A Discussion of FSB - Hausdorff Property on 
FS- Cartesian Product Topological Space

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Abstract: For any nonempty family \( \{(B_{i}, \tau_{i})\} \) of FSB-Hausdorff Spaces. The FSB-Cartesian product topological space is also an FSB-Hausdorff Space.

Index Terms: FSB-Set, FSB-Subset, (b, β) object, FSB-Point, FSB-Topological Space.

I. INTRODUCTION

Axiom choice is not true in the theory of L-Fuzzy sets. Nistla V.E.S Murthy [10] proved Axiom Choice of fuzzy sets in his theory of F-sets. Vaddiparthi Yogeswara [2] etc... developed the theory of F-sets with the goal of introducing the complement of a fuzzy set which was not satisfactorily explained by previous relevant theories. Also, Vaddiparthi Yogeswara, Biswail Rath, Ch. Rama Sanyasi Rao, K.V. Uma Kameswari, D. Raghu Ram introduced the concept of FSB-topological Space on a given F-set of an F-set and also they introduced FSB-subspace in the same paper. F-points and F-point set FSP(\( \mathcal{W} \)) are introduced by that Vaddiparthi Yogeswara etc... [2] and based on F-set theory they defined a pair of relations between P(FSP(\( \mathcal{W} \))) and \( \mathcal{L}(\mathcal{W}) \). Here FSP(\( \mathcal{W} \)) stands for F-Point set of \( \mathcal{W} \) and \( \mathcal{F}(\mathcal{W}) \) stands for collection of all FPs of \( \mathcal{W} \) and P(FSP(\( \mathcal{W} \))) is power set of FSP(\( \mathcal{W} \)) and proved one of them as an 'A' complete homomorphism and other is 'V'-complete homomorphism and searched some properties of these relations between complemented constructed crisp sets and FS-complemented sets through these homomorphism and ultimately they proved a representation theorem connecting FSB-subsets of \( \mathcal{A} \) to crisp subsets of FSP(\( \mathcal{A} \)) via homomorphisms. In this paper we introduce the concepts of \( T_{1} \)-Space and Hausdorff Space on an FSB-topological Space via these representation theorems and we give an example. For a given non-empty family of compact FS-topologyspaces, we prove in this paper their FS-Cartesian Product space is also compact. F-Set, F-set functions etc... in brief are explained in first four sections of this paper. 'U' and '\( \cap \)' stands for natural set union and F-union and Similarly ' \( \cap \)' \( M_{A} \) and \( L_{A} \) stands for largest element of a given complete Boolean Algebra \( L_{A} \). For all lattice theoretic and relevant Properties one can refer [5],[8],[15],[16],[17].

SECTION 1

1.1 FSB-set: A four tuple of the form \( \mathcal{W} = (W_{1}, W, \overline{W}, (\mu_{W_{1}}, \mu_{W_{2}}), L_{W}) \) is an FSB-set iff, \( W \subseteq W_{1} \subseteq U \)

(1) \( L_{W} \) is a complete Boolean Algebra

(2) \( \mu_{W_{1}}: W_{1} \rightarrow L_{W}, \mu_{W_{2}}: W \rightarrow L_{W} \) are mappings such

(3) \( \overline{W}: W \rightarrow L_{W} \) is defined by \( \overline{W}x = \mu_{W_{1}}x \land (\mu_{W_{2}}x)^{c} \) for each \( x \in W \)

Where \( W \) is a non-void subset of some universal set \( U \)

1.2 FSB-subset: Suppose \( \mathcal{W} = (W_{1}, W, \overline{W}, (\mu_{W_{1}}, \mu_{W_{2}}), L_{W}) \) and \( \mathcal{U} = (U_{1}, U, \overline{U}, (\mu_{U_{1}}, \mu_{U_{2}}), L_{U}) \) are two FSB-sets.

We say \( \mathcal{U} \) is an FSB-subset of \( \mathcal{W} \), in symbol, We write \( \mathcal{U} \subseteq \mathcal{W} \), iff

(1) \( U_{1} \subseteq W_{1}, W \subseteq U \)

(2) \( L_{U} \) is a complete subalgebra of \( L_{W} \) or \( L_{U} \leq L_{W} \)

(3) \( \mu_{U_{1}} \leq \mu_{W_{1}}|U_{1} \), and \( \mu_{U_{2}}|W \geq \mu_{W_{2}} \)

1.2 FSB-union: Let \( \mathcal{U} = (U_{1}, U, \overline{U}, (\mu_{U_{1}}, \mu_{U_{2}}), L_{U}) \), \( \mathcal{V} = (V_{1}, V, \overline{V}, (\mu_{V_{1}}, \mu_{V_{2}}), L_{V}) \subseteq \mathcal{W} \).

Then, \( \mathcal{U} \cup \mathcal{V} = \mathcal{P} = (P, \overline{P}, (\mu_{P_{1}}, \mu_{P_{2}}), L_{P}) \), where

(1) \( P_{1} = U_{1} \cup V_{1} \), \( P = U \cup V \)

(2) \( L_{P} = L_{U} \cup L_{V} = \text{The complete subalgebra generated} \)

(3) \( \mu_{U_{1}} \cup \mu_{V_{1}} \)

(4) \( \mu_{P_{1}}: P_{1} \rightarrow L_{P} \) is defined by \( \mu_{P_{1}}x = (\mu_{U_{1}}x \lor \mu_{V_{1}}x) \)

(5) \( \mu_{P_{2}}: P \rightarrow L_{P} \) is defined by \( \mu_{P_{2}}x = \mu_{U_{2}}x \land \mu_{V_{2}}x \land \)}
1.4Fs-intersection: Let \( U = (U_1, U_2, U_3, U_4, U_5, U_6) \) and 
\( \mathcal{V} = (V_1, V_2, V_3, V_4, V_5, V_6) \) be elements of \( \mathcal{W} \) with the properties:
(i) \( U_1 \cap V_1 = U \cup V \)
(ii) If \( \mu_1, \mu_2 \geq 0 \), then for each \( x \in W \), \( \mu_1 \leq \mu_2 \).

Then, \( U \cap \mathcal{V} = Q = (Q_1, Q_2, Q_3, Q_4, Q_5, Q_6) \), where
(1) \( Q_1 = U_1 \cap V_1 \), \( Q = U \cup V \)
(2) \( L_1 = L_1 \cap L_2 = L_1 \cap L_3 \)
(3) \( \mu_1 Q_1 : Q_1 \rightarrow L_1 \) is defined by
\[ \mu_1 Q_1 x = \mu_1 U x \]
\( \mu_2 Q : Q \rightarrow L_1 \) is defined by
\[ \mu_2 Q x = (\mu_2 U Y \mu_2 V) x \]
\( Q : Q \rightarrow L_1 \) is defined by
\[ Q x = \mu_1 Q x \times (\mu_2 Q x) = (\mu_2 U Y \mu_2 V) x \]

3.3. Fs-point: The equivalence class corresponding to \((b, \beta)\) is denoted by \( \chi_b^\beta \) or \((b, \beta)\).
We define this \( \chi_b^\beta \) as an Fs-point of \( A \). Set of all Fs-points of \( A \) is denoted by FSP(\( W \)).

SECTION-4

4.0 Definition: An Fs-B topological space \((B, \mathfrak{X})\) is to be a T1-Space iff \( x_0^b \) is closed for any \( x_0^b \) \( \in A \).

SECTION-5

5.1 Definition of Fs-Cartesian Product: Let the family \( \{A_j\}_{j \in J} \) be defined by 
\( (A_j)_{j \in J} \) and also note for each \( \Pi_{j \in J} A_j = X = A^J \) where \( A_j = A, \forall j \in J \)

5.2 Theorem: \( \Pi_{j \in J} X_j \cap \Pi_{j \in J} X_j = \Pi_{j \in J} (X_j \cap X_j) \)

Proof: \( \Pi_{j \in J} X_j = \Pi_{j \in J} (X_j \cap X_j) \) is the product of \( (H_j)_{j \in J} \)
\[ U = \Pi_{j \in J} H_j \exists (\Pi_{j \in J} (P_j)_{j \in J}) \]
\[ L_U = \Pi_{j \in J} L_{H_j} \exists (\Pi_{j \in J} (\pi_j)_{j \in J}) \]
\[ (b, \beta) - Object \]

3.1 Definition: Let \( b, \beta \in L_A \) such that \( \beta \neq \beta \).
We define a \((b, \beta)\)-object, defined by \( (b, \beta) \) itself as follows:
\[ \mu_{1(b, \beta)} = \mu_{1(b, \beta)} \times \mu_{2(b, \beta)} \]
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\[ \mu_{1(b, \beta)} = \mu_{1(b, \beta)} \times \mu_{2(b, \beta)} \]

3.2 Relation: For any \((b, \beta)\)-object \( V_1 = (V_1, V_2, V_3, V_4, V_5, V_6) \) and \( V_2 = (V_1, V_2, V_3, V_4, V_5, V_6) \) of \( W \), we say that \( V_1 \triangleq V_2 \) if and only if \( \mu_{1(b, \beta)} = \mu_{2(b, \beta)} \).

We can easily show that \( R(b, \beta) \) is an equivalence relation.
\[ \mu_{2V} = \prod_{j \in \mathcal{E}} \mu_{2K,j} \prod_{j \in \mathcal{E}} K_i \rightarrow \prod_{j \in \mathcal{E}} L_{K_i} \]

\[ (a_{ij})_{j\in\mathcal{E}} \rightarrow (\mu_{2K,j} p_j(a_{ij}))_{j\in\mathcal{E}} = (\mu_{2K,j} a_{ij})_{j\in\mathcal{E}} \]

\[ \tilde{v} = \prod_{j \in \mathcal{E}} K_j \rightarrow \prod_{j \in \mathcal{E}} L_j \]

\[ (a_{ij})_{j\in\mathcal{E}} \rightarrow (K_j p_j(a_{ij}))_{j\in\mathcal{E}} = (\tilde{\kappa}_j a_{ij})_{j\in\mathcal{E}} = \mu_{1K,j} a_{ij} \wedge (\mu_{2K,j} a_{ij})_{j\in\mathcal{E}} \]

**L.H.S.** \[ \cup \cap \mathcal{V} = (Y_1, Y_2, Y(\mu_{1V}, \mu_{2Y}), L_Y) \]

where \[ Y_1 = U (V_1 = \prod_{j \in \mathcal{E}} H_j \cap \prod_{j \in \mathcal{E}} K_j) = \prod_{j \in \mathcal{E}} (H_j \cap K_j) \]

\[ Y = U (V_1 = \prod_{j \in \mathcal{E}} H_j) \cup (\prod_{j \in \mathcal{E}} K_j) = \prod_{j \in \mathcal{E}} (H_j \cap K_j) \]

\[ L_Y = L_{\mathcal{U}} \cap L_Y \]

\[ \mu_{1V} = \mu_{1V} \wedge \mu_{1V} = (\mu_{1H,j})_{j\in\mathcal{E}} \wedge (\mu_{1K,j})_{j\in\mathcal{E}} \]

\[ \mu_{2V} = \mu_{2V} \vee \mu_{2V} = (\mu_{2H,j})_{j\in\mathcal{E}} \vee (\mu_{2K,j})_{j\in\mathcal{E}} \]

So we can proved that \[ (a_{ij})_{j\in\mathcal{E}} \rightarrow (\mu_{1V,j} p_j(a_{ij}))_{j\in\mathcal{E}} = (\mu_{1V,j} a_{ij})_{j\in\mathcal{E}} \]

\[ \mu_{2V} = \prod_{j \in \mathcal{E}} \mu_{2B,j} \rightarrow \prod_{j \in \mathcal{E}} L_{B_j} \]

\[ (a_{ij})_{j\in\mathcal{E}} \rightarrow (\mu_{2B,j} p_j(a_{ij}))_{j\in\mathcal{E}} = (\mu_{2B,j} a_{ij})_{j\in\mathcal{E}} \]

\[ \tilde{c} = \prod_{j \in \mathcal{E}} \tilde{B}_j \rightarrow \prod_{j \in \mathcal{E}} L_{B_j} \]

\[ (a_{ij})_{j\in\mathcal{E}} \rightarrow (\tilde{B}_j p_j(a_{ij}))_{j\in\mathcal{E}} = (\tilde{B}_j a_{ij})_{j\in\mathcal{E}} \]

\[ (\mu_{2V,j} a_{ij})_{j\in\mathcal{E}} = \tilde{B}_j \]

**5.3 Theorem:** \( (\prod_{i \in \mathcal{E}} B_j)^{-1} = \prod_{i \in \mathcal{E}} B_j \)

\[ \chi^a_i \in L.H.S. \Rightarrow C = (\chi^c_i \cap c \in \mathbb{C}, c \in c, x < c, y < c) \]

\[ \{x, c \in (c)_{i \in \mathcal{E}} \prod_{i \in \mathcal{E}} B_i, \gamma = (c)_{i \in \mathcal{E}} \in \prod_{i \in \mathcal{E}} B_i \}

\[ \Leftrightarrow \chi^a_i \text{ where } a = (a_{ij})_{i \in \mathcal{E}} \in \prod_{i \in \mathcal{E}} B_i, a = (a_{ij})_{i \in \mathcal{E}} \in \prod_{i \in \mathcal{E}} L_{B_i} \]

\[ \Leftrightarrow \chi^a_i \in \prod_{i \in \mathcal{E}} B_i \Rightarrow (\chi^a_i)_{i \in \mathcal{E}} \in \prod_{i \in \mathcal{E}} B_i \]

**5.4 Definition:** Define \( \chi^a_i = (\chi^a_i)_{i \in \mathcal{E}} \text{ where } a = (a_{ij})_{i \in \mathcal{E}}, a = (a_{ij})_{i \in \mathcal{E}} \]

**5.5 Theorem:** To prove Hausdorff Property of product space

Let \( \chi^a_i \chi^b_i \in \prod_{i \in \mathcal{E}} \mathbb{V}_i \) and \( \chi^a_i \neq \chi^b_i \) for at least one \( j_0 \in J \)

There exist a pair of disjoint open sets \( G_{j_{a,j}} \cap G_{j_{b,j}} \) in \( \mathbb{X} \)

Therefore there exist \( \mathbb{X} \cap \mathbb{X}' = \emptyset \) and \( \chi^a_{j_{a,j}} \in \mathbb{X}_{j_{a,j}} \), \( \chi^b_{j_{b,j}} \in \mathbb{X}_{j_{b,j}} \)

Defined \( \mathcal{P} = \prod_{i \in \mathcal{E}} P_i, \mathcal{Q} = \prod_{i \in \mathcal{E}} Q_i \cap V_i \in \mathcal{J}_i \cap \mathcal{Q}_{i_0} = G_{j_{a,j}} \)

Here \( \mathcal{P} \cap \mathcal{Q} = \prod_{i \in \mathcal{E}} P_i \cap \prod_{i \in \mathcal{E}} Q_i \cap \mathcal{Q}_{i_0} \)

\( \mathcal{Q} = \prod_{i \in \mathcal{E}} (P_i \cap Q_i) \cap (P_i \cap Q_{i_0}) \)

\( \mathcal{Q} = \mathcal{Q}_{i_0} \cap \mathcal{Q}_{i_0} \)

And observe that \( \chi^a_i \in \mathcal{P} = \prod_{i \in \mathcal{E}} P_i \)

\( \chi^b_i \in \mathbb{Q} = \prod_{i \in \mathcal{E}} Q_i \cap \mathcal{Q}_{i_0} \)

Here \( P, Q \) are defining \( F_s \) – sub basic open sets in Product topology.
REFERENCES