Fuzzy Soft Ternary $\Gamma$-Semirings-I

T. Satish, D. Madhusudhana Rao, M. Vasantha, K. Anuradha

Abstract: In this paper, we introduce the notions of FSTPSR (fuzzy soft ternary $\Gamma$-semiring), FSL(FSM, FR) T$\Gamma$ideal (fuzzy soft left, lateral, right $\Gamma$-ideal) over TFSR (ternary $\Gamma$-semiring). Further, we proved that the fuzzy $\Gamma$-soft set(FTSS) over TFSR is a FSTPSR (ideal) if its $\alpha$-level set is a soft TFSR (ideal) over TFSR for any $\alpha \in [0, 1]$. At the same time, we proved that union and intersection of two FSTPSR (ideals) are both FSTPSR (ideals) under certain conditions. Finally, the results that fuzzy soft image and fuzzy soft inverse image of FSTPSR (ideal) are both FSTPSR (ideals) under certain conditions are obtained.

Index Terms: T$\Gamma$SR, FSTPSR, AST$\Gamma$ideal, fuzzy soft image, fuzzy soft inverse image.

I. INTRODUCTION

In many of the branches of mathematics and other fields like sociology, environment, economics, engineering, medical science to solve complicated problems we have not been used classical methods, because the uncertainties appearing in these domains may be of various types. For dealing with there are four theories which are: (1) Theory of Probability, (2) Fuzzy Set Theory [1], (3) Interval Mathematics, (4) Rough Set Theory [2]. In this paper authors are dealing with some notions of fuzzy soft tools in T$\Gamma$SR.

II. PRELIMINARIES

Through out this paper T-$\Gamma$SR denoted as ternary $\Gamma$-semiring. For preliminaries about fuzzy concept refer the reference [6].

Def 2.1: Let $(q, P_1, \Gamma)$ be a soft set over $T$, $(q, P_1, \Gamma)$ is said to be a soft ternary $\Gamma$-semiring over $T$ if and only if $q(x)$ is a ternary $\Gamma$-sub-semiring of $T$ for all $x \in P_1$.

Def 2.2: Let $U$ be an initial universe. A set of parameters. Let $P(U)$ denotes the power set of $U \cup \emptyset \subseteq E$, A pair $(q, K)$ is known as a soft set over $U$, where $q$ is a mapping given by $q: K \rightarrow P(U)$. A soft set $(q, P_1)$ over a ternary $\Gamma$-semiring $T$ is called a soft ternary $\Gamma$-semiring over $T$ if, $(q, P_1) \circ (q, P_2) \subseteq (q, P_3)$. This is denoted by $(q, P_3, \Gamma)$, Let $(q, P_1, \Gamma), (r, P_2, \Gamma)$ two soft sets of a soft ternary $\Gamma$-semiring $(s, T, \Gamma)$ over a ternary $\Gamma$-semiring $T$. Now the addition of two soft sets is defined by $(q, P_1, \Gamma) \oplus (r, P_2, \Gamma) = (f, P_3, \Gamma)$, where $P_3 = P_1 \cup P_2$ and $f(x) = q(x) + r(x)$ $\forall x \in P_3$. For more preliminaries one can go through the references.

III. FUZZY SOFT TERNARY $\Gamma$-SEMIRING

In this section, $U$ refers to an initial universe, $E$ is a set of parameters, $S$ and $T$ are two ternary $\Gamma$-semiring.

Def 3.1: A triple $(q, P_1, \Gamma)$ is known as a fuzzy $\Gamma$-soft set(FTSS) over $T$, where $q: P_1 \rightarrow (T)$ is a mapping, $F(T)$ being the set of all fuzzy sets of $T$.

Note 3.2: Let $P_1$ be a non empty subset of $E$ and $F(U)$ be the collection of all fuzzy subsets of $T$ then the $\uparrow$ and $\downarrow$ are known as a fuzzy $\Gamma$-soft set(FTSS) over $U, \uparrow(T)$, where $q$ is a mapping given by $q: P_1 \rightarrow (U)$. For each $p_1 \subseteq P_1$, we denote $\uparrow p_1$ by $f_{p_1}$, which is a fuzzy set over $U$.

Def 3.3: Let $(q, P_1, \Gamma)$ be a FTSS over $T$. For each $\gamma \in [0, 1]$, the set $(q, P_1, \Gamma)^\gamma = (q^\gamma, P_1, \Gamma)$ is known as an $\gamma$-level set of $(q, P_1, \Gamma)$, where $q^\gamma = \{p \in P_1 | q(p) = \gamma\}$ for each $p \in P_1$. Obviously, $(q, P_1, \Gamma)^0 \subseteq (q, P_1, \Gamma)^1 \subseteq T$.

Def 3.4: Let $(p, P_1, \Gamma)$ and $(r, P_2, \Gamma)$ be two FTSS over $T$, $(q, P_1, \Gamma)$ is known as a fuzzy soft subset of $(r, P_2, \Gamma)$, denoted by $(p, P_1, \Gamma) \subseteq (r, P_2, \Gamma)$ if (i) $p \subseteq P_1$, (ii) for each $(p_1) \subseteq P_1$, $(p_1) \subseteq r(p_1)$.

Def 3.5: Let $(q, P_1, \Gamma)$ and $(r, P_2, \Gamma)$ be two FTSS over $T$ with $P_1 \cap P_2 \neq \emptyset$. The intersection of $(q, P_1, \Gamma)$ and $(r, P_2, \Gamma)$, denoted by $(q, P_1, \Gamma) \cap (r, P_2, \Gamma) = (s, P_3, \Gamma)$ is a fuzzy soft set $(s, P_3, \Gamma)$ over $T$, where $P_3 = P_1 \cap P_2$, and for each $(p_1) \subseteq P_1$, $(p_1) \subseteq r(p_1)$.

Def 3.6: Let $(q, K, \Gamma), (r, L, \Gamma)$ be two FTSS over $T$. Then the $\land$ of two FTSS $(q, K, \Gamma), (r, L, \Gamma)$ over $T$ is the fuzzy soft set $(s, M, \Gamma)$, where $M = K \cup L$, $\forall x \in M$.

Def 3.7: The collection of all FTSS over $U$ with parameters from $E$ is known as a fuzzy $\Gamma$-soft class(FTSC) and is denoted by $(U, E, \Gamma)$.

Def 3.8: Let $(q, K, \Gamma)$ be a FTSS over $T$, $(q, K, \Gamma)$ is known as a FSTT-$\Gamma$-semiring iff $q(k_1)$ is a fuzzy ternary $\Gamma$-subsemiring over $T$ for each $k_1 \in K$.

Def 3.9: A FTSS $(q, K, \Gamma)$ of a T-$\Gamma$-semiring, then $(q, K, \Gamma)$ is known as a fuzzy soft ternary $\Gamma$-sub-semiring(FSTT-SSR) of $T$ if $q_k (x\epsilon r/\lambda) \geq 0$.

Published By:
Blue Eyes Intelligence Engineering & Sciences Publication
min\{q(x), q(y), q(z)\}, for all \(k \in \mathbb{K}, x, y, z \in T, a, \beta \in \Gamma\).

**Def 3.10:** Let \((q, K, \Gamma)\) be a FTS\(SS\) over \(T\), \((q, K, \Gamma)\) is known as a FSLT\(SS) over \(T\) if \(x, y, z \in T, a, \beta \in \Gamma\).

**Note 3.11:** A FTS\(SS) \((q, K, \Gamma)\) is a T-SL\(R\) \(T\) if \(q_k(x, y, z) \geq q_k(x) \cup q_k(y) \cup q_k(z)\), for all \(k \in \mathbb{K}, x, y, z \in T, a, \beta \in \Gamma\).

**Example 3.12:** Let \(E = \{e_1, e_2, e_3\}, T = \{t_1, t_2, t_3\}\) and \(\Gamma = \{\alpha, \beta\}\) be a ternary \(\Gamma\)-semiring having the following multiplication table:

\[
\begin{array}{ccc|ccc}
& t_1 & t_2 & t_3 & t_1 & t_2 & t_3 \\
\hline
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 2 & 1 & 1 & 2 & 1 \\
3 & 1 & 1 & 3 & 1 & 1 & 3 \\
\end{array}
\]

Choose a FTS\(SS\) over \(T\) as: \((q, K, \Gamma) = \{e_i = \{t_{i_1}, t_{i_2}, t_{i_3}\} \} \). It is easy to verify that \(q(e_i)\) is a fuzzy left \(T\)-ideal over \(T\). Thus \((q, K, \Gamma)\) is a FSTS\(SS) over \(T\).

**Def 3.13:** Let \((q, K, \Gamma)\) be a fuzzy soft set over \(T\), \((q, K, \Gamma)\) is called a fuzzy soft \(T\)-ideal \((FSTI)\) if and only if \((q, K, \Gamma)\) is a fuzzysoft left, a fuzzy soft lateral \(T\)-ideal and a fuzzy soft right \(T\)-ideal over \(T\).

**Note 3.14:** A FTS\(SS) \((q, K, \Gamma)\) is a T-SL\(R\) \(T\) if \(q_k(x, y, z) \geq \max\{q_k(x), q_k(y), q_k(z)\}\), for all \(k \in \mathbb{K}, x, y, z \in T, a, \beta \in \Gamma\).

**Example 3.15:** Let \(E = \{e_1, e_2, e_3\}, T = \{t_1, t_2, t_3\}, \Gamma = \{\alpha\}\) be a T-SL\(R\) over the same multiplication table as Example 3.9. Choose a FTS\(SS\) over \(T\) as: \((q, K, \Gamma) = \{e_i = \{t_{i_1}, t_{i_2}, t_{i_3}\} \} \). It is easy to verify that \(q(e_i)\) and \(q(e_i)\) are both FTS\(SS) \(T\)-ideals over \(T\), at the same time, \(q(e_i)\) and \(q(e_i)\) are both STM\(T\)-ideals as well as FSM\(T\)-ideals over \(T\). Thus \((q, K, \Gamma)\) is a FTS\(SS) over \(T\).

**Th 3.16:** Let \((q, K, \Gamma)\) be a FTS\(SS) over \(T\), \((q, K, \Gamma)\) is a FSTS\(SS) iff \((q, K, \Gamma)\) is a soft \(T\)-SSR over \(T\) \(\forall \gamma \in \{0, 1\}\).

**Proof:** Suppose \((q, K, \Gamma)\) is a FSTS\(SS\). Then \(\forall \gamma \in \{0, 1\}, k \in \mathbb{K}, a \in \Gamma, a \in \Gamma, y \in \mathbb{R} \rightarrow q_k(y) \geq q_k(y) \cup q_k(y) \cup q_k(y)\), for all \(k \in \mathbb{K}, y \in \mathbb{R}, a \in \Gamma\).

Conversely, if \((q, K, \Gamma)\) is a soft \(T\)-SSR over \(T\) \(\forall \gamma \in \{0, 1\}\), then \(\forall \gamma \in \{0, 1\}, k \in \mathbb{K}, a \in \Gamma, a \in \Gamma, y \in \mathbb{R} \rightarrow q_k(y) \geq q_k(y) \cup q_k(y) \cup q_k(y)\), for all \(k \in \mathbb{K}, y \in \mathbb{R}, a \in \Gamma\).

**Th 3.17:** Let \((q, K, \Gamma)\) be a FSTS\(SS) over \(T\), \((q, K, \Gamma)\) is a FSTS\(SS) \((FSTI)\) if \((q, K, \Gamma)\) is a FSM\(T\) over \(T\) \(\forall \gamma \in \{0, 1\}\).

**Proof:** Suppose \((q, K, \Gamma)\) is a FSM\(T\) \(\forall \gamma \in \{0, 1\}, k \in \mathbb{K}, a \in \Gamma, a \in \Gamma, y \in \mathbb{R} \rightarrow q_k(y) \geq q_k(y) \cup q_k(y) \cup q_k(y)\), for all \(k \in \mathbb{K}, y \in \mathbb{R}, a \in \Gamma\).

Conversely, if \((q, K, \Gamma)\) is a FSM\(T) over \(T\) \(\forall \gamma \in \{0, 1\}\), then \(\forall \gamma \in \{0, 1\}, k \in \mathbb{K}, a \in \Gamma, a \in \Gamma, y \in \mathbb{R} \rightarrow q_k(y) \geq q_k(y) \cup q_k(y) \cup q_k(y)\), for all \(k \in \mathbb{K}, y \in \mathbb{R}, a \in \Gamma\).
Let \((q, P_1, \Gamma), (r, P_2, \Gamma)\) be any two GSTTI over a TR-SR T with \(P_1 \cap P_2 \neq \emptyset\). Then \((q, P_1, \Gamma) \wedge (r, P_2, \Gamma)\) is a FSTTI over T.

Proof: Since \((q, P_1, \Gamma), (r, P_2, \Gamma)\) be any two FSTTI over a TR-SR T with \(P_1 \cap P_2 \neq \emptyset\). Hence \((q, P_1, \Gamma) \wedge (r, P_2, \Gamma) = (s, P_3, \Gamma)\) and \(s(p) = q(p) \cap r(p) \forall p \in P_1\) is either empty or FTT-ideal of T. Consequently, \((s, P_3, \Gamma)\) is a FSTTI over T.

ACKNOWLEDGMENT

Authors are very grate full to SRKR Engineering College, Bhimavaram, W.G. District, A.P. for their support to publish this paper.

REFERENCES


AUTHORS PROFILE

T. Satish completed his M.Sc. and M.Phil from Acharya Nagarjuna University, Guntur, Andhra Pradesh, India. Now he is Pursuing Ph.D under the guidance of Dr. D. Madhusudhana Rao from Acharya Nagarjuna University, Guntur. He is working as an Asst. Professor in Mathematics, in the department of Mathematics, SRKR Engineering College, Bhimavaram, Andhra Pradesh, India. He is published more than 8 research papers in popular international Journals to his credit.

Dr. D. Madhusudhana Rao completed his M.Sc. from Osmania University, Hyderabad, Telangana, India. M. Phil. from M. K. University, Madurai, Tamil Nadu, India. Ph.D. from Acharya Nagarjuna University, Andhra Pradesh, India. He joined as Lecturer in Mathematics, in the department of Mathematics, VSR & NVR College, Tenali, A. P. India in the year 1997, after that he promoted as Head, Department of Mathematics, VSR & NVR College, Tenali. At present he is working as professor of Mathematics. He helped more than 10 Ph.D’s, 4 M’Phills and at present he guided 5 Ph. D. Scholars in the department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur, A. P. He received Andhra Pradesh State Best Teacher Award from Govt. of Andhra Pradesh in the year 2018. He also received National Excellent Award in the same year. A major part of his research work has been devoted to the use of semigroups, Gamma semigroups, duo gamma semigroups, partially ordered gamma semigroups and ternary semigroups, Gamma semirings and ternary semirings, Near rings etc. He is acting as peer review member to the “British Journal of Mathematics & Computer Science”. He published more than 150 research papers in different International Journals in the last two academic years.

M. Vasantha She is working as an Assistant Professor in the Department Mathematics, DNR Engineering College, Bhimavaram, A. P. INDIA. She completed her M.Phil. in Madhurai Kamaraj University, Tamil Nadu, India. She is pursuing Ph.D. under the guidance of Dr. D. Madhusudanarao in K. L.University. She published more than 8 research papers in popular international Journals to her credit. Her areas of interests are ternary semirings, ordered ternary semirings, semirings and topology. Presently she is working on Ternary F-semigroup.

K. Anuradha Completed her M.Sc and M.Phil from Acharya Nagarjuna University, Guntur, Andhra Pradesh, India...She is working as a lecturer in Mathematics in the department of Mathematics, Andhra Loyola Degree College, Vijayawada, Andhra Pradesh, India.