ in $[0, \omega]$, $0 \le \omega \le 1$,

ii. $\mu_{\tilde{A}}(x) = 0, \quad \forall\, x \in (-\infty, p_1]$,

iii. $\mu_L(x) = L_{\tilde{A}}(x)$ is strictly increasing on $[p_1, p_2]$,

iv. $\mu_{\tilde{A}}(x) = \omega, \quad \forall\, [p_2, p_3]$ and ω is a constant $0 < \omega \le 1$,

v. $\mu_R(x) = R_{\tilde{A}}(x)$ is strictly decreasing on $[p_3, p_4]$,

$\mu_{\tilde{A}}(x) = 0, \quad \forall\, x \in [p_4, \infty)$ where p_1, p_2, p_3, p_4 are real numbers such that $p_1 < p_2 \le p_3 < p_4$.

Definition 2.2. Let p_1, p_2 and p_3 be real numbers with $p_1 < p_2 < p_3$, then the TFN $\tilde{A} = (p_1, p_2, p_3)$ is the fuzzy number with membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p_1}, & x \in [p_1, p_2] \\ \dfrac{p_3 - x}{p_3 - p_2}, & x \in [p_2, p_3] \\ 0, & x < p_1 \text{ and } x > p_3 \end{cases}$$

And $\mu_{\tilde{A}}(p_2) = 1$ for a normalized TFN and p_2 need not be in the middle of $'p_1'$ and $'p_3'$.

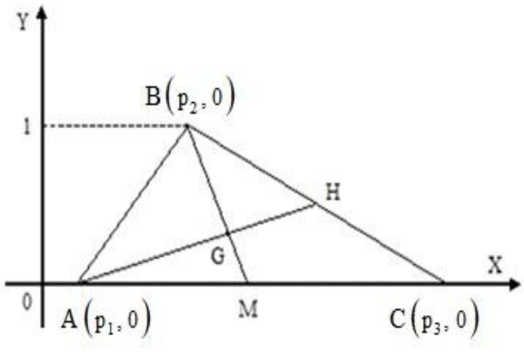

Figure (2.2) COG of the TFN

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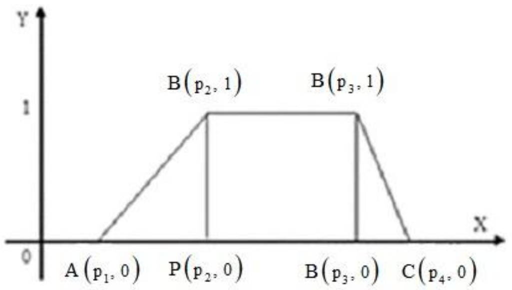
Definition 2.3. Generalized TrFN (GTrFN)

A GTrFN $\tilde{A} = (p_1, p_2, p_3, p_4; \omega)$ is defined by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < p_1 \\ L_{\tilde{A}}(x) = \dfrac{w(x - p_1)}{(p_2 - p_1)} & \text{for } p_1 \le x \le p_2 \\ \omega & \text{for } p_2 \le x \le p_3 \\ R_{\tilde{A}}(x) = \dfrac{w(x - p_4)}{(p_3 - p_4)} & \text{for } p_3 \le x \le p_4 \\ 0 & \text{for } p_4 < x \end{cases}$$

where $\mu_{\tilde{A}}(x)$ is piecewise linear, $0 \le \omega \le 1$ is a constant and p_1, p_2, p_3, p_4 are real numbers and $L_{\tilde{A}}(x) : [p_1, p_2] \to [0, \omega]$, $R_{\tilde{A}}(x) : [p_3, p_4] \to [0, \omega]$ are two strictly monotonic and continuous functions from $\mathbb{R} \to [0, \omega]$.

Figure (2.3)



Definition 2.4. A fuzzy set $\tilde{A}$ is said to be ***normal*** if $\ni$ an element 'x' such that $\mu_{\tilde{A}}(x) = 1$.

Definition 2.5. If $\omega = 1$ in GTrFN, we have ***normalized*** TrFN $\tilde{A} = (p_1, p_2, p_3, p_4; 1)$ and can be simply denoted by $\tilde{A} = (p_1, p_2, p_3, p_4)$.

Definition 2.6. Given the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ the representation $\tilde{A} \le \tilde{B}$ (or $\ge$) iff $\tilde{A}_\ell^x \le \tilde{B}_\ell^x$ and $\tilde{A}_r^x \le \tilde{B}_r^x$ (or $\ge$), $\forall x \in [0, 1]$. Two fuzzy numbers which satisfy the above relation are said to be ***comparable*** otherwise ***non-comparable.***

Definition 2.7. Let $\tilde{A} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ and $\tilde{B} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ be two TrFNs then,

$\tilde{A} + \tilde{B} = (\lambda_1 + \gamma_1, \lambda_2 + \gamma_2, \lambda_3 + \gamma_3, \lambda_4 + \gamma_4)$ is the ***sum***,

$\tilde{A} - \tilde{B} = \tilde{A} + (-\tilde{B}) = (\gamma_1 - \lambda_4, \gamma_2 - \lambda_3, \gamma_3 - \lambda_2, \gamma_4 - \lambda_1)$ is the ***difference*** where $-\tilde{B} = (-\gamma_4, -\gamma_3, -\gamma_2, -\gamma_1)$ is the ***opposite*** of $\tilde{B}$.

Definition 2.8. For any real number 'τ', the ***scalar operation*** for the TrFN $\tilde{A} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ is

$\tau + \tilde{A} = (\tau+\lambda_1, \tau+\lambda_2, \tau+\lambda_3, \tau+\lambda_4)$,

$\tau\tilde{A} = (\tau\lambda_1, \tau\lambda_2, \tau\lambda_3, \tau\lambda_4)$, if $\tau > 0$ and

$\tau\tilde{A} = (\tau\lambda_4, \tau\lambda_3, \tau\lambda_2, \tau\lambda_1)$, if $\tau < 0$.

This definition can be applied to TFNs also.

Definition2.9.

Let $\tilde{A}_i = (\lambda_{1i}, \lambda_{2i}, \lambda_{3i}, \lambda_{4i})$, $i = 1, 2, ..., n$ be TrFNs / TFNs, where $n \ge 2$. Then the mean value of $\tilde{A}_i$ is described by $\tilde{A} = \frac{1}{n}(\tilde{A}_1 + \tilde{A}_2 + \cdots + \tilde{A}_n)$. And we require the results found in [11, 13].

Result 2.10. Let $D = \{[r_1, r_2], r_1 \le r_2 \text{ and } r_1, r_2 \in \mathbb{R}\}$ be the set of all closed, bounded intervals in $\mathbb{R}$.

Result 2.11. Let $A = [p_1, p_2]$ & $B = [q_1, q_2]$ be in D.

Then A = B if $p_1 = q_1$ and $p_2 = q_2$

Now, the rank from COG of TrFN is employed for the hypothesis test.

3. RANK OF TrFNs FROM COG:

In this section, the COG of TrFNs is used for finding ranks and these ranks are considered as defuzzifying tool. Firstly, the COG of TFN is derived:

Proposition 3.1. The COG of the TFN $\tilde{A} = (p_1, p_2, p_3)$ for which the coordinates (X, Y) are given by [17],

$$X = \frac{a + b + c}{3}, \quad Y = \frac{1}{3}.$$

Proposition 3.2. Consider the TrFN .The COG of TrFN $\tilde{A} = (p_1, p_2, p_3, p_4)$ for which the coordinates (X, Y) are given by the formulae [17] $X = \dfrac{p_3^2 + p_4^2 - p_1^2 - p_2^2 + p_4 p_3 - p_2 p_1}{3(p_3 + p_4 - p_1 - p_2)}$

and $Y = \dfrac{2p_3 + p_4 - p_1 - 2p_2}{3(p_3 + p_4 - p_1 - p_2)}$

Corollary:

The rank of the TFN $\tilde{A} = (p_1, p_2, p_3)$ as well as TrFN $\tilde{A} = (p_1, p_2, p_3, p_4)$ is calculated by the method of Root Sum Square [7, 8] $R(\tilde{A}) = \sqrt{X^2 + Y^2}$ --- (*) .

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4. RESULTS & DISCUSSIONS

Example 4.1. The life length of two brands A and B of gear wheels in a machinary plant is observed. Due to some inevitable situations, the recorded data are found as TrFNs which are given below. And the manufacturer of brand A claims that the life length of A is higher than that of B. To test the claim, let us assume the observed data follow normal distribution with equal variances. We test the significance of the difference between two brands of gear wheels at 5% level of significance (los).

$\tilde{A}$ **: (4, 4.5, 5, 6), (3.5, 4, 5, 6.5), (5, 5.5, 5.8, 6), (5.5, 5.8, 6, 6.5), (3, 3.5, 4, 5)**

$\tilde{B}$ **: (5, 6.5, 7, 8), (4, 4.5, 5, 6), (5.5, 7, 8, 8.5), (5, 6, 6.5, 7), (6, 6.5, 7, 8), (6, 7.5, 8.5, 9)**

Solution:

Here, the ranks are calculated by the formula (*) of corollary of proposition-3.2 which are given below:

$\left(R_{\tilde{A}}\left(x_i\right);\ i=1, 2, ..., 5\right)$:

4.9163, 4.8098, 5.5767, 5.9738, 3.9205

$\left(R_{\tilde{B}}\left(y_j\right);\ j=1, 2, ..., 6\right)$:

6.6062, 4.9163, 7.2204, 6.1131, 6.9116, 7.7196

Null hypothesis H0: The mean life length of the two brands of gear wheels are same.

That is $H_0: \mu_1 = \mu_2$.

Alternative hypothesis HA: The mean life length of brand A is higher than that of brand B.

That is, $H_A: \mu_1 > \mu_2$.

Here, the calculated mean and variance values are

$R_{\tilde{A}}\left(\overline{x}\right) = 5.0394$, $R_{\tilde{B}}\left(\overline{y}\right) = 6.5812$; $s^2_{\tilde{A}} = 0.6204$, and $s^2_{\tilde{B}} = 0.9611$ respectively.

And the calculated value of 't' is given by

$$t = \left(R_{\tilde{A}}\left(\overline{x_i}\right) - R_{\tilde{B}}\left(\overline{y_j}\right)\right) / \left(\sqrt{\frac{s^2_{\tilde{A}}}{m} + \frac{s^2_{\tilde{B}}}{n}}\right) = -2.8918$$

From t-table, it is seen that $T_\alpha = 1.833$ at 5% los with 9 degrees of freedom. Here, $|t| > T_\alpha$ (right tailed test).

$\Rightarrow$ We reject the null hypothesis $\tilde{H}_0$.

So, the alternative hypothesis H_A is accepted.

Therefore, the manufacturer's claim that the life length of brand A is higher than that of B is true.

5. CONCLUSION

Although better decision can be arrived by using the method of COG of TrFN, it is not an ultimate solution to all problems. Nevertheless it has opened the door for the test of hypothesis involving TrFNs through their defuzzified forms such as rank from COG of TrFNs and of course it needs further refinement and research to arrive a finer result.

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