

Software Reliability Growth Model (SRGM) Based on Inverse Half Logistic Distribution

Pushpa Latha Mamidi, R. Subba Rao, R N V Jagan Mohan

Abstract: In the recent studies most of the researchers are studying the Reliability of the Software using probability growth models, which are established to detect the fault portion and used for the reliability estimation in the software system. In the present, recent contributions we considered Inverse Half Logistic Distribution (IHL) as an SRGM along with several reliability performance measures. The foremost requirement of software reliability model is developed concerning to IHL. An NHPP is used to establish the estimation procedures and finding the system reliability using IHL.

Index Terms: IHL, Reliability estimation, Mean value function, Maximum likelihood estimation

I. INTRODUCTION

There are several quality attributes which may represent any software system, such as reliability, maintainability, security, availability etc. Among all other attributes and features for predicting and measuring the product quality, software reliability is the most vibrant feature. To remove and to identify the errors in software one may use a process called debugging. The sequence of fault occurrences and absence of error will be exploited by providing an approximate value and the quality of failure part of the software. For preparing the software reliability models the concept of probability is essential.

In the earlier studies of researchers, many growth models are studied the reliability of the software systems., some of them are Goel and Okumoto [1979] Musa [1980], Ramamurthy and Bastani [1982] explained status of the software reliability and perspectives, Crow and Basu [1988] developed the reliability growth estimation with missing data-II for AMSAA model, Malaiya *et al.* [1992] explained the predictability of Software-Reliability models, Wood [1996] discussed about the prediction of software reliability, Zhang *et al.* [2003] studied imperfect debugging systems, Pham [2005] developed an SRGM for Generalized logistic distribution, Kantam & Subba Rao [2009] studied a reliability growth model with reference to Pareto distribution, Smitha Chowdary *et al.* [2015] discussed Burr type III software reliability growth model, Rawat *et al.* [2017] explained software reliability growth modeling for Agile Software development, Akilandeswari and Saavithri [2018] developed the algorithm for Pareto type III order

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statistics distribution and its parameters are estimated by unconstrained optimization technique.

The main interest of several software studies is to upgrade the performance of the software. We may study this performance in two categories: the first one is to emphasize the experiential analysis of data collected from software systems; the other type is quantitative assessment of software performance. The aim of reliability study is to ensure software operations without failures by the customer while applying some engineering procedures to minimize the errors in the software. The research findings of this paper are reported in six sections; the interpretation and construction of mean value function for the assumed NHPP are given in section II. Parametric estimation of IHL based on the time between failure data is discussed in section III. Section IV deals with the application of proposed model for a live software failure data. A comparative study is made with the other existing models is presented in section V, and conclusions are given in section VI

II. DEVELOPMENT OF IHL AS SRGM

An NHPP of a failure count model is described here. Consider a failure time process $\{N(t), t > 0\}$, which observes the cumulative failures in the counting process of any software system at time t

Also at the initial time $t=0$, we observe no failures so that $N(0) = 0$. It is also assumed that the process has independent increments, which means that for any different time points t_1, t_2, \dots, t_n ($t_1 < t_2 < \dots < t_n$) the 'n' random variables $N(t_1), \{N(t_2) - N(t_1)\} \dots \{N(t_n) - N(t_{n-1})\}$ is independent.

Let $m(t)$ is the expected number of software failures by the time 't'. Since the expected number of errors remaining in the system at any time is finite i.e., $m(t)$ is bounded and non-decreasing function of 't' with the initial and boundary conditions.

$$m(t) = 0 \text{ as } t = 0$$

$$= a \text{ as } t \rightarrow \infty$$

Here 'a' is the mean number of failures that are to be identified.

Let $N(t)$ be a probability mass function (p.m.f.) of Poisson distribution with parameter $m(t)$, i.e.,

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!}$$



Clearly, N(t) is an NHPP. Therefore the random behavior of software failure phenomenon is explained through the process of N(t). In literature we may found various time domain models which gives the stochastic failure process by an NHPP with variety of mean value functions m(t). The p.d.f., c.d.f. and mean value function of Inverse Half Logistic Distribution are respectively

$$f(x) = \frac{2be^{-b/x}}{x^2(1+e^{-b/x})^2}, x > 0, b > 0$$

$$F(x) = \frac{2e^{-b/x}}{1+e^{-b/x}}, x > 0, b > 0$$

$$m(t) = \frac{2ae^{-b/t}}{1+e^{-b/t}} \tag{2.1}$$

Where m(t)/a is the c.d.f. of IHLD

$$\lim_{t \rightarrow \infty} P\{N(t) = x\} = \lim_{t \rightarrow \infty} \frac{[m(t)]^x \cdot e^{-m(t)}}{x!} = \frac{e^a \cdot a^x}{x!}$$

is a Poisson model with parameter 'a'

Suppose $\bar{N}(t)$ indicates the number of software failures left in the system at a time 't.'

$$\bar{N}(t) = N(\infty) - N(t)$$

$$E[\bar{N}(t)] = E[N(\infty)] - E[N(t)]$$

$$\Rightarrow a - m(t) = a - a \left[\frac{2e^{-b/t}}{1+e^{-b/t}} \right] = \frac{a(1 - e^{-b/t})}{1 + e^{-b/t}}$$

Let the time between (k - 1)th and kth failures of the software product be S_k and the time up to the kth failure is X_k then the probability that S_k beyond a real number s, given that the total time up to the (k - 1)th failure is equal to x, is

$$P\{S_k > s / X_{k-1} = x\} = e^{-[m(x+s) - m(x)]} \tag{2.2}$$

The expression (2.2) is known as Software Reliability

I. PARAMETERIC ESTIMATION

The expressions to predict the parameters of IHLD are developed for a time between failure data.

The successive errors in the software product connected to the Non Homogeneous Poisson Process N(t) will be denoted as S₁, S₂, S₃, ----.

$$\sum_{k=1}^n S_k, k = 1, 2, 3, \dots$$

Let $X_k =$ denotes the time to failure k
Assume that there are 'n' time instants first, second, third ...

nth failures and call them as x_1, x_2, \dots, x_n and it is a different instance of a life testing experiment in which only a single item is put to the test, and its successive failures are recorded alternatively separated by error detections and debugging.

The constants named a and b appeared in m(t) in (2.1) and are known as parameters of IHLD. If a and b are known one can assess the reliability of the software, if they are unknown they have to be estimated from the data. The expressions are derived for estimating 'a' and 'b' for the IHLD model.

The likelihood function (L) is

$$L = e^{-m(x_i)} \cdot \prod_{i=1}^n m'(x_i) \tag{3.1}$$

Values of a, b that would maximize the likelihood function is called Maximum Likelihood Estimators (MLEs) and the method is known as Maximum Likelihood Estimation (MLE).

$$L = e^{-\frac{2ae^{-b/x_i}}{1+e^{-b/x_i}}} \cdot \prod_{i=1}^n \frac{2abe^{-b/x_i}}{x_i^2(1+e^{-b/x_i})^2} \tag{3.2}$$

Taking logarithms on both sides of L, to get the log-likelihood equation and the unknown values of 'a' and 'b' are obtained

$$\log L = \frac{-2ae^{-b/x_i}}{1+e^{-b/x_i}} + n[\log 2ab] + \sum_{i=1}^n \log\left(\frac{1}{x_i^2}\right) + \sum_{i=1}^n \log(e^{-b/x_i}) + \sum_{i=1}^n \log(1+e^{-b/x_i})^{-2} \tag{3.3}$$

The unknown values of 'a' and 'b' can be obtained from the following likelihood equations.

$$\frac{\partial \log L}{\partial a} = 0, \frac{\partial \log L}{\partial b} = 0$$

$$\frac{\partial \log L}{\partial a} = 0 \Rightarrow a = \frac{n(1+e^{-b/x_i})}{2e^{-b/x_i}} \tag{3.4}$$

$$\frac{\partial \log L}{\partial b} = 0 \Rightarrow \frac{2ae^{-b/x_i}}{x_n(1+e^{-b/x_i})^2} + \frac{n}{b} - \sum_{i=1}^n \frac{1}{x_i} + 2 \sum_{i=1}^n \frac{e^{-b/x_i}}{x_i(1+e^{-b/x_i})} = 0 \tag{3.5}$$

$$\frac{z_i e^{-z_i}}{1 + e^{-z_i}} = \alpha_i + \beta_i z_i, \quad i = 1, 2, \dots, n-1$$

$$\frac{z_n}{1 + e^{-z_n}} = \gamma_n + \delta_n z_n$$

$$\text{Where } z_i = \frac{b}{x_i}, i = 1, 2, \dots, n$$

Equation (3.5) for b can be reduced to

$$b \left[n \frac{\delta_n}{x_n} - \sum_{i=1}^n \frac{1}{x_i} + 2 \sum_{i=1}^n \frac{\beta_i}{x_i} \right] = -n\gamma_n - n - 2 \sum_{i=1}^n \alpha_i$$

$$\Rightarrow b = \frac{n\gamma_n + n + 2 \sum_{i=1}^n \alpha_i}{\sum_{i=1}^n \frac{1}{x_i} - n \frac{\delta_n}{x_n} - 2 \sum_{i=1}^n \frac{\beta_i}{x_i}} \tag{3.6}$$

MMLE for b is obtained by solving equation (3.6), and by substituting this value in equation (3.4), we get the predicted value of 'a'. Equation (2.2) can be used for the estimate of software reliability.



II. ILLUSTRATION

The procedures narrated and derived in Sections two and three are explained by software failure data taken from Jelinski and Moranda [1972]. The data is originally taken from the U.S. Navy Fleet computer programming centre, and consist of the errors in the development of software for the real-time, multi-computer complex which forms the core of the Naval Tactical Data System (NTDS). The NTDS software consisted of some 38 different modules. Each module is supposed to follow three stages; the production (development) phase, the test phase, and the user phase. The data are based on the trouble reports or software anomaly reports for one of the larger modules, denoted as A-module. The time (days) between software failures and additional information for this module are summarized in the following

Table 1: Software Failure Data

Error Number	Time between Errors	Cumulative Time
n	S _k days	$x_n = \sum s_k$
Production (Checkout)Phase		
1	9	9
2	12	21
3	11	32
4	4	36
5	7	43
6	2	45
7	5	50
8	8	58
9	5	63
10	7	70
11	1	71
12	6	77
13	1	78
14	9	87
15	4	91
16	1	92
17	3	95
18	3	98
19	6	104
20	1	105
21	11	116
22	33	149
23	7	156
24	91	247
25	2	249
26	1	250
Test Phase		
27	87	337
28	47	384
29	12	396
30	9	405
31	135	540
User Phase		
32	258	798
Test Phase		
33	16	814
34	35	849

From the above table, we can observe that there are 26 errors during the production phase and test phase contains five

errors, user phase is identified with one error, and the test phase contains two more errors obtained after the user phase. The errors in production phase considered as an ordered random sample whose sample size is 26 and are assumed to follow IHL D with 'b' as a parameter. At b = 1 this assumption is confirmed by the method of QQ plot correlation. The ML estimates so obtained for standard IHL D are

$$\hat{a} = 29.13457, \quad \hat{b} = 54.00338$$

The predicted value at any time x more than 250 days using equation (2.2), the reliability function is given below.

$$P[S_k > s / X_{k-1} = x] = e^{-[m(x+s)-m(x)]}$$

$$RS_{27} / X_{26}(250 / 50) = e^{-[m(50+250)-m(250)]} = 0.59491 .$$

III. COMPARISON

It is noticed that the ML estimate of a & b for our considered SRGM IHL D approach $\hat{a} = 29.1345$ and $\hat{b} = 54.00338$. For the same data, Goel and Okumoto [1979] recorded the ML estimates $\hat{a} = 33.99$ & $\hat{c} = 172.71157$. Smitha Chowdary *et al.* [2015] for Burr type III distribution obtained ML estimates of b & c are $\hat{a} = 34.46571$, $\hat{b} = 1.76364$ and $\hat{c} = 1.81012$. Among these three models by consideration of an SRGM structure that provides highest value for a likelihood function is taken as a good fit. There may exist several criteria for the comparison of a good fit. For the considered three models we are approaching the likelihood function technique. The likelihood function values are evaluated, and these are

1. For Goel & Okumoto [1979] structure

$$L\left[a, \frac{1}{\sigma} / S\right] = n \ln a + n \ln b - a\left(1 - e^{-bx}\right) - b \sum_{k=1}^n S_k = -142.69730$$

2. For Smitha Chowdary *et al.* [2015] structure

$$\log L = \frac{-a}{(1+t_n^{-c})^b} + \sum_{i=1}^n [\log a + \log b + \log c - (c+1) \log t_i - (b+1) \log(1+t_i^{-c})] = -168.07041$$

3. For IHL D SRGM structure

$$\log L = \frac{-2ae^{-b/x_n}}{1+e^{-b/x_n}} + n[\log 2ab] + \sum_{i=1}^n \log\left(\frac{1}{x_i^2}\right) + \sum_{i=1}^n \log(e^{-b/x_i}) + \sum_{i=1}^n \log(1+e^{-b/x_i})^{-2} = -87.30125$$

From the above three structures IHL D values are better than other two structures. Hence IHL D is a good model than the other two models.

IV. CONCLUSION

In this paper we considered IHL D as an SRGM model, m(t) and M L estimators of the parameters are calculated to assess the software r(t). Live software failure data is considered. ML estimates are compared for Goel and Okumoto [1979], Smitha Chowdary *et al.* [2015], with considered model. It is observed that our model IHL D has high value of likelihood than that of Goel and Okumoto [1979] and Smitha Chowdary *et al.* [2015] claiming to be a better fit.

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