A Discussion of FSB - Hausdorff Property on FS- Cartesian Product Topological Space

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Abstract: ***For anynonempty family*** $\{(\mathcal{B}_i, \mathfrak{T}_i)\}$ ***of FsB-HausdorffSpaces. The Fs-Cartesianproduct topological space is also an FsB-Hausdorff Space.***

Index Terms: ***Fs-Set, Fs-Subset, (b,β) object, Fs-Point, FsB-Toplogical Space.***

I. INTRODUCTION

Axiom choice is not true in the theory of L-Fuzzy sets .Nistla V.E.S Murthy [10] proved Axiom Choice of fuzzy sets in his theory of F-sets. VaddiparthiYogeswara[2] etc … developed the theory of Fs-sets with the goal of introducing the complement of a fuzzy set which was not satisfactorily explained by previous relevant theories .Also VaddiparthiYogeswara, BiswaiitRath ,Ch.RamaSanyaasiRao ,K.V.UmaKameswari, D.Raghu Ram introduced the concept of FsB-topological Space on a given Fs –subset of an Fs-set and also they introduced FsB-subspace in the same paper .Fs-points and Fs-point set FSP($\mathcal{W}$) are introduced by VaddiparthiYogeswara etc…[2] and based on Fs-set theory they defined a pair of relations between P(FSP($\mathcal{W}$))and $\mathcal{L}(\mathcal{W})$. Here FSP($\mathcal{W}$) stands for Fs-Point set of $\mathcal{W}$,$\mathcal{L}(\mathcal{W})$ stands for collection of allFs-subsets of $\mathcal{W}$ and P(FSP($\mathcal{W}$)) is power set of FSP($\mathcal{W}$) and proved one of them is a '$\wedge$'- complete homomorphism and other is '$\vee$'- complete homomorphism and searched some properties of these relations between complemented constructed crisp sets and Fs-complemented sets through these homomorphism and ultimately they proved a representation theorem connecting Fs-subsets of $\mathcal{A}$ to crisp subsets of FSP($\mathcal{A}$) via homomorphisms. In this paper we introduce the concepts of T_1- Space and Haudorff Space on an Fs-B topological Space via these representation theorems and we give an example . For a given non-empty family of compact Fs-toplogicalspaces , we prove in this paper their Fs-Cartesian Product space is also compact. Fs-Sets, Fs- Set functions etc… in brief are explained in first four sections of this paper. '⊔' and '⊓' sands for natural set union and Fs-union and Similarly '⊓'. M_A or 1_A sands for largest element of a given complete Boolean Algebra L_A. For all lattice theoretic and relevant Properties one can refer [5],[8],[15],[16],[17].

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SECTION-1

1.1 **Fs-set:** A four tuple of the form $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$ is an Fs-set iff, $W \sqsubseteq W_1 \sqsubseteq U$

(1) L_W is a complete Boolean Algebra

(2) $\mu_{1W_1}: W_1 \to L_W$,$\mu_{2W}: W \to L_W$ are mappings such that

(3) $\overline{W}$: W→ L_W is defined by

$\overline{W}x = \mu_{1W_1}x \wedge (\mu_{2W}x)^c$ for each $x \in W$

Where W is a non-void subset of some universal set U

1.2 Fs-subset: Suppose $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$ and $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$ are two Fs-sets.

We say $\mathcal{U}$ is anFs-subset of $\mathcal{W}$, in symbol,

We write $\mathcal{U} \sqsubseteq \mathcal{W}$, iff

(1) $U_1 \sqsubseteq W_1$, $W \sqsubseteq U$

(2) L_U is a complete subalgebra of L_W or $L_U \leq L_W$

(3) $\mu_{1U_1} \leq \mu_{1W_1}|U_1$, and $\mu_{2U}|W \geq \mu_{2W}$

1.2 **Fs-union :**Let$\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V) \sqsubseteq \mathcal{W}$.

1.3 Then,

$\mathcal{U} \sqcup \mathcal{V} = \mathcal{P} = (P_1, P, \overline{P}(\mu_{1P_1}, \mu_{2P}), L_P)$,where

(1) $P_1 = U_1 \sqcup V_1$, $P = U \sqcap V$

(2) $L_P = L_U \curlyvee L_V$ = The complete subalgebra generated

(3) y $L_U \sqcup L_V$

(4) $\mu_{1P_1}: P_1 \to L_P$ is defined by

$$\mu_{1P_1}x = (\mu_{1U_1} \curlyvee \mu_{1V_1})x$$

$\mu_{2P}: P \to L_P$is defined by

$\mu_{2P}x = \mu_{2U}x \wedge$
$\mu_{2V}x$ and

$\overline{P}: P \to L_P$ is defined by

$$\overline{P}x = \mu_{1P_1}x \wedge (\mu_{2P}x)^c$$

1.4Fs-intersection: Let $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$ and

$\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V) \sqsubseteq \mathcal{W}$ with the properties:

(i) $U_1 \sqcap V_1 \sqsupseteq U \sqcup V$

(ii) $\mu_{1U_1}x \wedge \mu_{1V_1}x \geq (\mu_{2U} \vee \mu_{2V})x$ for each $x \in W$ said Then,

$\mathcal{U} \sqcap \mathcal{V} = \mathcal{Q} = (Q_1, Q, \overline{Q}(\mu_{1Q_1}, \mu_{2Q}), L_Q)$, where

(1) $Q_1 = U_1 \sqcap V_1$, $Q = U \sqcup V$

(2) $L_Q = L_U \wedge L_V = L_U \sqcap L_V$

(3) $\mu_{1Q_1}: Q_1 \to L_Q$ is defined by

$$\mu_{1Q_1}x = \mu_{1U_1}x \wedge \mu_{1V_1}x$$

$\mu_{2Q}: Q \to L_Q$ is defined by

$$\mu_{2Q}x = (\mu_{2U} \vee \mu_{2V})x$$

$\overline{Q}: Q \to L_Q$ is defined by

$$\overline{Q}x = \mu_{1Q_1}x \wedge (\mu_{2Q}x)^c.$$

SECTION-2

2.1 FsB-ToplogicalSpace : Suppose $\mu_{1w_1} = 1, \mu_{2w} = 0$ in $\mathcal{W}$. $\mathfrak{T} \subseteq \mathcal{L}(\mathcal{W})$ is said to be FsB-toplogy if, and only if

1) $(\mathcal{V}_i)_{i\in I} \subseteq \mathfrak{T} \Rightarrow \bigcup_{i\in I}\mathcal{V}_i \in \mathfrak{T}$

2) $(\mathcal{V}_i)_{i\in I}$, I is finite set $\Rightarrow \bigcap_{i\in I}\mathcal{V}_i \in \mathfrak{T}$.

The pair $(\mathcal{A}, \mathcal{T})$ is called an FsB-topological space. Elements of $\mathfrak{T}$ are called FsB-open ses or FsB-open subset of $\mathcal{A}$.

SECTION-3

$(\mathbf{b}, \boldsymbol{\beta})$- Obiect

3.1Definition Let $b \in A, \beta \in L_A$ such that $\beta \leq \overline{A}b$. we define a (b,β)-obiect, denoted by (b,β) itself as follows

for $A \subseteq B \subseteq B_1 \subseteq A_1, L_B \leq L_A$, such that $\mu_{1B_1}x, \mu_{2B}x \in L_B(b,\beta) = (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$

$$\mu_{1B_1}x = \begin{cases} \mu_{2A}x, & x \neq b, x \in A \\ \beta \vee \mu_{2A}b, & x = b \\ \alpha, & x \notin A, x \in A_1 \end{cases} \quad \text{And } \mu_{2B}x = \begin{cases} \mu_{2A}x, & x \in A \\ \alpha, & x \notin A, x \in B \end{cases}$$

Here $\alpha \in L_A$ is fixed and $\alpha \leq \mu_{1A_1}x, \forall x \in A_1$

3.2 $\mathbf{R}(\mathbf{b}, \boldsymbol{\beta})$Relation: For any (b,β)obiects $\mathcal{V}_1 = (V_{11}, V_1, \overline{V}_1(\mu_{1V_{11}}, \mu_{2V_1}), L_{V_1})$ and $\mathcal{V}_2 = (V_{12}, V_2, \overline{V}_2(\mu_{1V_{12}}, \mu_{2V_2}), L_{V_2})$ of $\mathcal{W}$, we say that $\mathcal{V}_1 R(b,\beta)\mathcal{V}_2$ if, and only if $\mu_{1V_{11}}x = \mu_{2V_1}x, x \neq b$ and $\forall x \in V_1$ and $\mu_{1V_{12}}x = \mu_{2V_2}x, x \neq b$ and $\forall x \in V_2$ and $\mu_{1V_{11}}b = \mu_{1V_{12}}b = \beta \vee \mu_{2A}b$ and $\mu_{2V_1}b = \mu_{2V_2}b = \mu_{2W}b$.

We can easily show that $R(b,\beta)$ is an equivalence relation

3.3. Fs-point : The equivalence class corresponding to (b,β) is denoted by χ_b^β or (b,β). We define this χ_b^β is an Fs- point of $\mathcal{A}$.Set of all Fs-point of $\mathcal{A}$ is denoted by $FSP(\mathcal{W})$.

SECTION-4

4.0 Definition: An Fs-B topological space $(\mathcal{B}, \mathfrak{T})$ is to be T_1-Space iff $\{\chi_b^\beta\}$ is closed for any $\chi_b^\beta \in \mathcal{A}^\sim$.

4.1 Definition : An Fs-B topological space $(\mathcal{B}, \mathfrak{T})$ is said to be T_2-Space or FsB- Hausdroff space or simply Hausdroff space iff $\chi_{a_1}^{\alpha_1}, \chi_{a_2}^{\alpha_2} \in \mathcal{A}^\sim$ such that $\chi_{a_1}^{\alpha_1} \neq \chi_{a_2}^{\alpha_2}$ then there exists a pair of disiointFs-open sets $\mathcal{G}_1$ and $\mathcal{G}_2$ such that $\chi_{a_1}^{\alpha_1} \in \mathcal{G}_1^\sim$, $\chi_{a_2}^{\alpha_2} \in \mathcal{G}_2^\sim$.

SECTION-5

5.1 Definition of**Fs**-Cartesian Product$\prod_{j\in\mathbb{J}}\mathcal{A}_j$ of the family $(\mathcal{A}_j)_{j\in\mathbb{J}}$ can be referred from [2] and also note for each $\prod_{j\in\mathbb{J}}\mathcal{A}_j = \mathcal{X} = \mathcal{A}^I$ where $\mathcal{A}_j = \mathcal{A}, \forall j \in \mathbb{J}$

5.2 Theorem: $\prod_{j\in\mathbb{J}}\mathcal{H}_j \cap \prod_{j\in\mathbb{J}}\mathcal{K}_j = \prod_{j\in\mathbb{J}}(\mathcal{H}_j \cap \mathcal{K}_j)$

Proof: $\prod_{j\in\mathbb{J}}\mathcal{H}_j = \mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, where

$U_1 = \prod_{j\in\mathbb{J}}H_{1j} \ni \left(\prod_{j\in\mathbb{J}}H_{1j}, (P_{1j})_{j\in\mathbb{J}}\right)$is the product of $(H_{1j})_{j\in\mathbb{J}}$

$U = \prod_{j\in\mathbb{J}}H_j \ni \left(\prod_{j\in\mathbb{J}}H_j, (P_j)_{j\in\mathbb{J}}\right)$is the product of $(H_j)_{j\in\mathbb{J}}$

$L_U = \prod_{j\in\mathbb{J}}L_{H_j} \ni \left(\prod_{j\in\mathbb{J}}L_{H_j}, (\pi_j)_{j\in\mathbb{J}}\right)$is the product of $(L_{H_j})_{j\in\mathbb{J}}$

$\mu_{1U_1} = \prod_{j\in\mathbb{J}}\mu_{1H_{1j}}: \prod_{j\in\mathbb{J}}H_{1j} \longrightarrow \prod_{j\in I}L_{H_j}$

$$(a_j)_{j\in\mathbb{J}} \mapsto \left(\mu_{1H_{1j}}P_{1j}(a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{1H_{1j}}a_j\right)_{j\in\mathbb{J}}$$

$\mu_{2U} = \prod_{j\in\mathbb{J}}\mu_{2H_j}: \prod_{j\in\mathbb{J}}H_j \longrightarrow \prod_{j\in\mathbb{J}}L_{H_j}$

$$(a_j)_{j\in\mathbb{J}} \mapsto \left(\mu_{2H_j}P_j(a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{2H_j}a_j\right)_{j\in\mathbb{J}}$$

$\overline{U} = \prod_{j\in\mathbb{J}}\overline{H_\iota}: \prod_{j\in\mathbb{J}}H \longrightarrow \prod_{j\in\mathbb{J}}L_{H_j}$

$$(a_j)_{j\in\mathbb{J}} \mapsto \left(\overline{H_j}P_j(a_j)_{j\in\mathbb{J}}\right) = \left(\overline{H_j}a_j\right)_{j\in\mathbb{J}} = \left[\mu_{1H_{1j}}a_j \wedge (\mu_{2_j}a_j)^c\right]_{j\in\mathbb{J}}$$

$\prod_{j\in\mathbb{J}}\mathcal{K}_j = \mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$,where

$V_1 = \prod_{j\in\mathbb{J}}K_{1j} \ni (\prod_{j\in\mathbb{J}}K_{1j}, (P_{1i})_{j\in\mathbb{J}})$is the product of $(K_{1i})_{j\in\mathbb{J}}$

$V = \prod_{j\in\mathbb{J}}K_j \ni \left(\prod_{j\in\mathbb{J}}K_j, (P_j)_{j\in\mathbb{J}}\right)$is the product of $(K_j)_{j\in\mathbb{J}}$

$L_V = \prod_{j\in\mathbb{J}}L_{K_j} \ni \left(\prod_{j\in\mathbb{J}}L_{K_j}, (\pi_j)_{j\in\mathbb{J}}\right)$is the product of $(L_{K_j})_{j\in\mathbb{J}}$

$\mu_{1V_1} = \prod_{j\in\mathbb{J}}\mu_{1K_{1j}}: \prod_{j\in\mathbb{J}}K_{1j} \longrightarrow \prod_{j\in\mathbb{J}}L_{K_j}$

$$(a_j)_{j\in\mathbb{J}} \mapsto \left(\mu_{1K_{1j}}P_{1j}(a_i)_{j\in\mathbb{J}}\right) = \left(\mu_{1K_{1j}}a_j\right)_{j\in\mathbb{J}}$$

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$\mu_{2V} = \prod_{j\in\mathbb{J}} \mu_{2K_j} : \prod_{j\in\mathbb{J}} K_i \longrightarrow \prod_{j\in\mathbb{J}} L_{K_i}$

$(a_i)_{j\in I} \longmapsto \left(\mu_{2K_j} P_j (a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{2K_j} a_j\right)_{j\in\mathbb{J}}$

$\bar{V} = \prod_{j\in\mathbb{J}} \overline{K_j} : \prod_{j\in\mathbb{J}} K_j \longrightarrow \prod_{j\in\mathbb{J}} L_{K_j}$

$(a_i)_{j\in\mathbb{J}} \longmapsto (\overline{K_j} P_j (a_i)_{j\in\mathbb{J}}) = (\overline{K_j} a_j)_{j\in\mathbb{J}} = \left[\mu_{1K_{1j}} a_j \wedge \left(\mu_{2K_j} a_j\right)^c\right]_{j\in\mathbb{J}}$

L.H.S= $\mathcal{U} \cap \mathcal{V} = \mathcal{Y} = (Y_1, Y, \bar{Y}(\mu_{1Y_1}, \mu_{2Y}), L_Y)$ where

$Y_1 = U_1 \cap V_1 = \prod_{j\in\mathbb{J}} H_{1j} \cap \prod_{j\in\mathbb{J}} K_{1j} = \prod_{j\in\mathbb{J}} (H_{1j} \cap K_{1j})$

$Y = U \cup V = (\prod_{j\in\mathbb{J}} H_j) \cup (\prod_{j\in\mathbb{J}} K_i) = \prod_{j\in\mathbb{J}} (H_i \cap K_i)$

$L_Y = L_U \cap L_V$

$\mu_{1Y_1} = \mu_{1U_1} \wedge \mu_{1V_1} = \left(\mu_{1H_{1j}}\right)_{j\in\mathbb{J}} \wedge \left(\mu_{1K_{1j}}\right)_{j\in\mathbb{J}}$

$= \left(\mu_{1H_{1j}} \wedge \mu_{1K_{1j}}\right)_{j\in\mathbb{J}}$

$\mu_{2Y} = \mu_{2U} \vee \mu_{2V} = \left(\mu_{2H_j}\right)_{j\in\mathbb{J}} \vee \left(\mu_{2K_j}\right)_{j\in\mathbb{J}}$

$= \left(\mu_{2H_j} \vee \mu_{2K_j}\right)_{j\in\mathbb{J}}$

$\mathcal{H}_j \cap \mathcal{K}_j = \mathcal{W}_j$

$\mathcal{W}_j = \left(W_{1j}, W_j, \overline{W}_j \left(\mu_{1W_{j1}}, \mu_{2W_j}\right), L_{W_j}\right)$

$\prod_{j\in\mathbb{J}} \mathcal{W}_j = \mathcal{X} = (X_1, X, \bar{X}(\mu_{1X_1}, \mu_{2X}), L_X)$,where

$X_1 = \prod_{j\in\mathbb{J}} W_{1j}$ such that $\left(\prod_{j\in I} W_{1j}, (P_{1j})_{j\in\mathbb{J}}\right)$is the product of $(W_{1j})_{j\in\mathbb{J}}$

$X = \prod_{j\in\mathbb{J}} W_j$ such that $\left(\prod_{j\in\mathbb{J}} W_j, (P_j)_{j\in\mathbb{J}}\right)$is the product of $(W_j)_{j\in\mathbb{J}}$

$L_X = \prod_{j\in\mathbb{J}} L_{W_j}$ such that $\left(\prod_{j\in\mathbb{J}} L_{W_j}, (\pi_j)_{j\in\mathbb{J}}\right)$is the product of $\left(L_{W_j}\right)_{j\in\mathbb{J}}$

$\mu_{1X_1} = \prod_{j\in\mathbb{J}} \mu_{1W_{1j}} : \prod_{j\in\mathbb{J}} W_{1j} \longrightarrow \prod_{j\in\mathbb{J}} L_{W_j}$

$(a_j)_{j\in\mathbb{J}} \longmapsto \left(\mu_{1W_{1j}} P_{1j} (a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{1W_{1j}} a_j\right)_{j\in\mathbb{J}}$

$\mu_{2X} = \prod_{j\in\mathbb{J}} \mu_{2W_j} : \prod_{j\in\mathbb{J}} W_j \longrightarrow \prod_{j\in\mathbb{J}} L_{W_j}$

$(a_j)_{j\in\mathbb{J}} \longmapsto \left(\mu_{2W_j} P_j (a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{2W_j} a_j\right)_{j\in\mathbb{J}}$

$\bar{X} = \prod_{j\in\mathbb{J}} \overline{W_i} : \prod_{j\in\mathbb{J}} W_j \longrightarrow \prod_{j\in\mathbb{J}} L_{W_j}$

$(a_j)_{j\in\mathbb{J}} \longmapsto \left(\overline{W_j} P_j (a_j)_{j\in\mathbb{J}}\right) = (\overline{W_j} a_j)_{j\in\mathbb{J}} = \left[\mu_{1W_{1j}} a_j \wedge \left(\mu_{2W_j} a_j\right)^c\right]_{j\in\mathbb{J}}$

So we can proved that $\prod_{j\in\mathbb{J}} \mathcal{H}_i \cap \prod_{j\in\mathbb{J}} \mathcal{K}_i = \prod_{j\in\mathbb{J}} (\mathcal{H}_j \cap \mathcal{K}_j)$

Let $\{(\mathcal{B}_j, \mathfrak{T}_j)\}$ Housdorff space

$\prod_{j\in\mathbb{J}} \mathcal{B}_j = \mathcal{C} = (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$,where

$C_1 = \prod_{j\in\mathbb{J}} B_{1j} \ni \left(\prod_{j\in\mathbb{J}} B_{1j}, (P_{1j})_{j\in\mathbb{J}}\right)$is the product of $(B_{1j})_{j\in\mathbb{J}}$

$C = \prod_{j\in\mathbb{J}} B_j \ni (\prod_{j\in\mathbb{J}} B_j, (P_\ni)_{j\in\mathbb{J}})$is the product of $(B_i)_{j\in\mathbb{J}}$

$L_C = \prod_{j\in\mathbb{J}} L_{B_j} \ni \left(\prod_{j\in\mathbb{J}} L_{B_j}, (\pi_j)_{j\in\mathbb{J}}\right)$is the product of $\left(L_{B_j}\right)_{j\in\mathbb{J}}$

$\mu_{1C_1} = \prod_{j\in\mathbb{J}} \mu_{1B_{1j}} : \prod_{j\in\mathbb{J}} B_{1j} \longrightarrow \prod_{j\in\mathbb{J}} L_{B_j}$

$(a_j)_{j\in\mathbb{J}} \longmapsto \left(\mu_{1B_{1j}} P_{1j} (a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{1B_{1j}} a_j\right)_{j\in\mathbb{J}}$

$\mu_{2C} = \prod_{j\in\mathbb{J}} \mu_{2B_j} : \prod_{j\in\mathbb{J}} B_j \longrightarrow \prod_{j\in\mathbb{J}} L_{B_j}$

$(a_j)_{j\in\mathbb{J}} \longmapsto \left(\mu_{2B_j} P_j (a_j)_{j\in\mathbb{J}}\right) = \left(\mu_{2B_j} a_j\right)_{j\in\mathbb{J}}$

$\bar{C} = \prod_{j\in\mathbb{J}} \overline{B_j} : \prod_{j\in\mathbb{J}} B_j \longrightarrow \prod_{j\in\mathbb{J}} L_{B_i}$

$(a_j)_{j\in\mathbb{J}} \longmapsto \left(\overline{B_j} P_j (a_j)_{j\in\mathbb{J}}\right) = (\overline{B_j} a_j)_{j\in\mathbb{J}} = \left[\mu_{1B_{1j}} a_j \wedge \left(\mu_{2B_j} a_j\right)^c\right]_{j\in\mathbb{J}} = (\overline{B_j})_{j\in\mathbb{J}}$

5.3 Theorem : $(\prod_{i\in I} \mathcal{B}_j)^{\sim} = \prod_{i\in I} \mathcal{B}_j^{\sim}$

$\chi_a^\alpha \in$ L.H.S $\Rightarrow \mathcal{C}^{\sim} = \{\chi_c^\gamma / c \in C, \gamma \in L_c, \gamma \leq \bar{C}c\}$

$\{\chi_c^\gamma / c = (C_i)_{i\in I} \epsilon \prod_{i\in I} B_i, \gamma = (\gamma_i)_{i\in I} \in \prod_{i\in I} L_{B_i},$

$\gamma = (\gamma_j)_{j\in I} \leq \bar{C}c = (\bar{B}_j C_j)_{j\in I} \}$

$\Leftrightarrow \chi_a^\alpha$where a $= (a_i)_{i\in I} \in \prod_{i\in I} B_i, \alpha = (\alpha_i)_{i\in I} \in \prod_{i\in I} L_{B_i}$, $\alpha = (\alpha_j)_{j\in I} \leq \bar{C}a$

$= (\bar{B}_j a_j)_{j\in I}$

$\Leftrightarrow a_j \in B_j, \alpha_j \in L_{B_j}, \alpha_j \leq \bar{B}_j a_j$

$$\Leftrightarrow \chi_{a_j}^{\alpha_j} \in \mathcal{B}_j^{\sim} \Rightarrow \left(\chi_{a_j}^{\alpha_j}\right)_{j\in I} \in \prod_{j\in I} \mathcal{B}_j^{\sim}$$

5.4 Definition: Define $\chi_a^\alpha = \left(\chi_{a_i}^{\alpha_i}\right)_{i\in I}$

where a= $(a_j)_{j\in J}, \alpha = (\alpha_j)_{j\in J}$

5.5 Theorem:

To prove Hausdroff Property of product space

Let $\chi_a^\alpha, \chi_b^\beta \in (\prod_{i\in I} \mathcal{V}_i)^{\sim} = \prod_{i\in I} \mathcal{V}_i^{\sim}$ and $\chi_a^\alpha \neq \chi_b^\beta$

$\Rightarrow \chi_{a_{j_0}}^{\alpha_{j_0}} \neq \chi_{b_{j_0}}^{\beta_{j_0}}$for at least one $j_0 \in J$

There exist a pair of disiointFs – open sets $\mathcal{G}_{j_{01}}, \mathcal{G}_{j_{02}}$ in $\mathcal{V}_j$ there exist

$\mathcal{G}_{j_{01}} \cap \mathcal{G}_{j_{02}} = \Phi_A$ and $\chi_{a_{j_0}}^{\alpha_{j_0}} \in \mathcal{G}_{j_{01}}^{\sim}, \chi_{b_{j_0}}^{\beta_{j_0}} \in \mathcal{G}_{j_{02}}^{\sim}$

Defined $\mathcal{P} = \prod_{j\in J} \mathcal{P}_j, \mathcal{P}_j = \mathcal{V}_i \forall j = j_0, \mathcal{P}_{j_0} = \mathcal{G}_{j_{01}}$,

$\mathcal{Q} = \prod_{j\in I} \mathcal{Q}_j, \mathcal{Q}_i = \mathcal{V}_i \forall i = j_0, \mathcal{Q}_{j_0} = \mathcal{G}_{j_{02}}$

Here $\mathcal{P} \cap \mathcal{Q} = \prod_{j\in I} \mathcal{P}_j \cap \prod_{j\in J} \mathcal{Q}_j$

$= \prod_{j\in I} (\mathcal{P}_j \cap \mathcal{Q}_j)$

$= \prod_{j\neq j_0} (\mathcal{P}_j \cap \mathcal{Q}_j) (\mathcal{P}_{j_0} \cap \mathcal{Q}_{j_0})$

$= \Phi_A$

And observe that $\chi_a^\alpha \in \mathcal{P} = \prod_{i\in I} \mathcal{P}_i$

$\chi_b^\beta \in \mathcal{Q} = \prod_{i\in I} \mathcal{Q}_i$

Here P ,Q are defining Fs – sub basic open sets in Product topology.

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REFERENCES

1. VaddiparthiYogeswara, G.Srinivas and BiswaiitRath ,A Theory of Fs-sets, Fs-Complements andFs-De Morgan Laws,IIARCS,Vol- 4, No. 1Sep-Oct 2013.
2. VaddiparthiYogeswara ,BiswaiitRath, Ch.RamasanyasiRao, K.V. Umakameswari, D. RaghuRamFs-Sets, Fs-Points, and A Representation Theorem, InternationalIournalof Control Theory and Applications (IICTA), Volume 10(07), 2017, pp. 159-170
3. VaddiparthiYogeswara ,Biswaiit Rath, Ch. Ramasanyasi Rao, D.Raghu Ram Some Properties of Associates of Subsets of FSP-PointsTransactions on Machine Learning and ArificialIntelligence, 2016 ,Volume-4,Issue-6.
4. Vaddiparthi Yogeswara ,Biswaiit Rath, Ch.RamasanyasiRao,K.V.Umakameswari, D.RaghuRam Fs-Setand Theory of FsB-TopologyMathematical Sciences International Research Journal, 2016 ,Volume-5,Issue-1, Page No-113-118
5. G.F.Simmons, Introduction to topology and Modern Analysis, McGraw-Hill international Book Company
6. Iames Dugundii, Topology, Universal Book Stall, Delhi.
7. George I. Klir and Bo Yuan ,Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected PaperbyLotfi A. Zadeh ,Advances in Fuzzy Systems-Applications and Theory, Vol-6,World Scientific Steven Givant• Paul Halmos, Introduction to Boolean algebras,Springer
8. I.A.Goguen ,L-Fuzzy Sets, IMAA,Vol.18, P145-174,1967.
9. Nistala V.E.S. Murthy, Is the Axiom of Choice True for Fuzzy Sets?, IFM, Vol 5(3),P495-523, 1997, U.S.A
10. VaddiparthiYogeswara, BiswaiitRathandS.V.G.Reddy, A Study of Fs-Functions andProperties of Images of Fs-Subsets Under
11. Various Fs-Functions. MS-IRI, Vol-3,Issue-1
12. VaddiparthiYogeswara, BiswaiitRath, Ch.RamaSanyasiRao,K.V.UmaKameswariGeneralized Definition of Image of an Fs-Subset under an Fs-function-Resultant Properties ofImagesMathematical Sciences International Research Iournal,2015,Volume -4, Spl Issue, 40-56
13. L.Zadeh, Fuzzy Sets, Information and Control,Vol.8,P338-353,1965
14. Nistala V.E.S. Murthy, f-Topological Spaces Proceedings of The National Seminar on Topology, Category Theory and their applications to Computer Science, P89-119, March 11-13, 2004, Department of Mathematics, St Ioseph's College, Irinialaguda, Kerala (organized by the Kerala Mathematical Society. Invited Talk).
15. Szasz, G., An Introduction to Lattice Theory, Academic Press, New York.
16. Garret Birkhoff, Lattice Theory, American Mathematical Society Colloquium publicationsVolume-xxv
17. Thomas Iech ,Set Theory, The Third Millennium Edition revised and expanded, Springer

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