Fuzzy Soft Ternary Γ-Semirings-I

T. Satish, D. Madhusudhana Rao, M. Vasantha, K. Anuradha

Abstract*: *In this paper, we introduce the notions of FSTΓ-SR (fuzzy soft ternary Γ-semiring), FSL(FSM, FSR) TΓ-ideal (fuzzy soft (left, Lateral, right)TΓ-ideal) over TΓ-SR (ternary Γ-semiring). Further, we proved that the fuzzy Γ-soft set(FΓSS) over TΓ-SR is a FSTΓ-SR (ideal) iff its α-level set is a soft TΓ-SR (ideal) over TΓ-SR for any α ∈ [0, 1]. At the same time, we proved that union and intersection of two FSTΓ-SR (ideals) are both FSTΓ-SR (ideals) under certain conditions. Finally, the results that fuzzy soft image and fuzzy soft inverse image of FSTΓ-SR (ideal) are both FSTΓ-SR (ideals) under certain conditions are obtained.

***Index Terms*: *TΓ-SR, FSTΓ-SR, ASTΓ-ideal, fuzzy soft image, fuzzy soft inverse image*.**

I. INTRODUCTION

In many of the branches of mathematics and other fields like sociology, environment, economics, engineering, medical science to solve complicated problems we have not being used classical methods, because the uncertainties appearing in these domains may be of various types. For dealing with there are four theories which are: (1) Theory of Probability, (2) Fuzzy Set Theory[1], (3) Interval Mathematics, (4) Rough Set Theory [2]. In this paper authors are dealing with some notions of fuzzy soft tools in TΓ-SR.

II. PRELIMINARIES

Through out this paper T-ΓSR denoted as ternary Γ-semiring. For preliminaries about fuzzy concept refer the reference [6].

Def 2.1: Let (q, P_1, Γ) be a soft set over T, (q, P_1, Γ) is said to be a soft ternary Γ-semiring over T if and only if $q(x)$ is a ternary Γ-sub-semiringof T for all $x \in P_1$.

Def 2.2: Let U be an initial universe,Ea set of parameters. Let $P(U)$ denotes the power set of U,$\emptyset \neq K \subseteq E$,A pair ($q$, K) is known as a soft set over U,where $q$is a mapping given by q:$K \longrightarrow P(U)$.A soft set (q, P_1) over a ternary Γ-semiring T is called a soft ternaryΓ-semiring over T if,(q, P_1)o(q, P_1)o(q, P_1)⊆ (q,P_1). This is denoted by (q, P_1, Γ).Let (q, P_1, Γ), (r, P_2, Γ) two soft sets of a soft ternary Γ-semiring (s, T, Γ) over a ternary Γ-semiring T. Now the addition of two soft sets is defined by (q, P_1, Γ) ⊕(r, BP_2, Γ) = (f, P_3, Γ), where $P_3 = P_1 \cap P_2$ and $f(x) = q(x) + r(x)$ $\forall x \in P_3$. For more preliminaries one can go through the references.

III. FUZZY SOFT TERNARY Γ-SEMIRING

In this section, U refers to an initial universe, E is a set of parameters, S and T are two ternary Γ-semiring.

Def 3.1: A triple (q, P_1, Γ) is known as a fuzzy Γ-soft set(FΓSS) over T, where q:$P_1 \rightarrow$ F(T) is a mapping, F(T) being the set of all fuzzy sets of T.

Note 3.2: Let P_1 be a non empty subset of E and F(U) be the collection of all fuzzy subsets of U then the tipple(q , P_1, Γ) is called a fuzzy Γ-soft set (FΓSS) over U,where q is a mapping given by, q : $P_1 \rightarrow$F(U).For each$p_1 \in P_1$, we denote f (p_1) by f_{p_1} , which is a fuzzy set over U.

Def 3.3: Let (q, P_1, Γ) be a FΓSS over T. For each $\gamma \in [0, 1]$, the set $(q, P_1, \Gamma)^{\gamma} = (q^{\gamma}, P_1, \Gamma)$is called an γ-level set of (q, P_1, Γ), where $q^{\gamma}(p_1) = \{t \in T \,/ q(p_1)(t) \geq \gamma\}$ for each $p_1 \in P_1$.Obviously, $(q, P_1, \Gamma)^{\gamma}$ is a soft set over T.

Def 3.4: Let(q, P_1, Γ)and (r, P_2, Γ)be two FΓSS over T, (q, P_1, Γ)is known as a ***fuzzy soft subset*(FΓSSS)** of (r, P_2, Γ),denoted by (q, P_1, Γ)⊆(r, P_2, Γ)if (i) $P_1 \subseteq P_2$, (ii) for each $p_1 \in P_1$,$q(p_1) \leq r(p_1)$.

Def 3.5: Let (q, P_1, Γ)and(r, P_2, Γ)be two FΓSS over T with$P_1 \cap P_2 \neq \emptyset$. The intersection of ($q$, P_1, Γ)and (r, P_2, Γ), denoted by (q, P_1, Γ) ∧ (r, P_2, Γ)= (s, P_3, Γ)is a fuzzy soft set (s, P_3, Γ)over T, where$P_3 = P_1 \cap P_2$, and for each $p_3 \in P_3$,$s(p_3) = q(p_3) \wedge r(p_3)$

Def 3.6: Let (q, K, Γ), (r, L, Γ) be two FΓSS over T. Then the ∨ of two FΓSS (q, K, Γ), (r, L, Γ) over T is the fuzzy soft set (s, M, Γ), where $M = K \cup L$, $\forall v \in M$,

$$s(v) = \begin{cases} q(v) \text{ if } v \in K - L \\ r(v) \text{ if } v \in L - K \\ q(v) \cup r(v) \text{ if } v \in K \cap L. \end{cases}$$

We write (q, K, Γ) ∨ (r, L, Γ) = (s, M, Γ).

Def 3.7: The collection of all FΓSS over U with parameters from E is known as a ***fuzzy* Γ*-soft class*(FΓSC)** and is denoted by $\left(U, E, \Gamma\right)$.

Def 3.8: Let (q, K, Γ)be a FΓSS over T, (q, K, Γ)is known as a FSTΓ-SR iff $q(k_1)$ is a fuzzy ternary Γ-subsemiring over T for each $k_1 \in K$.

Def 3.9: A FΓSS (q, K, Γ)of a TΓ-SR T, then (q, K, Γ) is known as a fuzzy soft ternary Γ-sub-semiring(FSTΓ-SSR) of T if q_k $(x\alpha y\beta z)$ ≥

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min{$q(x)$,$q(y)$, $q(z)$}, for all k ∈K, x, y, z ∈ T,α, β∈Γ.

Def 3.10: Let(q, K, Γ) be a FΓSS over T, (q, K, Γ) is known as a ***FSL(FSM, FSR) TΓ-ideal*** iff $q(k_1)$ is a FSL(FSM, FSR) TΓ-ideal l over T ∀ k_1∈K.

Note 3.11: A FΓSS (q, K, Γ) of a TΓ-SR T is known as a ***FSL(FSM, FSR) TΓ-ideal*** of T if $q_k(x\alpha y\beta z) \geq q_k(z)$ ($q_k(x\alpha y\beta z) \geq q_k(y)$, $q_k(x\alpha y\beta z) \geq q_k(x)$) ∀ k ∈K, x, y, z ∈ T, α, β ∈Γ.

Example 3.12: Let E = {e_1, e_2, e_3},T= {t_1, t_2, t_3} and Γ = {α }be a ternary Γ-semiring having the following multiplication table:

+	t_1	t_2	t_3
t_1	t_1	t_1	t_1
t_2	t_3	t_2	t_2
t_3	t_3	t_3	t_2

α	t_1	t_2	t_3
t_1	t_1	t_3	t_3
t_2	t_3	t_2	t_3
t_3	t_3	t_3	t_3

Choose a FΓSS over T as: (q, K, Γ)= {e1= $\left\{t_{1_{0.1}}, t_{2_{0.4}}, t_{3_{0.8}}\right\}$ }. It is easy to verify that $q(e_1)$ is a fuzzy left TΓ-ideal over T. Thus (q, K, Γ)is a FSLTΓ-ideal over T.

Def 3.13: Let (q, K, Γ) be a fuzzy soft set over T, (q, K, Γ) is called a ***fuzzy soft TΓ-ideal*(FSTΓI)** if and only if (q, K, Γ) is a fuzzysoft left, a fuzzy soft lateral TΓ-ideal and a fuzzy soft right TΓ-ideal over T.

Note 3.14: A FΓSS (q, K, Γ) of a TΓ-SR T is known as a **FSTΓI** of T if $q_k(x\alpha y\beta z) \geq$ Max { $q_k(x)$, $q_k(y)$, $q_k(z)$}for all k ∈K, x, y, z ∈ T, α, β∈Γ.

Example 3.15: Let E = {e_1, e_2, e_3}, T= {t_1, t_2, t_3}, Γ = {α } be a TΓ-SR having the same multiplication table as Example 3.9.Choose a FΓSS over T as: (q, K, Γ)= {e1= $\left\{t_{1_{0.1}}, t_{2_{0.3}}, t_{3_{0.5}}\right\}$, e3= $\left\{t_{1_{0.2}}, t_{2_{0.4}}, t_{3_{0.8}}\right\}$ }. It is easy to verify that $q(e_1)$ and $q(e_3)$ are both FLTΓ-ideals over T, at the same time, $q(e_1)$, $q(e_3)$ are both FMTΓ-ideals as well as FRTΓ-ideals over T. Thus(q, K, Γ)is a FSTΓI over T.

Th 3.16: Let (q, K, Γ) be a FΓSS over T, (q, K, Γ) is a FSTΓ-SR iff (q, K, Γ)$^\gamma$ is a soft TΓ-SR over T ∀ γ ∈ [0, 1].

***Proof*:** Suppose (q, K, Γ) is a FSTΓ-SR. For each γ ∈ [0, 1], k_1∈K, α , β ∈Γ, y_1, y_2, y_3∈$q^\gamma(k_1)$, then $q(k_1)(y_1) \geq \gamma$, $q(k_1)(y_2) \geq \alpha$ and $q(k_1)(y_3) \geq \alpha$. According to Def 3.7, $q(k_1)$ is a fuzzy ternary Γ-sub-semiring over T, thus $q(k_1)(y_1\alpha y_2 \beta y_3) \geq$ min{$q(k_1)(y_1)$, $q(k_1)(y_2)$, $q(k_1)(y_3)$} ≥ γ. This implies that $y_1 \alpha y_2 \beta y_3$ ∈$q^\gamma(k_1)$, i.e., $q^\gamma(k_1)$is a ternary Γ-sub-semiring over T. According to Definition 2.4, (q, K, Γ)$^\gamma$ is a soft ternary Γ-semiring over T for each γ ∈ [0, 1].

Conversely, if (q, K, Γ)$^\gamma$ is a soft TΓ-SR over T ∀ γ ∈ [0, 1], For each k_1∈K, α , β ∈Γ, y_1, y_2, y_3∈T, let γ = min{$q(k_1)(y_1)$, $q(k_1)(y_2)$, $q(k_1)(y_3)$}, then y_1, y_2, y_3∈$q^\gamma(k_1)$. Since $q^\gamma(k_1)$is a ternary Γ-sub-semiringover T, then $y_1 \alpha y_2 \beta y_3$ ∈$q^\gamma(k_1)$. This means that $q(k_1)(y_1 \alpha y_2 \beta y_3) \geq \gamma$ = min{$q(k_1)(y_1)$, $q(k_1)(y_2)$, $q(k_1)(y_3)$}, i.e.,$q(k_1)$ is a FTΓ-SSR over T. According to Def 3.7, (q, K, Γ) is a FSTΓ-SR over T.

Th 3.17:Let (q, K, Γ) be a FΓSS over T, (q, K, Γ)is a FSL(FSM, FSR) TΓ-ideal iff (q, K, Γ)$^\gamma$ is a SL(SM, SR) TΓ-ideal over T ∀ γ ∈ [0, 1].

***Proof*:** Suppose (q, K, Γ) is a GSLTΓ-ideal. For each γ ∈ [0, 1],k_1∈K, α , β ∈Γ , y_0, y_1∈$q^\gamma(k_1)$, y ∈ T, then $q(k_1)(y_0) \geq \gamma$, $q(k_1)(y_1) \geq \gamma$. According to Def 3.8,$q(k_1)$ is a FLTΓ-ideal over T, thus $q(k_1)(y\alpha y_0 \beta y_1) \geq q(k_1)(y_0) \geq \gamma$. This means that $y\alpha y_0 \beta y_1$∈$q^\gamma(k_1)$, i.e., $q^\gamma(k_1)$is a left TΓ-ideal over T. Thus (q, K, Γ)$^\gamma$ is a SLTΓ-ideal over T ∀ γ ∈ [0, 1].

.Conversely, if (q, K, Γ)$^\gamma$ is a left TΓ-ideal over T ∀ γ ∈ [0, 1]. For each k_1∈K, α , β ∈Γ, y_0∈T, letγ = $q(k_1)(y_0)$, then y_0∈$q^\gamma(k_1)$. Since $q^\gamma(k_1)$is a left TΓ-ideal over T, then $y\alpha y_0 \beta y_1$∈$q^\gamma(k_1)$for each y, y_1∈T. This means that $q(k_1)(y\alpha y_0 \beta y_1) \geq q(k_1)(y_0) \geq \gamma = q(k_1)(y_0)$, i.e., $q(k_1)$ is a FLTΓ-ideal over T, according to Def 3.8, (q, K, Γ) is a FSLTΓ-ideal over T. FSM(FSR)TΓ-ideal is similar.

Th 3.18: Let (q, K, Γ) be a FΓSS over T, (q, K, Γ) is a FSTΓ-ideal iff (q, K, Γ)$^\gamma$ is a soft TΓ-ideal over T ∀ γ ∈ [0, 1].

Th 3.19 :: Let (q, P_1, Γ), (r, P_2, Γ) be two FSTΓ-SR over T. If $P_1 \cap P_2 \neq \emptyset$. Then ($q$, P_1, Γ) ∧(r, P_2, Γ) is a FSTΓ-SR over T.

***Proof*:** By definition (s, P_3, Γ) = (q, P_1, Γ) ∧ (r, P_2, Γ), where $P_3 = P_1 \cap P_2$, $s(p_3) = q(p_3) \cap r(p_3)$ ∀p_3∈P_3.As $q(p_3)$, $r(p_3)$ are TΓ-SSR of T, therefore either $q(p_3) \cap r(p_2) = \emptyset$ or $q(p_1) \cap r(p_2)$ is a TΓ-SSR of T. Consequently, (s, P_3, Γ) is a STΓ-SR over T. ∴ (q, P_1, Γ) ∧ (r, P_2, Γ) is a FSTΓ-SR over T.

Note 3.20: Generally, the union of two FSTΓ-SR over T is not a FTΓ-SR over T. For this consider the following example.

Example 3.21: Let E = {e_1, e_2, e_3} and T= {t_1, t_2, t_3}andΓ = { α } be a ternary Γ-semiring as Example 3.9. Choose two FΓSS over T respectively as(q, K, Γ) = {e1= $\left\{t_{1_{0.1}}, t_{2_{0.8}}, t_{3_{0.4}}, t_{4_{0.2}}\right\}$ }, (r, P, Γ)= {e1= $\left\{t_{1_{0.9}}, t_{2_{0.3}}, t_{3_{0.5}}, t_{4_{0.7}}\right\}$ }. Obviously, (q, K, Γ) and (r, P, Γ)are both fuzzy soft ternary Γ-semi rings over T. Let (q, K, Γ) ∨ (r, P, Γ)=(s, Q, Γ),

Bydefinition 3.4, Q= K∪ P= {e_1} and $s(e_1) = q(e_1) \vee r(e_1)$ = $\left\{t_{1_{0.9}}, t_{2_{0.8}}, t_{3_{0.5}}, t_{4_{0.2}}\right\}$. Since $s(e_1)(t_1\alpha t_2 \beta t_3) = s(e_1)(t_4)$ = 0.2<0.5= min{$s(e_1)(t_1)$,$s(e_2)(t_1)$, $s(e_1)(t_3)$}, this implies that $s(e_1)$ is not a TΓ-SSR over T. Thus the union of two FSTΓ-SRs is not a FSTΓ-SR.

Th 3.22: (q, P_1, Γ) and (r, P_2, Γ)be any two FSTΓI over a TΓ-SR T. Then (q, P_1, Γ) ∨ (r, P_2, Γ)is a FSTΓI over T.

Proof: Since (q, P_1, Γ) ∨ (r, P_2, Γ) = (s, P_3, Γ) where, $P_3 = P_1 \cup P_2$. Suppose $p_3 \in P_1 - P_2$ then $s(p_3) = q(p_3)$, if $p_3 \in P_2 - P_1$ then $s(p_3) = r(p_3)$ and suppose $p_3 \in P_1 \cap P_2$ then $s(p_3) = q(p_3) \cup r(p_3)$. In all the cases $s(p_3)$ is a FTΓ-ideal of T. ∴, (s, P_3, Γ) is a FSTΓI over T.

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Th 3.23: Let (q, P_1, Γ), (r, P_2, Γ) be any two GSTΓI over a TΓ-SR T with $P_1 \cap P_2 \neq \emptyset$. Then $(q, P_1, \Gamma) \wedge (r, P_2, \Gamma)$ is a FSTTΓI over T.

Proof: Since (q, P_1, Γ), (r, P_2, Γ) be any two FSTΓI over a TΓ-SR T with $P_1 \cap P_2 \neq \emptyset$. Hence $(q, P_1, \Gamma) \wedge (r, P_2, \Gamma) = (s, P_3, \Gamma)$ and $s(p_3) = q(p_3) \cap r(p_3)\ \forall p_3 \in P_3$ is either empty or FTΓ-ideal of T. Consequently, (s, P_3, Γ) is a FSTΓI over T.

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