A Comparison of Fuzzy Game Matrix Solutions using Defuzzification Methods with a New Method

D.V.L.Prasanna, R.Subba Rao, A.V.S.N. Murthy, G.V.Ramana

***Abstract*: In our present research work, a fuzzy game matrix is solved using PM technique.The solutionof fuzzy game matrix using PM technique is compared with the solutions using first maximum, first minimum, centroid and centroid of centroid methods of defuzzification.**

***Index Terms*: Saddle point,Principle of dominance, Arithmetic method, Method of Subgames, Fuzzy matrix game, Defuzzification, Centroid method,Centroid of centroid method.**

I. INTRODUCTION

Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." Game theory studies the general principles that explain how people and organizations act in strategic situations. It is an important tool used in economics, political science, and psychology, as well as logic, computer science, and biology [3,4]. Fuzzy logic is used to deal unclear situations. It is not connected with fixed or exact value but generally it deals with approximate or probabilistic value. In this, the situation lies possibly between 0 and 1 which is known as membership function. This came into existence by the extension of Boolean logic in which the situation is either true or false i.e., 1 and 0 respectively [5,7]. A fuzzy matrix is a matrix where elements of it lies in the closed interval [0,1]. The concept of a fuzzy matrix was introduced by Kim and Roush[1].In recent times Fuzzy matrices (FMs) remains as a broad subject in modeling, uncertain situations occurred in science, automata theory, logic of binary relations, medical diagnosis etc[2].The method of creating a crisp quantity fuzzy is called as Fuzzification. Defuzzification is the change of a fuzzy quantity to a specific quantity. R. Senthil kumara and D. Keerthana are discussed the solution of fuzzy game matrix using first maximum method of defuzzification in [7].L.N.PradeepkumarRallabandi et al studied the improved consistency ratio for pairwise comparison matrix in analytic hierarchy process(2016).

In this paper we presented the preliminary concepts/existing methods for defuzzification and the new approach established by the authors known as PM technique is given in section 2. Three examples are considered and the solutions are evaluated using all the methods mentioned in section 3.A table of comparison between all the methods given in section 4 with the suitable graphs.

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II. PRELIMINARIES AND PM TECHNIQUE

2.1 Definition: The saddle point of a payoff matrix is defined as the position of an element in the payoff matrix which is minimum in its row and maximum in its column and the value of the game is the gain at this point.

2.2 Definition: Principle of dominance is if one pure strategy of a player is better or superior than another.one (irrespective of the strategy employed by his opponent), then the inferior strategy may be simply ignored by assigning a zero probability while searching for optimal strategies.

2.3 Arithmetic method: It is a technique for obtaining optimal strategies for each player in 2×2 games without saddle point.

The steps involved in this method are 1. Find the difference of two numbers in column I and put it under the column II, neglecting the negative sign if occurs. 2. Observe the difference of two numbers in column II and put it under column I, neglecting the negative sign if occurs. 3. Similarly repeat the above two steps for rows also. The values that are obtained from the above steps are called oddments. These are the frequencies with which the players must use their courses of action intheir optimal strategies.

2.4 Method of sub games: This method is used for $2 \times norm \times 2$ games. This method subdivides the given $2 \times norm \times 2$ game into a number of 2×2 subgames, each of which is then solved and optimal strategies are determined. While solving subgames first check for saddle point. If saddle point exists that will be the value of the game. If saddle point doesn't exist, then solve the subgame by arithmetic or algebraic method.

2.5 Theorem[6]: For any 2×2 two- person zero –sum game without any saddle point, having payoff matrix for player A as

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	Player Q	
Player P	Q_1	Q_2
P_1	Ψ_{11}	Ψ_{12}
P_2	Ψ_{21}	Ψ_{22}

Then the optimal strategies are determined by $x = \frac{\Psi_{22}-\Psi_{21}}{(\Psi_{11}+\Psi_{22})-(\Psi_{12}+\Psi_{21})}$, $y = \frac{\Psi_{22}-\Psi_{12}}{(\Psi_{11}+\Psi_{22})-(\Psi_{12}+\Psi_{21})}$ and the value of the game $V = \frac{\Psi_{11}\Psi_{22}-\Psi_{12}\Psi_{21}}{(\Psi_{11}+\Psi_{22})-(\Psi_{12}+\Psi_{21})}$

2.6 Definition: Let P and Q be the two players with strategies $P_i (i = 1,2, \dots, m), Q_j (j = 1,2, \dots, n)$ and the choice of each player choosen his strategies from amongst the pure strategies. Let us assume player A is gainer and player B is the looser. The payoffs p_{ij} given in the matrix are gains of P from player Q. Then the form of **payoff matrix of fuzzy game matrix** is

		Player Q			
Player P		Q_1	Q_2	. . .	Q_n
	P_1	(u_{11},v_{11},w_{11})	(u_{12},v_{12},w_{12})	. . .	(u_{1n},v_{1n},w_{1n})
	P_2	(u_{21},v_{21},w_{21})	(u_{22},v_{22},w_{22})		(u_{2n},v_{2n},w_{2n})
	⋮	⋮	⋮	. . .	⋮
	P_m	(u_{m1},v_{m1}, w_{m1})	(u_{m2},v_{m2},w_{m2})	. . .	(u_{mn},v_{mn},w_{mn})

2.7 Definition: The defuzzification of a fuzzy set is a method of 'rounding it off' from its position in the unit hyper cube to the immediate vertex. Various types of defuzzification methods includes Maximum membership principle, Centroid method, Weighed average method, Mean maximum membership, center of sums, First(or last) of maxima etc.

2.8 Definition: Centroid method of a plane figure is the arithmetic mean position of all points in the space.

2.9 Definition: Centroid of centroid method of a plane figure is the arithmetic mean of arithmetic mean positions of all points in the space.

All the above mentioned methods are existing and the maximum payoff matrices will be calculated according to their methods and processes. The authors are established a newapproach calledas PM technique for finding the defuzzification, when a set of n fuzzy numbers are given.

2.10PM technique: If$x_1, x_2, \dots, x_n$ are n fuzzy numbers of the data and $x_1 \le x_2 \le \dots \le x_n$ then the defuzzification of $x_1, x_2, \dots, x_n$ is defined as follows

$\frac{x_1+x_2+\dots+x_n}{n-1} = L$, the value of L will be considered as

$$\begin{cases} x_n, & if L \le x_n \\ L, & if x_n < L < 1 \\ 1, & if L > 1 \end{cases}$$

III. EVALUATION OF METHODS THROUGH DIFFERENT EXAMPLE

Example 3.1: Let usconsider a 3×3 fuzzy game matrix problem having two players P with strategies P_1, P_2, P_3 and Q with strategies Q_1, Q_2, Q_3 respectively.

		Player Q		
Player P		Q_1	Q_2	Q_3
	P_1	(0.62,0.5,0.1)	(0.8,0.5,0.2)	(0.1,0.25,0.5)
	P_2	(0.12,0.3,0.4)	(0.2,0.5,0.7)	(0.42,0.2,0.3)
	P_3	(0.9,0.15,0.16)	(0.1,0.2,0.3)	(0.3,0.2,0.1)

Case (i): Using First maxima defuzzification method(selecting maximum element the given membership values), the given problem is converted to

		Player Q			Row min
Player P		Q_1	Q_2	Q_3	
	P_1	0.62	0.8	0.5	0.5
	P_2	0.4	0.7	0.42	0.4
	P_3	0.9	0.3	0.3	0.3
Col max		0.9	0.8	0.5	

Maximin=0.5; Minimax=0.5 and hence Minimax= Maximin. Therefore saddle point exists and Value of the game=0.5

Case (ii): Using first minimum method(selecting minimum element in the given membership values),the given problem is converted to

		Player Q			Row min
Player P		Q_1	Q_2	Q_3	
	P_1	0.1	0.2	0.1	0.1
	P_2	0.12	0.2	0.2	0.12
	P_3	0.15	0.1	0.1	0.1
Col Max		0.15	0.2	0.2	

Here, Maximin=0.15;Minimax=0.12 and hence Minimax≠ Maximin.

Therefore there is no saddlepoint.

By using principle of dominance, the matrix is reduced to

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	Q_1	Q_2
P_2	0.12	0.2
P_3	0.15	0.1

Using Theorem 2.5, value of the game= $\frac{(0.12x0.1)-(0.2x0.15)}{(0.12+0.1)-(0.2+0.15)}$ =0.138

Case (iii): Using Centroid method of defuzzification the given problem is reduced to

		Player Q			Row min
		Q_1	Q_2	Q_3	
Player P	P_1	0.4066	0.5	0.2833	0.2833
	P_2	0.273	0.4666	0.3066	0.273
	P_3	0.4033	0.2	0.2	0.2
Colmax		0.4066	0.5	0.3066	

Here, Maximin=0.3066;Minimax=0.2833;

Minimax≠ Maximin.

Therefore saddle point does not exist.

Then by the principle of dominance the above game is reduced to

	Q_1	Q_3
P_1	0.4066	0.2833
P_2	0.273	0.3066

Using Theorem 2.5, value of the game = $\frac{(0.4066X0.3066)-(0.2833X0.273)}{(0.4066+0.3066)-(0.2833+0.273)} = 0.3016$

Case (iv): Using centroid of centroid method, the given problem is converted to

		Player Q			Row min
		Q_1	Q_2	Q_3	
Player P	P_1	0.43	0.5	0.275	0.275
	P_2	0.28	0.475	0.28	0.280.2
	P_3	0.34	0.2	0.2	
Col max		0.43	0.5	0.28	

Here, Maximin=0.28; Minimax=0.28;
Minimax= Maximin.
Therefore saddle point exists and value of the game=0.28

Case (v): By PM technique, the given problem is converted to

		Player Q			Row min
		Q_1	Q_2	Q_3	
Player P	P_1	0.62	0.8	0.5	0.5
	P_2	0.41	0.7	0.46	0.41
	P_3	0.9	0.3	0.3	0.3
Col max		0.9	0.8	0.5	

Maximin=0.5;Minimax=0.5
Minimax = Maximin
Saddle point exists and Value of the game=0.5

Example 3.2: Let us consider a 3×3 fuzzy game matrix problem having two players P with strategies P_1, P_2, P_3 and Q with strategies Q_1, Q_2, Q_3 respectively.

		Player Q		
		Q_1	Q_2	Q_3
Player P	P_1	(0.1,0.15,0.2)	(0.18,0.2,0.25)	(0.3,0.4,0.5)
	P_2	(0.47,0.4,0.2)	(0.3,0.5,0.18)	(0.4,0.3,0.6)
	P_3	(0.5,0.2,0.2)	(0.1,0.27,0.15)	(0.3,0.6,0.62)

Example 3.3: Let us consider a 4×4 fuzzy game matrix problem having two players P with strategies P_1, P_2, P_3 ,$P_4$and Q with strategies Q_1, Q_2, Q_3,Q_4 respectively.

		Player Q			
		Q_1	Q_2	Q_3	Q_4
Player P	P_1	(0.3,0.5,0.2)	(0.5,0.1,0.6)	(0.4,0.2,0.6)	(0.1,0.3,0.6)
	P_2	(0.2,0.8,0.3)	(0.4,0.6,0.7)	(0.3,0.2,0.25)	(0.14,0.12,0.1)
	P_3	(0.16,0.3,0.12)	(0.24,0.1,0.35)	(0.4,0.6,0.15)	(03,0.2,0.1)
	P_4	(0.5,0.16,0.4)	(0.3,0.4,0.62)	(0.1,0.14,0.5)	(0.2,0.1,0.4)

For examples 3.2 and 3.3 the maximum payoff is calculated using all the methods including the new approach PM technique. The results are mentioned in table 4.1.

IV. COMPARISON AND CONCLUSION:

From the table below, it has been observed that the given fuzzy game has maximum payoff in PM technique comparing to the remaining defuzzification methods. The graphs for all the methods to the examples are given in figures 4.1, 4.2 and 4.3.

Table 4.1

Method	Maximum method	Minimum method	Centroid method	Centroid of centroid method	**PM** method
Ex 1	0.5	0.138	0.3016	0.28	0.5
Ex 2	0.47	0.18	0.33	0.3675	0.5
Ex 3	0.539	0.1	0.333	0.325	0.542

Figure 4.1

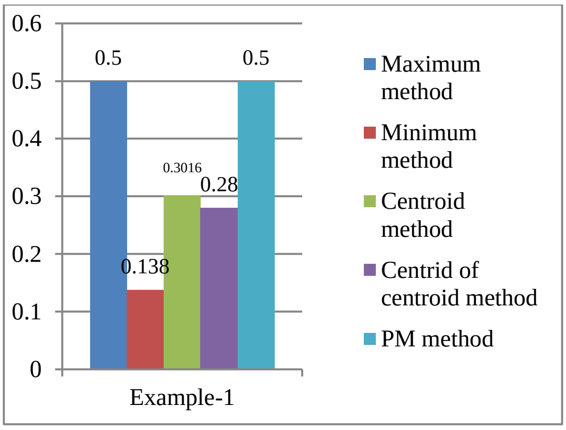


Figure 4.2

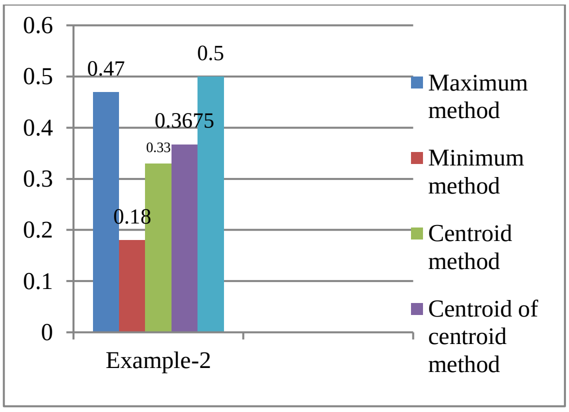
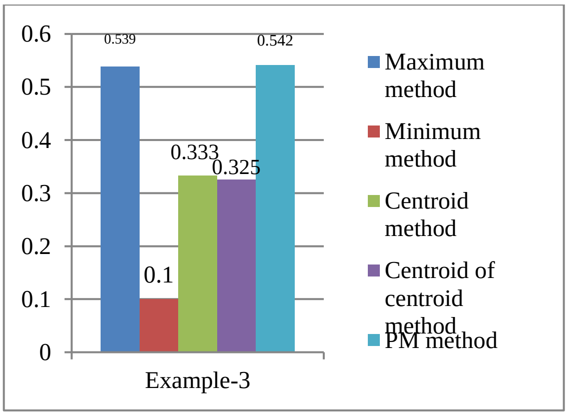


Figure 4.3



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