Abstract: For any nonempty family \((B_i,X_i)\) of compact FSB-Topological Spaces, the corresponding Fs-product space is also compact.

Index Terms: Fs-Set, Fs-Subset, \((b,b)\) object, Fs-Point, FSB-Topological Space.

I. INTRODUCTION

Axiom choice is not true in the theory of L-Fuzzy sets. Nistla V.E.S Murthy [10] proved Axiom Choice of fuzzy sets in his theory of F-sets. Vaddiparthi Yogeswara [2] etc. ... developed the theory of Fs-sets with the goal of introducing the complement of a fuzzy set which was not satisfactorily explained by previous relevant theories. Also Vaddiparthi Yogeswara, Biswajit Rath, Ch. Rama Sanyaas Rao, K.V. Uma Kameswari, D. Raghu Ram introduced the concept of FSB-topological Space on a given Fs-subset of an Fs-set and also they introduced FsB-subspace in the same paper. Fs-points and Fs-point set \(FSP(\mathcal{W})\) are introduced by Vaddiparthi Yogeswara etc. [2] and based on F-sets theory they defined a pair of relations between \(P(FSP(\mathcal{W}))\) and \(\mathcal{L}(\mathcal{W})\). Here \(FSP(\mathcal{W})\) stands for Fs-set point of \(\mathcal{W}\), \(\mathcal{L}(\mathcal{W})\) stands for collection of all Fs-subsets of \(\mathcal{W}\) and \(P(FSP(\mathcal{W}))\) is power set of \(FSP(\mathcal{W})\) and proved one of them is a ‘A’-complete homomorphism and other is ‘V’-complete homomorphism and searched some properties of these relations between complemented constructed crisp sets and Fs-complemented sets through thesehomomorphism and ultimately they proved a representation theorem connecting Fs-subsets of \(\mathcal{W}\) to crisp subsets of \(FSP(\mathcal{W})\) via homomorphism. For a given non-empty family of compact Fs-topological spaces, we prove in this paper their Fs-Cartesian Product space is also compact. Fs-Sets, Fs-Set functions etc. in brief are explained in first four sections of this paper. ‘\(U\)’ and ‘\(\cap\)’ stands for natural set union and Fs-union and Similarly ‘\(\cap\)’. \(M_A\) or \(1_A\) stands for largest element of a given complete Boolean Algebra \(L_A\). For all lattice theoretic and relevant Properties one can refer [5],[8],[15],[16],[17].

SECTION-1

1.1 Fs-set: A four tuple of the form \(\mathcal{W} = (W_1, W_2, W_3, \mu W_1, \mu W_2, \mu W_3, L_\mathcal{W})\) is an Fs-set iff, \(W \subseteq W_1 \subseteq U\)

(1) \(L_\mathcal{W}\) is a complete Boolean Algebra

(2) \(\mu W_1: W_1 \rightarrow L_\mathcal{W}, \mu W_2: W \rightarrow L_\mathcal{W}\) are mappings such that \(\mu W_1 | W \geq \mu W_2\)

(3) \(W: W \rightarrow L_\mathcal{W}\) is defined by \(Wx = \mu W_1 | x \land (\mu W_2 | x)^c\) for each \(x \in W\)

Where \(W\) is a non-void subset of some universal set \(U\).

1.2 Fs-subset: Suppose \(\mathcal{W} = (W_1, W_2, W_3, \mu W_1, \mu W_2, \mu W_3, L_\mathcal{W})\) and \(U = (U_1, U_2, U_3, (\mu U_1, \mu U_2), L_U)\) are two Fs-sets. We say \(U\) is an Fs-subset of \(\mathcal{W}\), in symbol, we write \(U \subseteq \mathcal{W}\), iff

(1) \(U_1 \subseteq W_1, U_2 \subseteq U_2, U_3 \subseteq U_3\)

(2) \(L_U\) is a complete subalgebra of \(L_\mathcal{W}\) or \(L_U \subseteq L_\mathcal{W}\)

(3) \(\mu U_1 \leq \mu W_1 | U_1, \mu U_2 | W \geq \mu W_2\)

1.3 Arbitrary Fs-unions and arbitrary Fs-intersections

For any \((U_i)_{i \in I}, U_i = (U_{i1}, U_{i2}, U_{i3}, \mu U_{i1}, \mu U_{i2}), L_{U_i}) \subseteq \mathcal{W} = (W_1, W_2, W_3, \mu W_1, \mu W_2, \mu W_3, L_\mathcal{W})\), \(i \in I\)

(1) \(i \in I, U_i = \varphi_{\mathcal{W}}\), for \(i \neq \varphi\)

(2) If \(I \neq \varnothing\), \(\bigcup_{i \in I} U_i = U = (U_1, U_2, U_3, \mu U_{i1}, \mu U_{i2}), L_U),\) where

(a) \(U_1 = \bigcup_{i \in I} U_{i1}, U = \bigcup_{i \in I} U_i\)
Given $\mathcal{V}=(V_1, V, \bar{V}(\mu_{1V}, \mu_{2V}), L_V)$. We define $\text{Fs-complement of } \mathcal{A}$ in $\mathcal{L}$, denoted by $\mathcal{V}^{\mathcal{A}}$ for $\mathcal{V}=\mathcal{W}$ and $L_V=L_V$ as $\mathcal{V}^{\mathcal{A}}=\mathcal{U}(U_1, U, U(\mu_{1U}, \mu_{2U}), L_U)$, where

(a) $U_1=C_A V_1 = V_1$ $\cup$ $U$, $W, U = V = \mathcal{W}$ where $V_1 = W_1 - V_1$

(b) $U_1 = L_W$

(c) $\mu_{1U_1}$ : $U_1 \rightarrow L_W$ is defined by $\mu_{1U_1} x = M_A$

$\mu_{2U_1} : W \rightarrow L_W$ is defined by $\mu_{2U_1} x = \bar{V} x = \mu_{1V} x \land (\mu_{2V} x)^c$

$\bar{V} : W \rightarrow L_W$ is defined by $\bar{V} x = \mu_{1V} x \land (\mu_{2V} x)^c$

1.19.2 Definition

1) $\forall U_i = W$, for $i = \varphi$

2) Suppose $\bigcap_{i \in I} U_i \neq \bigcup_{i \in I} U_i$, $V = \bigcup_{i \in I} U_i$

(a) $V_i = \bigcap_{i \in I} U_{i1}$, $V = \bigcup_{i \in I} U_i$

(b) $L_Y = \bigwedge_{i \in I} U_i$

(c) $\mu_1V_i : V_i \rightarrow L_V$ is defined by $\mu_1V_i x = \left(\bigwedge_{i \in I} \mu_{1U_{i1}}\right)x$

$\mu_{2V} : V \rightarrow L_V$ is defined by $\mu_{2V} x = \left(\bigwedge_{i \in I} \mu_{2U_{i1}}\right)x$

$\bar{V} : V \rightarrow L_V$ is defined by $\bar{V} x = \mu_{1V} x \land (\mu_{2V} x)^c$

$\varrho_2$ is an Type-II Void set and is denoted by $\varrho_2$

SECTION-2

(b, $\beta$) Object

2.1 Definition Let $\alpha \in A, \beta \in A$, such that $\beta \leq \bar{A}b$. We define a (b, $\beta$) object, denoted by (b, $\beta$) itself as follows

For $A \subseteq B \subseteq B_1 \subseteq A_2, \beta \subseteq L_B$, such that $\mu_{1B} x, \mu_{2B} x \in L_B$,

$\mu_{2B} x = \left(\bigwedge_{\alpha \in A_1, \beta \subseteq L_B} \bar{B} \beta \mu_{2B} x, x = \beta\right)$

And $\mu_{2B} x = \left(\bigwedge_{\alpha \in A_1, \beta \subseteq L_B} \bar{B} \beta \mu_{2B} x, x = \beta\right)$

Here $\alpha \in A_1$ is fixed and $\alpha \subseteq \mu_{1A} x, \forall x \in A_1$
\[ \mu_{1B_1}b = \mu_{1B_2}b = \beta \lor \mu_{2A}b \land \mu_{2B}b = \mu_{2B}b = \mu_{2A}b. \]

We can easily show that \( R(b, \beta) \) is an equivalence relation.

2.3 Fs-point: The equivalence class corresponding to \( (b, \beta) \) is denoted by \( \chi^b_\beta \) or \( (b, \beta) \). We define this \( \chi^b_\beta \) is an Fs-point of \( \mathcal{A} \). Set of all Fs-point of \( \mathcal{A} \) is denoted by FSP(\( \mathcal{A} \)).

2.4 Definition: For any \( \mathcal{V} \subseteq \mathcal{W} \), define \( \mathcal{V}^- = \{ \chi^b_\beta \in \mathcal{V}, b \in \mathcal{W}, \beta \leq \mathcal{V} \} \) otherwise.

\[ \text{Hence } \chi^b_\beta \in \mathcal{V}^- \text{ for any } \mathcal{V} \subseteq \mathcal{W} \text{ if } \mathcal{V} \text{ exists.} \]

3.1 FsB-Topological Space: Suppose \( \mu_{A_1} = 1, \mu_{2A} = 0 \) in \( \mathcal{A} \). \( \mathcal{X} \subseteq \mathcal{L}(\mathcal{W}) \) is said to be FsB-topology if, and only if

1. \( (B_i)_{i \in I} \subseteq \mathcal{X} \Rightarrow \bigcup_{i \in I} B_i \in \mathcal{X} \)
2. \( (B_i)_{i \in I}, I \text{ is finite set } \Rightarrow \bigcap_{i \in I} B_i \in \mathcal{X} \). The pair \( (\mathcal{A}, \mathcal{X}) \) is called an FsB-topological space.

Elements of \( \mathcal{X} \) are called FsB-open sets or FsB-open subset of \( \mathcal{A} \).

3.2 FsB-Product topological Space: \( S = \prod_{i \in I} \mathcal{G}_i \) with \( \mathcal{G}_i \) is an open \( \mathcal{A}_i \).

Every component of RHS of \( \mathcal{A}_i \) for each \( j \neq 1 \) and at the jth place \( \mathcal{G}_i \) is there.

\( \mathcal{S} = \{ S \} \) is called defining FsB-open subbase for FsB-product topology on \( \prod_{i \in I} \mathcal{A}_i \).

The FsB-open base \( \mathcal{B} = \{ \prod_{i \in I} \mathcal{G}_i | \mathcal{G}_i = \mathcal{A}_i \text{ for all } i \in 1 - \{i, j, k, i_0 \} \} \) is called defining FsB-open base for the FsB-topology generated by \( \mathcal{B} \).

The \( \mathcal{B} = \{ \prod_{i \in I} F_i | F_i = \mathcal{A}_i \text{ for all } i \neq i_0, S_{i_0} \text{ is closed in } B_{i_0} \} \) is called defining FsB-closed sets for the Product topology.

The FsB-topology on \( \prod_{i \in I} \mathcal{A}_i \) generated by \( \mathcal{B} \) is called FsB-product topology.

Let \( \mathcal{A}_i = (A_{1i}, A_{2i}, \bar{A}_i(\mu_{A_{1i}}, \mu_{A_{2i}}), L_{Ai}) \) be a family of FsB-topological spaces.

Let \( \prod_{i \in I} \mathcal{A}_i \) be Fs-Cartesian Product of the family \( \{ \mathcal{A}_i \}_{i \in I} \).

Let \( \mathcal{G} = \{ S \} \) where \( S = \prod_{i \in I} \mathcal{G}_i \) where \( \mathcal{B}_i = \{ \mathcal{A}_i \mid i \neq i_0 \} \) and \( \mathcal{G}_i \) be FsB-open in \( \mathcal{A}_i, i \in I \).

Then \( \chi_{\alpha_{i0}} \neq 0 \text{ in } B_{i_0} \). Then \( (\chi_{\alpha_{i0}})_{i \in I} \in (\prod_{i \in I} B_i)^- = (\prod_{i \in I} B_i)^-(3.4) \).

Hence \( (\chi_{\alpha_{i0}})_{i \in I} \in (\bigcap_{j \in J} F_j)^- \). So, that \( \bigcap_{j \in J} F_j \) is non-empty in \( \prod_{i \in I} B_i \). Hence \( \prod_{i \in I} B_i \) is compact.

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3.5 Theorem: \((\mathcal{B}, \mathcal{X})\) is compact if and only if every non empty family of FsB-sub basic closed sets with finite intersection property has nonempty intersection.

Proof: Sufficient to prove that every non-empty family of defining FsB-sub basic closed sets with finite intersection property has non-empty intersection.

Consider \( \{ F_j \} \), a non-empty family of non-empty defining FsB-sub basic closed sets in \( (\mathcal{B}, \mathcal{X}) \).

Then for each \( j \in J \), \( F_j = \prod_{i \in I} F_{ji} \) where \( F_{ji} = \{ B_i, i \neq j \} \) for all \( i \in I \neq i_0 \). \( F_{ji_0} \), a sub basic closed set in \( B_{i_0} \).

Then \( \prod_{i \in I} F_{ji} = F_{ji_0} \) is non-empty FsB-sub basic closed set in \( B_{i_0} \).

In particular, \( \prod_{i \in I} F_j = F_{ji_0} \) is non-empty FsB-sub basic closed set in \( B_{i_0} \).

Hence \( \mathcal{F}_{ji_0} \) \( j \in J \) is a nonempty family of nonempty FsB-closed sets in \( B_{i_0} \).

Also, every finite subfamily of \( \mathcal{F}_{i_0} \) has nonempty intersection.

Since \( B_{i_0} \) is compact, we have \( \mathcal{F}_{i_0} = \bigcap_{j \in J} F_{ji_0} \) is non-empty.

Fix \( \chi_{\alpha_{i0}} \neq 0 \text{ in } B_{i_0} \). Then \( (\chi_{\alpha_{i0}})_{i \in I} \in (\prod_{i \in I} B_i)^- = (\prod_{i \in I} B_i)^-(3.4) \).

Hence \( (\chi_{\alpha_{i0}})_{i \in I} \in (\bigcap_{j \in J} F_j)^- \). So, that \( \bigcap_{j \in J} F_j \) is non-empty in \( \prod_{i \in I} B_i \).

Hence \( \prod_{i \in I} B_i \) is compact.
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