

# Linear Quadratic Regulator for three Interacting Cylindrical Tank Control

K. Anbumani, R. Rani Hemamalini

**Abstract:**In this paper, optimal control for three interacting cylindrical process was designed to control the level of process tanks. This paper describes the theoretical base and practical application of Linear Quadratic Regulatory (LQR) manage for a non-rectangular framework with controlled components and three methodology yields. as a rule, state remarks controller is expected for restricting country change with insignificant control quality. LQR is a top-quality multivariable comments system which is organized assembled totally generally as for the u .s .a .furthermore, control weighting systems. The general execution of controller is advanced by methods for proper choice of Q, R grid. The most pleasurable country remarks option is picked to transport the states to zero and integrator controller is added to transport the respect the set segment. The test outcomes admish that the made control plans gems agreeably.

**Keywords:**Three Interacting cylindrical tank Process, Linear Quadratic Regulator, State feedback control.

## I. INTRODUCTION

ideal manipulate principle is growing device in bleeding aspect method control. In ideal manage speculation, the maths assortments and headway strategies joined used to define method for controlled variable with the very last goal that cost restriction like control essentialness and nation instability is constrained [1][2][3]. at the same time as doubtful model insightful manage method is used for the most element on an increasing number of raised sum and the presentation development of decrease degree PID circles gives improved execution in multivariable structures. in any case, Linear quadratic Gaussian manage gives perfect and floor-breaking control approach to Multivariable approach [4][5]. Direct Quadratic Gaussian association is a super control speculation which has various application on top of things building hassle. This LQG technique by using using and large used in medical method controllers, in nuclear electricity plants and motor manipulate structures. In Linear Quadratic Regulator (LQR) version, the ensuing manipulate law are straight away with recognize to country variable. The control regulation is surely no longer hard to figure and perform well at the same time as the shape is obtainable to aggravations [6]. So the only of a kind case of controller is organized the use of department giant [7].

The paper has been dealt with as seeks after: section II rapidly depict the three accomplice cylinder fashioned tank system interplay exam and Linear Quadratic controller plan method has been stated in section III. Propagation effects

and controller execution exam are showed up and referred to in section IV. final closures are given in portion V.

## II. THREE INTERACTING CYLINDRICAL TANK PROCESS DESCRIPTION

three spherical and empty tanks with identical skip-sectional place (A) related with channels of same go-sectional vicinity (an) is taken into consideration. the two tanks are related via a working collectively pipe with valves (HV1 and HV2). The correspondence of method can be changed by way of position of this valve (HV1 and HV2). Hand Valves, HV1 and HV2 are used to change the state of affairs of the valve once steady the position need to no longer be modified. changing the area will bring about unique components thusly undertaking controller disillusionment. It has an archive to preserve water and that is given through the directs to the tanks through manage valves. sport plans for water influx and flood are given at the pinnacle and base of the tank independently. Portal valves put on the overflowing of the tank1, 2 and 3 are related to hold up the component of water within the tanks. Differential pressure transmitter (DPT) is used for assessing the stature of the tanks. DPT measures the base weight made by using the use of water degree and it offers stature to the extent milliamps. The objective of this proposed paintings is to govern the dimension in all of the 3 tanks via fluctuating the inflow  $F_{in1}$  and  $F_{in2}$  of the number one and the 1/3 tank and along those strains retaining up the stature of the tanks. Schematic chart of 3 running collectively tank technique is confirmed up in determine 1.

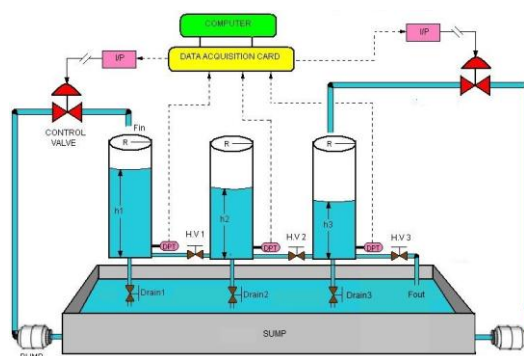


Figure 2.1: Three Interacting Tank System with two Inputs

Hence the differential equations describing the system are given as,

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# LINEAR QUADRATIC REGULATOR FOR THREE INTERACTING CYLINDRICAL TANK CONTROL

$$\frac{dh_1(t)}{dt} = -\text{sgn}(h_1 - h_2) \frac{\beta_{12}\alpha_{12}}{A_1} \sqrt{2g(h_1(t) - h_2(t))} + \frac{K_1}{A_1} u_1 \beta_1 \alpha_1 \sqrt{2g(h_1(t))} - \frac{\beta_1 \alpha_1}{A_1} \sqrt{2g(h_1(t))} \quad (1)$$

$$\frac{dh_2(t)}{dt} = \text{sign}(h_1 - h_2) \frac{\beta_{12}\alpha_{12}}{A_2} \sqrt{2g(h_1(t) - h_2(t))} - \text{sign}(h_2 - h_3) \frac{\beta_{23}\alpha_{23}}{A_2} \sqrt{2g(h_2(t) - h_3(t))} - \frac{\beta_2 \alpha_2}{A_2} \sqrt{2g(h_2(t))} \quad (2)$$

$$\frac{dh_3(t)}{dt} = \text{sign}(h_2 - h_3) \frac{\beta_{23}\alpha_{23}}{A_3} \sqrt{2g(h_2(t) - h_3(t))} - \frac{\beta_3 \alpha_3}{A_3} \sqrt{2gh_3(t)} + \frac{K_2}{A_3} u_2 \quad (3)$$

- $h_i$  Level of tank i (cm)
- $u_i$  Control input to control valve  $cv_i$  (v)
- $A_i$  Area of tank i (cm)
- $\alpha_i$  Cross section area of pipe connecting tank i (cm<sup>2</sup>)
- $\beta_i$  Valve co-efficient of between tank i
- $\beta_{ij}$  Valve ratio between tank i and tank j.
- $K_i$  Gain of valve  $cv_i$  (cm<sup>3</sup>/vs)
- $g$  Gravity

Numerical models of machine were advanced for abundance reasons. they may be attempted to help inside the comprehension of preliminary data, to are sitting tight for the effects of modifications of system data or working condition, to assemble most profitable structure or walking

conditions and for supervise limits. in any case the essential issue in showing is the gadget components ought to be gotten commonly little need in exhibiting the technique. The dynamic model of the methodology has been gotten from the item program of fundamental physical and substance essentials to the system, using a conventional logical showing procedure. in view of the proximity of non-direct terms inside the conditions, the conditions ought to at first be linearized. The country space recognition with the machine is essentially more unmistakable perfect than a few different affirmation, on account of the over the top participation settled on the subsystems (i.e., tanks).

The process parameters are given in Table 1.

**Table 1: System Parameters**

$A_1, A_2, A_3$ (cm <sup>2</sup> )	$\alpha_1, \alpha_2, \alpha_3$ (cm <sup>2</sup> )	$\beta_{12}$	$\beta_{23}$	$\beta_1, \beta_2, \beta_3$	$K_1, K_2$ (cm <sup>3</sup> /vs)
615.7522	5.0671	0.9	0.8	0.3	75

The open loop facts have become generated in the three interacting cylindrical tank device with the aid of various the influx charge via V1 and V2 open loop records generated. running factors found out from the enter output traits. utilizing Jacobian function for the differential first precept version, kingdom vicinity version is obtained.

**TABLE 2. OPERATING CONDITIONS AND THE CONVENTIONAL STATE SPACE AND TRANSFER FUNCTION MODEL OF THE THREE INTERACTING CYLINDRICAL TANK PROCESS**

Operating points	
$V_{1s}=5.5, V_{2s}=5.5, h_{1s}= 16.8, h_{2s}=16.34, h_{3s}=16.92$	
State space model	
$A = \begin{bmatrix} -0.2551 & 0.2418 & 0 \\ 0.2418 & -0.4467 & 0.1914 \\ 0 & 0.1914 & -0.2047 \end{bmatrix}$	
$B = \begin{bmatrix} 0.1218 & 0 \\ 0 & 0 \\ 0 & 0.1218 \end{bmatrix}$	
$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	
Transfer Function Matrix G(s)	
$\begin{bmatrix} \frac{0.1218 s^2 + 0.07935 s + 0.0066706}{s^3 + 0.9066 s^2 + 0.1626 s + 0.002016} & \frac{0.005638}{s^3 + 0.9066 s^2 + 0.1626 s + 0.002016} \\ \frac{0.02945 s + 0.006029}{s^3 + 0.9066 s^2 + 0.1626 s + 0.002016} & \frac{0.02331 s + 0.005949}{s^3 + 0.9066 s^2 + 0.1626 s + 0.002016} \\ \frac{0.005638}{s^3 + 0.9066 s^2 + 0.1626 s + 0.002016} & \frac{0.1218 s^2 + 0.08549 s + 0.006762}{s^3 + 0.9066 s^2 + 0.1626 s + 0.002016} \end{bmatrix}$	

### III. OPTIMAL LINEAR QUADRATIC REGULATORY CONTROL DESIGN

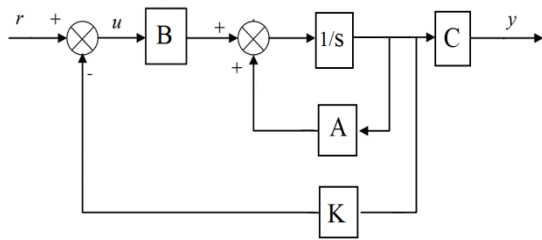


Fig .2 State feedback control scheme

nation can not be completely measurable in actual time tool, so the states are predicted based totally on the output. This regulator has country-vicinity equation is

$$\dot{\hat{x}} = A \hat{x} + B u \quad (4)$$

Estate estimated error  $e = x - \hat{x}$ ,

Dynamic equation of error is

$$\dot{e} = A x - A \hat{x} = A e \quad (5)$$

The error goes to zero asymptotically when the state matrix A is stable. If the state matrix A is unstable then error become uncontrollable and estimated states  $\hat{x}$  grows further apart from process state x.

Steps for Designing Linear Quadratic Gaussian Regulatory,

- i. Check for controllability and observability of the system.
- ii. Choose Q and R such that  $Q = M^T M$ , with (A, M) detectable, and  $R = R^T > 0$
- iii. This equation is the matrix algebraic Riccati equation (MARE), whose solution P is needed to compute the optimal feedback gain K.
- iv. Solve the Riccati equation  $PA + A^T P + Q - PBR^{-1}B^T P = 0$ , And compute  $K = R^{-1}B^T P$ , Simulate the initial response of  $\dot{x} = (A + BF)x$  for different initial conditions.
- v. If the transient response specifications and/or the magnitude constraints are not met, and again step 1 is considered to re-choose the value of Q and R.

The controllability of matrix is based upon on A and B matrix and observability of device is based upon on matrix A and C matrix. The device is said to be truly observable if all the states of device may be externally measured.

A system is controllable if condition W is satisfied

$$W: \text{rank} [B \ AB \ \dots \ A^{n-1}B] = n \quad (6)$$

And  $\text{rank}(W) = \text{Order}(A)$ , the system satisfies the condition W, therefore the system is controllable.

$$C_o = \begin{bmatrix} 0.1218 & 0 & -0.0311 & 0 & 0.0151 & 0.0056 \\ 0 & 0 & 0.0295 & 0.0233 & -0.0207 & -0.0152 \\ 0 & 0.1218 & 0 & -0.0249 & 0.0056 & 0.0096 \end{bmatrix} \quad (7)$$

$$\text{States uncontrollable} = \text{Order}(A) - \text{rank}(C_o) = 0 \quad (8)$$

Here in this case, number of uncontrollable states is 0 indicating that all the states are controllable. Therefore, the design of LQR controller is possible. The performance of LQR control is measured by performance index,

$$J(u) = \int_0^{\infty} \{x^T Q x + u^T R u\} dt \quad (9)$$

immaterial estimations of J(u) addresses unimportant effect of controller control 'u' and minimization of country variable changes in system. the decision of Q and R system is reliable and basic. The Q and R cross sections decided for insignificant charge of average all in all execution document j(u) in light of this that the minimization of rectangular of controlled data and minimization of square of nation factors instabilities. In MIMO technique, impacts among every data and yield elements shifts. The weightage for each controlled variable contrasts reliant on the impact of yield factors. The weighting systems Q, R are enduring for constraining the J(u). Q, R are the controller position parameters, enormous Q rebuffs vagrants of x, huge R rebuffs use of control advancement 'u'. The kingdom analysis bit of breathing space is constant at the irrelevant expense of J(u). This minimization of bit of leeway network 'adequate' find through clarifying Riccati condition [11] [12]. The controller response of this immediate quadratic controller might be by no means, affected by disturbance. at any rate the set part checking isn't achievable in LQR controller.

same weightage given to u. s.a.h1 and h2 of the tank and R system chose fundamentally reliant on the data weightage to restore US of america change and to constraining the controller quality.

$$Q_n = C^T C \quad (10)$$

$$R_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

The law of LQR controller is "u=-kx" that reduce the state fluctuation. Normally this gain is called state feedback gain or LQ-optimal gain. Control action is based on the state of process.

The optimal state feedback gain

$$K = \begin{bmatrix} 0.4896 & 0.3402 & 0.2448 \\ 0.2448 & 0.3186 & 0.5025 \end{bmatrix} \quad (12)$$

For servo tracking integrator is added and tuned by trail and method. The integrator gain  $K_i=1$  is fixed for better set point tracking.

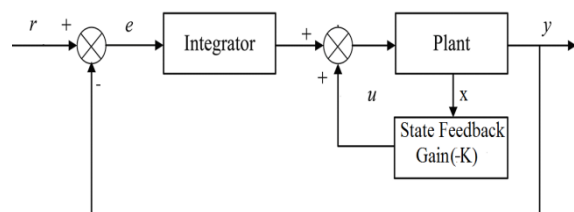


Fig.3 LQR with Integral control for servo tracking

### IV. ANALYSIS & RESULTS

Fourfold Tank framework was investigated with the resource of growing reenactment version. directly Sate space model is created via legitimate walking factor choice. The servo response of flexible LQG controller for the dimension in the tanks are appeared in parent four. figure 5



demonstrates the response underneath transferring unsettling have an effect on situations. parent 6 demonstrates the response of the tanks below servo-administrative conditions.

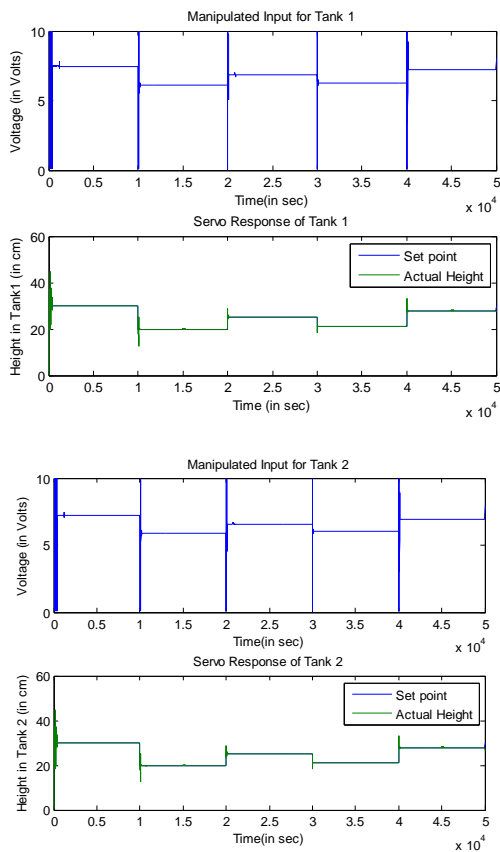


Figure 4. Servo response of the three interacting cylindrical tank process

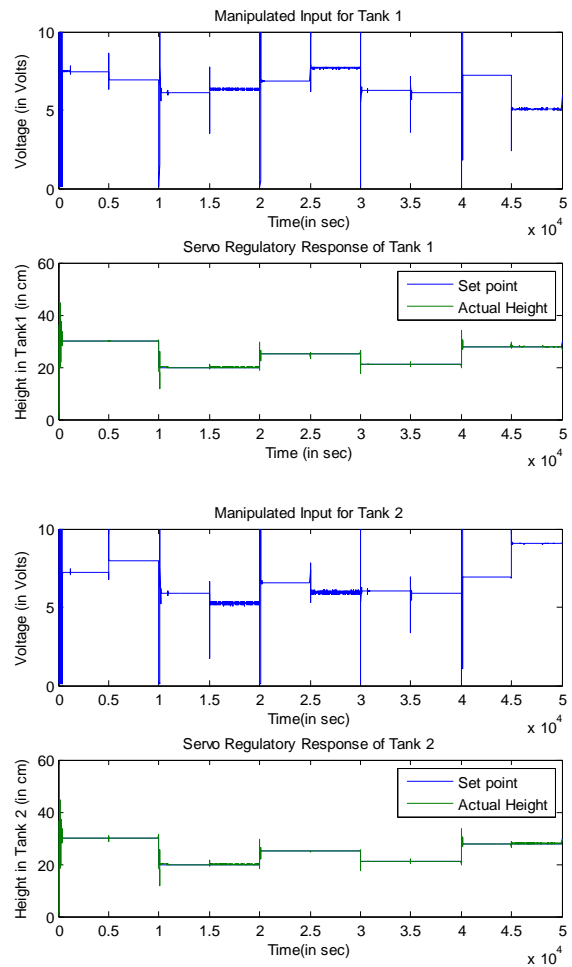


Figure 6. Servo-Regulatory response of the three interacting cylindrical tank process

V. CONCLUSION

A Linear Quadratic regulator (LQR) control design for three interacting cylindrical tank system has been investigated to achieve optimal control. The (LQR) designed by minimizing a typical all things considered execution rule reliant on comfort and controller power issues; be that as it may, the objective of this paper has been to examine the utilization of essential country remarks controllers in a non-square contraption. The integrator is passed on for servo after. there may be no expansive strategy to be had for choice of Q and R. straightforwardly here, basically reliant on the impact of states on way yield, the Q and R system picked. The disrupting impact expulsion of LQR controller is sure. The reenactment results exhibits that the method followed in this investigation work offers higher controller ordinary execution.

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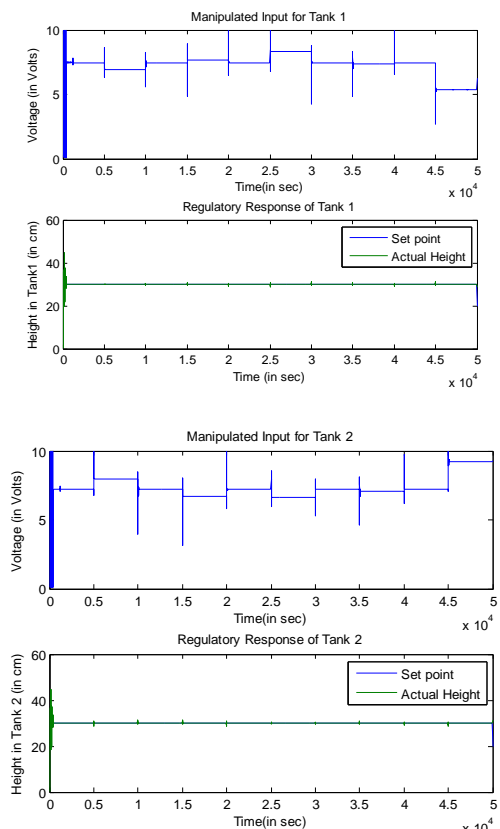


Figure 5. Regulatory response of the three interacting cylindrical tank process

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