MHD Mixed Convection at a Stretching Vertical Sheet with Convective Boundary Condition

K.S. Srinivasa Babu, A. Parandhama, R. Bhuvana Vijaya

Abstract*: *A numerical study of MHD mixed convection boundary layer flow at an exponentially stretching sheet with suction/injection has been presented here. Similarity solutions of the problem are obtained by the use of Nachtsheim-Swigert Scheme. Some of the important observations of the analysis are – Skin friction increases while Nusselt number diminishes as Prandtl number increases. In the suction region both Skin friction and Nusselt number increase in assisting flow case, while in the opposing flow case skin friction decreases and Nusselt number increases.

Index Terms*: *MHD, Mixed convection, Stretching surface, NS-Scheme.

I. INTRODUCTION

The studies of mixed convection flows or combinations of free and forced convection are very important in engineering applications. Also, the flow and heat transfer at a vertically stretching surfaces have large number of applications in industrial and manufacturing processes, namely bundle wrapping, extrusion of sheet material, hot rolling, wire rolling, metallurgical processes, glass blowing, plastic and rubber sheet manufacturing, drawing of plastic films, paper production, metal spinning, crystal growing etc. For all these applications, the rate of heat transfer at stretching surfaces decides the quality of the final product. In recent years, many researchers focused their attention on different types stretching surfaces such as, quadratic, exponential, linear and nonlinear variations in stretching velocity and distributions of temperature.

Many researchers concentrated on problems like MHD mixed convection at exponentially vertical surfaces and they got good results. Mainly they focused on how the local Nusselt number and local skin friction changes with the different other parameters. E. Magyari and B. Keller [1] studied the variations in heat and mass transfer in the boundary layers at an exponentially stretching continuous surface. E.M.A. Elbashbeshy [2] has discussed the influence of heat transfer over an exponentially stretching continuous surface with suction. M.Q. Al-Odat *et al.* [3] also discussed the behavior of thermal boundary layer on an exponentially stretching continuous surface with magnetic field effect. M.K. Partha *et al.* [4] paid their attention to investigate the effects of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Mixed convection heat transfer in the boundary layers on an exponentially stretching surface with magnetic field has been discussed by Dulal Pal [5]. The heat transfer characteristics in boundary layer of a viscous fluid at a linearly stretching surface with variable wall temperature was reported by K. Vajravelu and A. Hadjinicolaou [6]. B. Gebhart [7], B. Gebhart and J. Mollendorf [8] has been reported the viscous dissipation effects in natural convection. They have noticed that the effect is more predominant in mixed convection processes and vigorous natural convection. The mixed convection flow of Casson fluid over a stretching sheet with convective boundary conditions and hall effect has been studied by M. Bilal Ashraf *et al.* [9]. T. Hayat *et al.* [10] examined the unsteady three-dimensional MHD flow over exponentially stretching sheet along with conditions of slip analytically. The unsteady mixed convective heat transfer over a vertical stretching sheet with viscosity variation and viscous dissipation was discussed by Mohamed Abd El-Aziz [11]. A numerical solution of non-Newtonian nanofluid flow over a stretching sheet has been studied by S. Nadeem *et al.* [12]. They used Jeffrey fluid model in their study. The steady boundary layer flow of nanofluid at a stretching surface is investigated analytically by S. Nadeem and Ch. Lee [13]. N.M. Sarif *et al.* [14] obtained a numerical solution of flow and heat transfer over a stretching sheet with Newtonian heating. In their study they identified that the thermal boundary layer thickness strongly dependant on Prandtl number. Syed Muhammad Imran *et al.* [15] studied the mixed convection flow over an unsteady stretching surface with heat source in porous medium. It is similar to the earlier works but they concentrated on assisting and opposing buoyancies. The influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet has been reported by M. Sajid and T. Hayat [16].

In the present study, MHD Mixed convection boundary layer flow over an exponentially stretching permeable vertical sheet with convective boundary condition is analyzed numerically to obtain a similarity solution. The Nachtsheim-Swigert scheme [17] together with a shooting technique is used to obtain the solutions. Nachtsheim-Swigert scheme is utilized by many researchers [18]-[20] in analyzing similar problems. A formulation of the problem similar to that of Siti Suziliana Putri Mohamed Isa *et al.* [21] is utilized in this analysis, where, instead of the Reynolds number, a Péclet number is used to convert the governing equations in to a set of ordinary differential equations. This formulation helps in analyzing the problem in a different way. From this study, we noticed that the buoyancy and viscous dissipation have significant influence on the skin friction and heat transfer coefficient through

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parameters of the problem. The effects of local skin friction and local Nusselt number for different Prandtl numbers and for various values of other parameters of the problem are discussed with the help of graphs.

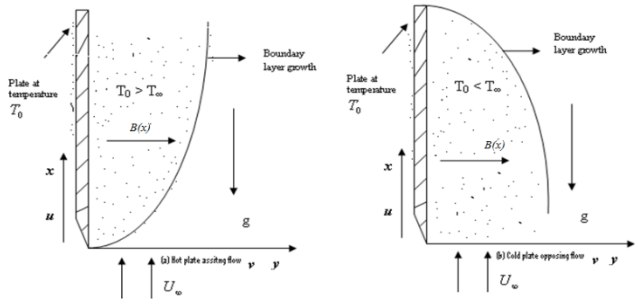
II. MATHEMATICAL FORMULATION

The steady two-dimensional boundary layer flow of an incompressible, viscous, and electrically conducting fluid at an exponentially permeable stretching sheet subjected to a transverse magnetic field $B(x)$ is considered. X-axis is taken to be parallel to the sheet in the direction of the movement of the sheet and the y-axis perpendicular to it. Under the Boussinesq and boundary layer approximations, the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{---} \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B^2(x)}{\rho}u \qquad \text{---} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\sigma}{\rho c_p}B^2(x)u^2+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho c_p}(T-T_\infty) \qquad \text{---} \qquad (3)$$



Schematic Diagram

The symbols are as given in [21] and the appropriate boundary conditions are as follows:

$$u=U_w(x)=U_0e^{(x/l)}, v=-V_w, -k\frac{\partial T}{\partial y}=h_0(T_0-T)$$

at $y=0$,

$$u\to 0\,, T\to T_\infty \quad \text{as} \quad y\to\infty \qquad \text{---} \qquad (4)$$

The following similarity variables are introduced into the governing equations:

$$\eta=\frac{y}{\sqrt{2}\,l}\left(\frac{U_0 l}{\alpha}\right)^{1/2}e^{(x/2l)}, \qquad \psi(x,y)=\sqrt{2}\,\alpha\left(\frac{U_0 l}{\alpha}\right)^{1/2}e^{(x/2l)}f(\eta),$$

$$T(x,y)=T_\infty+(T_0-T_\infty)e^{\left(\frac{ax}{2l}\right)}\theta(\eta) \qquad \text{---} \qquad (5)$$

where ψ is the conventional stream function and ‘a’ is a parameter of the temperature distribution $T(x,y)$.

Using in equations (1)-(3), one gets the equations

$$f'''+\frac{1}{Pr}(ff''-2(f')^2)-2\frac{Ha^2}{Re}f'+\frac{2\epsilon}{Pr}e^{aX/2}.e^{-2X}\theta=0 \qquad \text{---} \qquad (6)$$

$$\theta''-af'\theta+f\theta'+2Ec.\frac{Ha^2}{Re}e^{2X-aX/2}(f')^2+Ec.Pr\,e^{2X-aX/2}(f'')^2+2A^*e^{-X}\theta=0 \qquad \text{---} \qquad (7)$$

here a dash ($'$) denotes differentiation with respect to η and the boundary conditions reduce to

$$\text{at} \quad \eta=0; \qquad f'(\eta)=1, \qquad f(\eta)=V_w\sqrt{\frac{2l}{U\alpha}}\,e^{-x/2l}=-v_w, \qquad \theta(\eta)=\frac{1}{Bi}\theta'(\eta)+\frac{1}{e^{aX/2}}$$

$$\text{as}\ \eta\to\infty; \quad f'(\eta)\to 0, \ \theta(\eta)\to 0$$

In the above equations and boundary conditions, the notation is as in [21] and v_w is the suction / injection parameter.

III. SOLUTION PROCEDURE

The reduced Ordinary differential equations with the reduced boundary conditions will be solved by using Nachtsheim-Swigert Scheme [17]. The results will be discussed for several values of the parameters with the help of plots.

IV. RESULTS AND DISCUSSION

Solutions are obtained for many values of the Prandtl number and for many values of the other parameters. Behaviors of the skin friction, the Nusselt number are presented in the form of graphs and discussed as functions of the parameters.

The skin friction shows a significant increase as Prandtl number (Pr) increases upto 10 and beyond that approaches a constant value asymptotically (refer fig.-1). In the presence of magnetic field & suction, skin friction assumes small values.

From figures 2,3 it was noticed that the skin friction increases as the mixed convection parameter (ϵ) increases. This is true both in the case of assisting flow as well as opposing flow.

Form figure-4 it was noticed that when the plate is impermeable, the increase in skin friction is more significant for relatively larger values of ϵ.

From figure-5 it was observed that the Nusselt number diminishes significantly as Pr increases. However, in the presence of suction & magnetic field, it approaches a constant value with increasing values of Prandtl number.

Like skin friction, Nusselt number also increases as ϵ increases (refer fig.-6). The increase is significant in opposing flow when the plate is impermeable while it is significant in assisting flow for large Prandlt numbers & in the presence of suction.

In figures 2 & 7 the variations in skin friction & Nusselt number are shown when there is no magnetic field. In this case also skin friction & Nusselt number increases as ϵ increases.

The Nusselt number diminishes as Pr increases as well as suction parameter (v_w) increases (refer fig.-8). In fact, in the presence of suction, Nusselt number assumes larger values as compared to that of an impermeable plate and relatively larger values at an impermeable plate as compared to injection of fluid.

The skin friction increases as Pr and v_w. Values are relatively larger in injection region as compared to an impermeable plate which are inturn larger as compared to the case of suction (refer fig.-3).

For small Prandtl numbers, due to considerable changes in thermal conductivity there can be significant changes in velocity and hence significant changes in skin friction.

As Prandtl number increases, thermal conductivity can be expected to diminish and as a consequence the heat transfer coefficient can be expected to diminish.

For opposing flow (i.e., for negative values of ϵ), the buoyancy and the free flow

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oppose one another while in assisting flow, they assist one another. As a result there can be enhanced shear stress at the plate in case of assisting flow and opposite is a case for opposing flow (refer fig.-4).

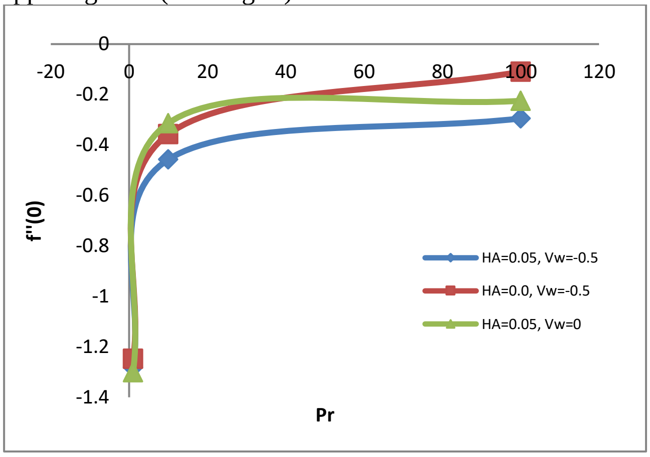

Fig.1 – Skin friction variations for $X = 0.5, a = -1,$ $Bi = 0.5, Ec = 0.1, A^* = 0.05, \epsilon = 1$

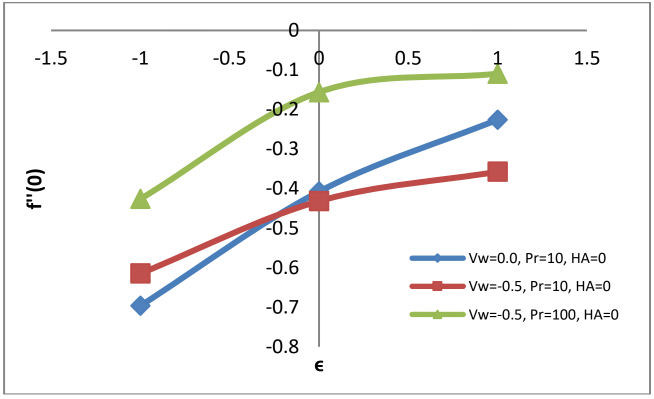

Fig.2 – Skin friction variations for $X = 0.5, a = -1,$ $Bi = 0.5, Ec = 0.1, A^* = 0.05, HA = 0$

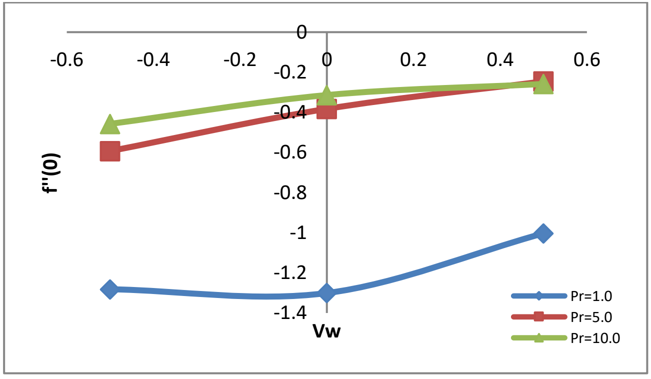

Fig.3 – Skin friction variations for $X = 0.5, a = -1,$ $Bi = 0.5, Ec = 0.1, A^* = 0.05, HA = 0.05, \epsilon = 1$

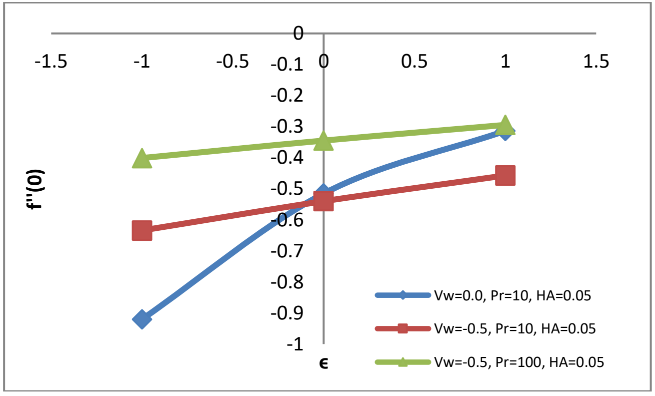

Fig.4 – Skin friction variations for $X = 0.5, a = -1,$ $Bi = 0.5, Ec = 0.1, A^* = 0.05, HA = 0.05$

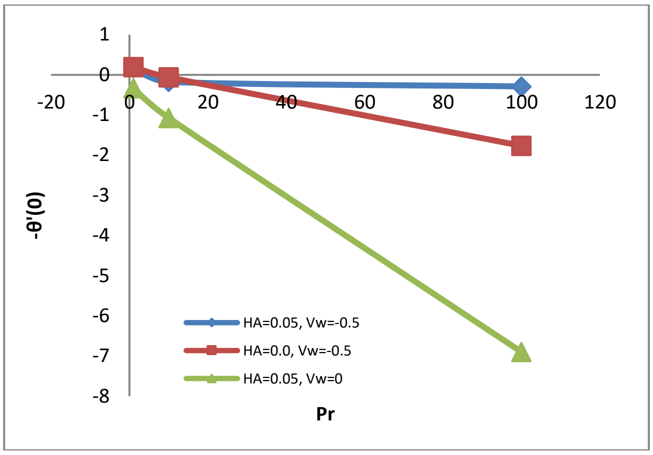

Fig.5 – Variations in Nusselt number for $X = 0.5,$ $a = -1, Bi = 0.5, Ec = 0.1, A^* = 0.05, \epsilon = 1$

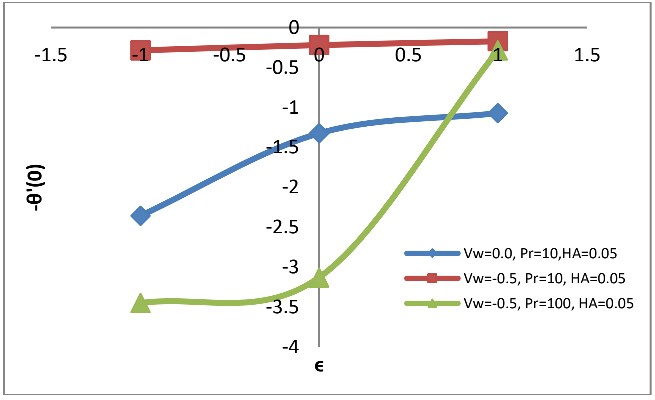

Fig.6 – Variations in Nusselt number for $X = 0.5,$ $a = -1, Bi = 0.5, Ec = 0.1, A^* = 0.05, HA = 0.05$

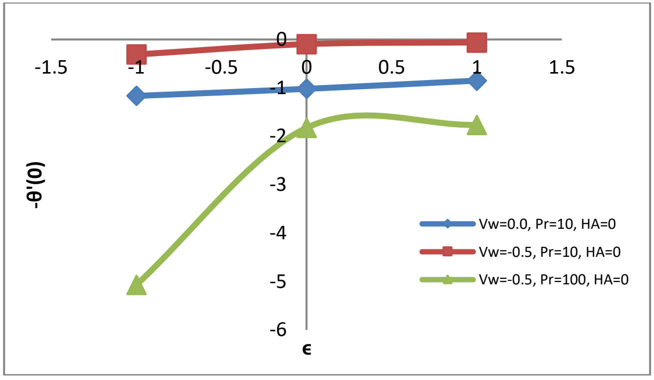

Fig.7 – Variations in Nusselt number for $X = 0.5,$ $a = -1, Bi = 0.5, Ec = 0.1, A^* = 0.05,$

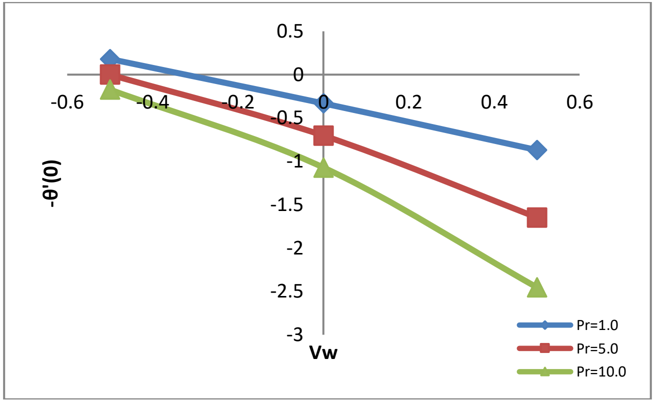

Fig.8 – Variations in Nusselt number for $X = 0.5,$ $a = -1, Bi = 0.5, Ec = 0.1, A^*$ $= 0.05, HA$ $= 0.05, \epsilon = 1$

V. CONCLUSION

In almost all the cases considered in the present work, skin friction assumes negative values. Similarly in almost all the cases considered the Nussult number also assumes negative values. Skin friction assumes larger values for higher Prandtl numbers, while opposite is the case with the Nussult number.

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