C*-Algebra Valued Fuzzy Soft Metric Space and Related Fixed Point Results by using Triangular A-Admissible Maps with Application to Nonlinear Integral Equations

D. Ram Prasad, G.N.V.Kishore, D.W.J. Victor, B.Srinuvasa Rao

Abstract*: *In this paper, we proved some interesting results related fixed point theory and we gave some examples to supporting our results.

Index Terms*: *C*- Algebra (A) Valued (V) Fuzzy (F) Soft (S) Metric (M) Space (S), Common fixed point, weakly compatible, coincidence point.

I. INTRODUCTION

Fixed point theory is an excellent sub fields of nonlinear functional analysis. It has been used in the several research areas of mathematics and nonlinear analysis. The fuzzy soft concept plays a vital role in many scientific and engineering applications. See ([1] – [22])

The main aim of this paper is to establish some results by using triangular α- admissible mappings under various contractive conditions.

For the basic definitions and other related lemmas are presented by the same authors in the ([21], [22])

II. RESULTS AND DISCUSSION

Definition 3.1: Let ($\tilde{E}$, $\tilde{C}$, $\widetilde{d_{C*}}$) be C*- Algebra Valued Fuzzy Soft MS, T: $\tilde{E} \rightarrow \tilde{E}$ be a given mapping. Then T is triangular ∝ −admissible mapping if there exist

$\propto : \tilde{E} \times \tilde{E} \rightarrow \tilde{C}_{+}$ such that

(S_0) $\propto(F_{e_1}, F_{e_2}) \succsim \tilde{I}_{\tilde{C}}$ implies $\propto(TF_{e_1}, TF_{e_2}) \succsim \tilde{I}_{\tilde{C}}$ for $F_{e_1}, F_{e_2} \in \tilde{E}$

(S_1) $\propto(F_{e_1}, F_{e_3}) \succsim \tilde{I}_{\tilde{C}}$ and $\propto(F_{e_2}, F_{e_3}) \succsim \tilde{I}_{\tilde{C}} \Rightarrow \propto(F_{e_1}, F_{e_2}) \succsim \tilde{I}_{\tilde{C}}$ for $F_{e_1}, F_{e_2}, F_{e_3} \in \tilde{E}$

Example 3.1: Let C ? R+ and E ? R+, let $\tilde{E}$ be a absolute fuzzy soft set, define T : $\tilde{E} \rightarrow \tilde{E}$ and

$$\propto(F_{e_1}, F_{e_2}) = \begin{cases} \tilde{I}_{\tilde{C}}, & if\ F_{e_1} \succsim F_{e_2} \\ \tilde{o}_{\tilde{C}}, & Otherwise \end{cases}$$

Then T is ∝ −triangular admissible map.

Lemma 3.1: Let ($\tilde{E}$, $\tilde{C}$, $\widetilde{d_{C*}}$) be a C*-algebra valued fuzzy soft metric space and T : $\tilde{E} \rightarrow \tilde{E}$ be a triangular ∝- admissible mapping . Assume that there exist $F_{e_0} \in \tilde{E}$ such that $\propto(F_{e_0}, TF_{e_0}) \succsim \tilde{I}_{\tilde{C}}$. Define a sequence { F_{e_n} } as $F_{e_{n+1}}$ =T F_{e_n} then $\propto(F_{e_m}, F_{e_n}) \succsim \tilde{I}_{\tilde{C}}$ for all m; n ∈ N with m < n.

Theorem 3.1: Let ($\tilde{E}$, $\tilde{C}$, $\widetilde{d_{C*}}$) be C* - Algebra Valued Fuzzy Soft MS which was complete. Let T be triangular ∝-admissible mapping on $\tilde{E}$. Suppose there exist a function τ : [0;∞) →[0; 1] such that, $\tau(\tilde{t}_n) \rightarrow \tilde{1}$ implies that $\tilde{t}_n \rightarrow \tilde{0}$ for any { $\tilde{t}_n$ } of positive fuzzy soft reals and

$$\left(\widetilde{d_{C*}}(TF_{e_1}, TF_{e_2}) + r\right)^{\propto(F_{e_1},\ TF_{e_1})\propto(F_{e_2},\ TF_{e_2})} \preceq \tilde{a}^{*}\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\, \widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a} + r$$

For all $F_{e_2}, F_{e_3} \in \tilde{E}$, where $\tilde{a} \in \tilde{C}$ with $\|\tilde{a}\|$<1 and r≥ 1. Suppose that if { F_{e_n} } is a sequence in $\tilde{E}$ such that $F_{e_n} \overset{\|\|.\|_{\tilde{C}}}{\longrightarrow} F_{e'}$, $\propto(F_{e_n}, F_{e_{n+1}}) \succsim \tilde{I}_{\tilde{C}}$ for all n, then $\propto(F_{e'}, TF_{e'}) \succsim \tilde{I}_{\tilde{C}}$. If there exist $F_{e_0} \in \tilde{E}$ such that $\propto(F_{e_0}, TF_{e_0}) \succsim \tilde{I}_{\tilde{C}}$, Then T has fixed point in $\tilde{E}$ which was unique.

Proof: Choose $F_{e_0} \in \tilde{E}$ such that $\propto(F_{e_0}, TF_{e_0}) \succsim \tilde{I}_{\tilde{C}}$. Define the sequence { F_{e_n} } $\in \tilde{E}$ by

$F_{e_n} = T^n F_{e_0}$=T$F_{e_{n-1}}$ ∀ n ∈ N. Since T is triangular ∝ − Admissible mapping and

$\propto(F_{e_0}, TF_{e_0}) \succsim \tilde{I}_{\tilde{C}}$, we deduce that

$\propto(F_{e_1}, F_{e_2}) = \propto(TF_{e_0}, T^2F_{e_0}) \succsim \tilde{I}_{\tilde{C}}$. By proceeding in this way, we get

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D.Ram Prasad, Research Scholar, Department of Mathematics, K L University, Vaddeswaram, Guntur - 522502, Andhra Pradesh, India. E-mail: ramprasadmphil09@gmail.com,

G.N.V.Kishore, Department of Mathematics, SRKR Engineering College, China Amiram, Bhimavaram, West Godavari District - 534 204, Andhra Pradesh, India..

E-mail: kishore.apr2@gmail.com

D.W.J. Victor, Department of Mathematics, A. M. A. L. College, Anakapalli, Vishakhapatnam District Andhra Pradesh, India.

Email: dwjvictor69@gmail.com

B. Srinuvasa Rao, Department of Mathematics, Dr. B. R. Ambedkar University, vizag srikakulam hwy, Ambedkar University Rd, Etcherla, Andhra Pradesh 532410, Email: srinivasabagathi@gmail.com

$\propto(F_{e_n}, TF_{e_n}) \gtrsim \tilde{I}_{\tilde{C}} \Rightarrow \|\propto(F_{e_n}, T F_{e_n})\| \geq 1$

$\forall$ n$\in N \cup \{0\}$. For convenience, we denote the element D = $\widetilde{d_{C*}}(F_{e_0}, F_{e_1})$ in $\tilde{C}$.

$$\widetilde{d_{C*}}(F_{e_n}, F_{e_{n+1}}) + r$$
$$= \widetilde{d_{C*}}(TF_{e_{n-1}}, TF_{e_n}) + r$$
$$\preceq \left(\widetilde{d_{C*}}(TF_{e_{n-1}}\ TF_{e_n}) + r\right)^{\propto(F_{e_{n-1}}, TF_{e_{n-1}})\propto(F_{e_n}, TF_{e_n})}$$
$$\preceq \left(\widetilde{d_{C*}}(TF_{e_{n-1}}, TF_{e_n}) + r\right)^{\|\propto(F_{e_{n-1}}, TF_{e_{n-1}})\propto(F_{e_n}, TF_{e_n})\|}$$

$$\preceq \tilde{a}^*\tau\left(\widetilde{d_{C*}}(F_{e_{n-1}}, F_{e_n})\right) \widetilde{d_{C*}}(F_{e_{n-1}}, F_{e_n})\tilde{a} + r$$

Then

$$\widetilde{d_{C*}}(F_{e_n}, F_{e_{n+1}}) \preceq \tilde{a}^*\tau(\widetilde{d_{C*}}(F_{e_{n-1}}, F_{e_n}))\widetilde{d_{C*}}(F_{e_{n-1}}, F_{e_n})\tilde{a}$$
$$\preceq (\tilde{a}^*)^2\tau(\widetilde{d_{C*}}(F_{e_{n-1}}, F_{e_n}))\tau(\widetilde{d_{C*}}(F_{e_{n-2}}, F_{e_{n-1}}))\widetilde{d_{C*}}(F_{e_{n-2}}, F_{e_{n-1}})\tilde{a}^2$$
$$\preceq (\tilde{a}^*)^n \prod_{i=1}^{n} \tau(\widetilde{d_{C*}}(F_{e_{i-1}}, F_{e_i}))\widetilde{d_{C*}}(F_{e_0}, F_{e_1})\tilde{a}^n$$
$$\preceq (\tilde{a}^*)^n \prod_{i=1}^{n} \tau(\widetilde{d_{C*}}(F_{e_{i-1}}, F_{e_i}))D\tilde{a}^n$$

So for n + 1 > m, we have

$$\widetilde{d_{C*}}(F_{e_{n+1}}, F_{e_m}) \preceq \widetilde{d_{C*}}(F_{e_{n+1}}, F_{e_n}) + \widetilde{d_{C*}}(F_{e_n}, F_{e_{n-1}}) + \ldots + \widetilde{d_{C*}}(F_{e_{m+1}}, F_{e_m})$$
$$\preceq (\tilde{a}^*)^n \prod_{i=1}^{n} \tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))D\tilde{a}^n + \ldots + (\tilde{a}^*)^m \prod_{i=1}^{m} \tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))D\tilde{a}^m$$
$$\preceq \sum_{k=m}^{n}\left(\prod_{i=1}^{k} \tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))(\tilde{a}^*)^k D\tilde{a}^k\right) \preceq \sum_{k=m}^{n}\left(\prod_{i=1}^{k} \tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))(\tilde{a}^*)^k D^{\frac{1}{2}} D^{\frac{1}{2}}\tilde{a}^k\right) \preceq \sum_{k=m}^{n}\left((D^{\frac{1}{2}}\tilde{a}^k)\right)$$
$$\preceq \sum_{k=m}^{n}\left(\prod_{i=1}^{k} \tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))\left|D^{\frac{1}{2}}\tilde{a}^k\right|^2\right)$$
$$\preceq \left\|\sum_{k=m}^{n}\prod_{i=1}^{k} \tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))\left|D^{\frac{1}{2}}\tilde{a}^k\right|^2\right\| \tilde{I}_{\tilde{C}}$$
$$\preceq \|D^{\frac{1}{2}}\|^2 \sum_{k=m}^{n}\prod_{i=1}^{k}\|\tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))\|\ \|\tilde{a}\|^{2k}\ \tilde{I}_{\tilde{C}}$$

By using the properties of the function τ and Lemma 36, we conclude that

$$\widetilde{d_{C*}}(F_{e_{n+1}}, F_{e_m}) \preceq \|D^{\frac{1}{2}}\|^2 \sum_{k=m}^{n}\prod_{i=1}^{k}\|\tau(\widetilde{d_{C*}}(F_{e_i}, F_{e_{i-1}}))\|\ \|\tilde{a}\|^{2k}\ \tilde{I}_{\tilde{C}} \to 0 \text{ as } m \to \infty$$

Therefore, $\{F_{e_n}\}_{n=1}^{\infty} \subseteq \tilde{E}$ is a Cauchy sequence in $\tilde{E}$. Since by completeness of $(\tilde{E}, \tilde{C}, \widetilde{d_{c*}})$, we have $F'_e \in \tilde{E}$ with $\lim_{n\to\infty} F_{e_n} = \lim_{n\to\infty} T F_{e_{n-1}} = F_{e'}$. That is $\|\widetilde{d_{C^*}}(F_{e_n}, F_{e'})\| \to 0$ as n→ ∞.

Suppose that $\{F_{e_n}\}_{n=1}^{\infty} \subseteq \tilde{E}$ such that $F_{e_n} \xrightarrow{\|\|.\|_{\tilde{C}}} F_{e'}$, $\propto(F_{e_n}, F_{e_{n+1}}) \gtrsim \tilde{I}_{\tilde{C}}$ for all n then $\propto(F_{e'}, TF_{e'}) \gtrsim \tilde{I}_{\tilde{C}} \Rightarrow \|\propto(F_{e'}, TF_{e'})\| \geq 1$, now we have,

$$\tilde{O}_{\tilde{C}} \preceq \widetilde{d_{C*}}(TF_{e'}, F_{e_{n+1}}) + r \preceq \left(\widetilde{d_{C*}}(TF_{e'}, TF_{e_n}) + r\right)^{\propto(F_{e'}, TF_{e'})\propto(F_{e_n}, TF_{e_n})} \preceq \left(\widetilde{d_{C*}}(TF_{e'}, TF_{e_n}) + r\right)^{\|\propto(F_{e'}, TF_{e'})\propto(F_{e_n}, TF_{e_n})\|}$$
$$\preceq \tilde{a}^*\tau(\widetilde{d_{C*}}(F_{e'}, F_{e_n}))\widetilde{d_{C*}}(F_{e'}, F_{e_n})\tilde{a} + r$$

That is

$$\tilde{O}_{\tilde{C}} \preceq \widetilde{d_{C*}}(TF_{e'}, F_{e_{n+1}}) \preceq \tilde{a}^*\tau(\widetilde{d_{C*}}(F_{e'}, F_{e_n}))\widetilde{d_{C*}}(F_{e'}, F_{e_n})\tilde{a}$$

Now using the triangle inequality, then we have

$$\tilde{O}_{\tilde{C}} \preceq \widetilde{d_{C*}}(TF_{e'}, F_{e'}) \preceq \widetilde{d_{C*}}(TF_{e'}, F_{e_{n+1}}) + \widetilde{d_{C*}}(F_{e_{n+1}}, F_{e'}) \preceq \tilde{a}^*\tau(\widetilde{d_{C*}}(F_{e'}, F_{e_n}))\widetilde{d_{C*}}(F_{e'}, F_{e_n})\tilde{a} + \widetilde{d_{C*}}(F_{e_{n+1}}, F_{e'})$$

Therefore, we have

$$0 \leq \|\widetilde{d_{C*}}(TF_{e'}, F_{e'})\| \leq \|\tilde{a}^*\tau(\widetilde{d_{C*}}(F_{e'}, F_{e_n}))\widetilde{d_{C*}}(F_{e'}, F_{e_n})\tilde{a} + \widetilde{d_{C*}}(F_{e_{n+1}}, F_{e'})\|$$
$$\leq \|\tilde{a}^*\|^2\|\tau(\widetilde{d_{C*}}(F_{e'}, F_{e_n}))\|\ \|\widetilde{d_{C*}}(F_{e'}, F_{e_n})\tilde{a}\| + \|\widetilde{d_{C*}}(F_{e_{n+1}}, F_{e'})\|$$
$$< \|\tilde{a}^*\|^2\|\tau(\widetilde{d_{C*}}(F_{e'}, F_{e_n}))\|\|\widetilde{d_{C*}}(F_{e'}, F_{e_n})\tilde{a}\| + \|\widetilde{d_{C*}}(F_{e_{n+1}}, F_{e'})\| \to 0 \text{ as } n \to \infty$$

Hence we get that $\widetilde{d_{C*}}(TF_{e'}, F_{e'}) = 0$ implies that $TF_{e'} = F_{e'}$.

Suppose $F_{e'}$ and $F_{e''}$ $(F_{e'} \neq F_{e''})$ are two fixed points of T. Then

$\propto(F_{e'}, TF_{e'}) \gtrsim \tilde{I}_{\tilde{C}}$ and $\propto(F_{e''}, TF_{e''}) \gtrsim \tilde{I}_{\tilde{C}}$, so that

$$\widetilde{d_{C*}}(F_{e'}, F_{e''}) + r \preceq \widetilde{d_{C*}}(TF_{e'}, TF_{e''}) + r \preceq \left(\widetilde{d_{C*}}(TF_{e'}, TF_{e''}) + r\right)^{\propto(F_{e'}, TF_{e'})\propto(F_{e''}, TF_{e''})} \preceq \tilde{a}^*\tau(\widetilde{d_{C*}}(F_{e'}, F_{e''}))\widetilde{d_{C*}}(F_{e'}, F_{e''})\tilde{a} + r$$

Therefore,

$\|\tau(\widetilde{d_{C*}}(F_{e'}, F_{e''}))\| \to 1$ thus

$\|\widetilde{d_{C*}}(F_{e'}, F_{e''})\| < \|\widetilde{d_{C*}}(F_{e'}, F_{e''})\|$,

It is contradiction. Hence T has the fixed point which was unique.

Example 3.2: Let U = [0; ∞) , and $\tilde{C} = M_2(R(C)^*)$, C = E = [0; 1] and $\tilde{E}$ be fuzzy soft set which was absolute. Define $\widetilde{d_{C*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ with $\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

Here i = inf{| $\mu^a F_{e_1}(s)$ - $\mu^a F_{e_2}(s)$ | / s∈ C }.

It is clearly $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ is a C*- Algebra Valued Fuzzy Soft MS Which is also complete. Now define T: $\tilde{E} \to \tilde{E}$ by

$$T F_e = \begin{cases} \frac{F_e}{4}, & if\ F_e \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1] \\ 2F_e & if\ F_e \in \{\frac{1}{3}, \frac{1}{2}\} \end{cases}$$ and

τ : [0;∞) →[0; 1] be defined as $\tau(t) = \frac{1}{3}$

Also we define $\propto: \tilde{E} \times \tilde{E} \to \tilde{C}_+$ by

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$$\propto(F_{e_1}, F_{e_2}) = \begin{cases} \tilde{I}_{\tilde{C}}, & if\ F_{e_1}, F_{e_2} \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1] \\ \tilde{o}_{\tilde{C}}, & Otherwise \end{cases}$$

First we shows that T is triangular $\propto$-admissible mapping.
Let $F_{e_1}, F_{e_2} \in \tilde{E}$, $\propto(F_{e_1}, F_{e_2}) \gtrsim \tilde{I}_{\tilde{C}}$ implies
$\|\propto(F_{e_1}, F_{e_2})\| \geq 1$, then
$F_{e_1}, F_{e_2} \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1]$
On the other hand, for all $F_e \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1]$, we have
$TF_e \leq 1$. It follows that
$\|\propto(TF_{e_1}, TF_{e_2})\| \geq 1$
Implies that $\propto(TF_{e_1}, TF_{e_2}) \gtrsim \tilde{I}_{\tilde{C}}$. Also, for all $F_{e_1}, F_{e_2}, F_{e_3} \in \tilde{E}$ such that $\propto(F_{e_1}, F_{e_3}) \gtrsim \tilde{I}_{\tilde{C}}$ and $\propto(F_{e_2}, F_{e_3}) \gtrsim \tilde{I}_{\tilde{C}} \Rightarrow$ $F_{e_1}, F_{e_2}, F_{e_3} \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1]$ and so $\propto(F_{e_1}, F_{e_2}) \gtrsim \tilde{I}_{\tilde{C}}$.
Therefore, the assertion holds. In reason of the above arguments, $\propto(0, T0) \gtrsim \tilde{I}_{\tilde{C}}$ implies
$\|\propto(0, T0)\| \geq 1$.
Now, if the sequence $\{F_{e_n}\}$ in $\tilde{E}$ such that $\propto(F_{e_n}, F_{e_{n+1}}) \gtrsim \tilde{I}_{\tilde{C}}$ $\forall$ n = 0; 1; 2; …………and
$F_{e_n} \xrightarrow{\|\|.\|_{\tilde{C}}} F_{e'}$ as n→ ∞, then $\{F_{e_n}\} \subseteq [0, \frac{1}{3}) \cup (\frac{1}{2}, 1]$ and hence $F_{e'} \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1]$. Which implies that $\propto(F_{e'}, TF_{e'}) \gtrsim \tilde{I}_{\tilde{C}} \Rightarrow \|\propto(F_{e'}, TF_{e'})\| \geq 1$.
Let $F_{e_1}, F_{e_2} \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1]$, we have

$$\left(\widetilde{d_{C*}}(TF_{e_1}, TF_{e_2}) + r\right)^{\propto(F_{e_1}, T F_{e_1})\propto(F_{e_2}, T F_{e_2})} \leq \widetilde{a^*}\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\, \widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a} + r$$

Otherwise, $\propto(F_{e_1}, T F_{e_1}) \propto (F_{e_2}, T F_{e_2}) = 0$ and we have

$$\left(\widetilde{d_{C*}}(TF_{e_1}, TF_{e_2}) + r\right)^{\propto(F_{e_1}, T F_{e_1})\propto(F_{e_2}, T F_{e_2})} = 1$$

$$\leq \widetilde{a^*}\tau\left(\widetilde{d_{C*}}(F_{e_1}, F_{e_2})\right)\widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a} + r.$$

Therefore it is clear that the conditions of Theorem 3.1 are satisfied and T has a fixed point.

Theorem 3.2.: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ be complete C*-Algebra Valued Fuzzy Soft MS. Let T be triangular $\propto$- Admissible mapping on $\tilde{E}$. Let that there exist a function
$\tau : [0;\infty) \to [0; 1]$ with $\tau(\tilde{t}_n) \to \tilde{1}$ implies that $\tilde{t}_n \to \tilde{0}$)
and
$\precsim 2^{\widetilde{a^*}\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a}}$
For all $\{\tilde{t}_n\}$ of positive fuzzy soft reals, $F_{e_2}, F_{e_3} \in \tilde{E}$, where $\tilde{a} \in \tilde{C}$ with $\|\tilde{a}\|$<1 and r≥ 1. Suppose that if $\{F_{e_n}\}$ on $\tilde{E}$ with $F_{e_n} \xrightarrow{\|\|.\|_{\tilde{C}}} F_{e'}$, $\propto(F_{e_n}, F_{e_{n+1}}) \gtrsim \tilde{I}_{\tilde{C}}$ for all n. Then
$\propto(F_{e'}, TF_{e'}) \gtrsim \tilde{I}_{\tilde{C}}$. If there exist $F_{e_0} \in \tilde{E}$ such that $\propto(F_{e_0}, TF_{e_0}) \gtrsim \tilde{I}_{\tilde{C}}$, Then T has a fixed point in $\tilde{E}$ which was unique.

$$TF_e = \begin{cases} \frac{F_e^2}{9}, & if\ F_e \in E - \{\frac{1}{3}, \frac{1}{2}\} \\ log F_e & if\ F_e \in \{\frac{1}{3}, \frac{1}{2}\} \end{cases} \quad \text{and}$$

$\tau : [0;\infty) \to [0; 1]$ be defined as $\tau(t) = \frac{1}{4}$
Also we define $\propto: \tilde{E} \times \tilde{E} \to \tilde{C}_+$ by $\propto(F_{e_1}, F_{e_2}) =$

$$\begin{cases} \tilde{I}_{\tilde{C}}, & if\ F_{e_1}, F_{e_2} \in E - \{\frac{1}{3}, \frac{1}{2}\} \\ \tilde{o}_{\tilde{C}}, & Otherwise \end{cases}$$

Clearly T is triangular $\propto$ - admissible mapping and satisfies all conditions of Theorem 3.2. From Theorem 3. 2. T has fixed point which is unique.

Theorem 3.3.: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ be Complete C*- Algebra valued Fuzzy Soft Metric Space, T be triangular $\propto$ - admissible mapping on $\tilde{E}$. Let $\tau : [0, \infty) \to [0, 1]$ be function with $\tau(\tilde{t}_n) \to \tilde{1}$ implies that $\tilde{t}_n \to \tilde{0}$ and

$$(\propto(F_{e_1}, T F_{e_1}) \propto (F_{e_2}, T F_{e_2}))\, \widetilde{d_{C*}}(TF_{e_1}, TF_{e_2}) \leq \widetilde{a^*}\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\, \widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a}$$

For all $\{\tilde{t}_n\}$ of positive fuzzy soft reals, $F_{e_2}, F_{e_3} \in \tilde{E}$, where $\tilde{a} \in \tilde{C}$ with $\|\tilde{a}\|$<1 and r≥ 1. Suppose that if $\{F_{e_n}\}$ is a sequence in $\tilde{E}$ such that
$F_{e_n} \xrightarrow{\|\|.\|_{\tilde{C}}} F_{e'}$, $\propto(F_{e_n}, F_{e_{n+1}}) \gtrsim \tilde{I}_{\tilde{C}}$ for all n, then $\propto(F_{e'}, TF_{e'}) \gtrsim \tilde{I}_{\tilde{C}}$. If there exist $F_{e_0} \in \tilde{E}$ such that $\propto(F_{e_0}, TF_{e_0}) \gtrsim \tilde{I}_{\tilde{C}}$. Then T has fixed point which is unique.

Example 3.4.: Let U = [0; ∞) & C = E = $\{\frac{1}{n} / n \in N\} \cup \{0\}$. $\tilde{E}$ be absolute fuzzy soft set, $\tilde{C} = M_2(R(C^*))$ and define $\widetilde{d_{C*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ as $\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$
here i = inf { | $\mu^a F_{e_1}(s)$ - $\mu^a F_{e_2}(s)$ | / s ∈ C }. Clearly $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ is a C* - Algebra Valued Fuzzy soft Metric Space and which was complete. Now define T: $\tilde{E} \to \tilde{E}$ as fuzzy soft set, $\tilde{C} = M_2(R(C^*))$ and define
$\widetilde{d_{C*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ by $\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$
here i = inf { | $\mu^a F_{e_1}(s)$ - $\mu^a F_{e_2}(s)$ | / s ∈ C }.
Clearly $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ is complete C* - Algebra Valued Fuzzy Soft Metric Space. Also define T: $\tilde{E} \to \tilde{E}$ as

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$$TF_e = \begin{cases} \frac{1-F_e^2}{6}, & if\ F_e \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1] \\ F_e & if\ F_e \in \{\frac{1}{3}, \frac{1}{2}\} \end{cases}$$

and $\tau : [0;\infty) \to [0; 1]$ be defined as $\tau(t) = \frac{1}{2}$

Also we define $\propto : \tilde{E} \times \tilde{E} \to \tilde{C}_+$ by

$$\propto(F_{e_1}, F_{e_2}) = \begin{cases} \tilde{I}_{\tilde{C}}, & if\ F_{e_1}, F_{e_2} \in [0, \frac{1}{3}) \cup (\frac{1}{2}, 1] \\ \tilde{o}_{\tilde{C}}, & Otherwise \end{cases}$$

Hence it is clear that the conditions of Theorem 3.1 are satisfied and T has a fixed point.

Application to Integral Equation

Let us consider the boundary value problem of ODE :

$$\begin{cases} \frac{-d^2 \ F_e(t)}{dt^2} = K(t, F_e(t)) \ \ if \ \ t \in [0, 1] \quad (3) \\ F_e(0) = F_e(1) = 0 \end{cases}$$

Where K: [0; 1] × R(C)+ → R(C)+ is continuous mapping. The Green's function to (3) is given by $I(t, s) = \begin{cases} t(1-s), & 0 \le t \le s \le 1 \\ s(1-t), & 0 \le s \le t \le 1 \end{cases}$

Let E = C = [0; 1] be Lebesgue measurable set, $\tilde{\tilde{E}} = L^\infty(C)$ be absolute fuzzy soft set, H = $L^2(C)$ be denoted by Hilbert Space. Clearly L(H) is a C*- algebra with usual norm. Also define $\widetilde{d_{C*}} : \tilde{E} \times \tilde{E} \to L(H)$ by

$\widetilde{d_{C*}}(F_{e_1} ; F_{e_2}) = M_{\inf\{|\mu^a F_{e_1}(s) - \mu^a F_{e_2}(s)| / s \in C\}}$

$\forall\ F_{e_1}, F_{e_2} \in \tilde{E}$ where M_h: H → H is any multi – valued operator defined by $M_h(\tau)$ = h τ for $\tau \in$ H. Then $(\tilde{E}$, L(H), $\widetilde{d_{C*}}$) is a complete C* - Algebra valued fuzzy Soft Metric Space.

Let Ψ: [0; ∞) → [0; ∞) be satisfying :

(i) Ψ is increasing ;

(ii) for each t > 0 , Ψ (t) < t;

(iii) $\tau(t) = \frac{\Psi(t)}{t} \in \mathcal{F}$ where

$\mathcal{F} = \{ \tau \ / \ \tau: [0;\infty) \to [0; 1] \}$ As example of such function, we can list the following ,

$\Psi(t) = \frac{t}{1+t}$ and , $\Psi(t) = \ln(1+t)$

We consider the following conditions:

(a) There is a function $\theta: R(C)^2{}_+ \to$ R(C)* such that for all t ∈E, for all $F_{e_1}, F_{e_2} \in$ R(C)+ with $\theta(F_{e_1}, F_{e_2}) \ge \tilde{0}$ and r∈ (0, 1) we have,

$$\inf\{ |K(t, F_{e_1}(s)) - K(t, F_{e_2}(s))| \ / s \in C\} \le r\Psi(\inf\{ |F_{e_1}(s) - F_{e_2}(s)| \ / s \in C \})$$

(b) There is $F_e \in L^\infty$(C) such that

$$\theta(F_e(t), \int_0^1 I(t,s)K(s, F_e(s)ds) \ge \tilde{0} \ \forall\ t \in E$$

(c) For all t ∈ E and for all $F_{e_1} ; F_{e_2} \in L^\infty(C)$,

$$\theta(F_{e_1}(t), F_{e_2}(t)) \ge \tilde{0} \Rightarrow$$

$$\theta(\int_0^1 I(t,s)K(s, F_{e_1}(s))ds, \int_0^1 I(t,s)K(s, (s)ds) \ge \tilde{0}$$

(d) For any cluster point $F_{e'}$ of sequence $\{F_{e_n}\} \in L^\infty(C)$ with $\theta(F_{e_n}(t), F_{e_{n+1}}(t)) \ge \tilde{0}$ and

$$\lim_{n\to\infty} inf\ \theta(F_{e_n}(t), F_{e'}(t)) \ge \tilde{0}$$

Theorem 3. 4. : Suppose that conditions (a) - (d) are satisfied. Then boundary value problem (3) has minimum one solution $F_{e'} \in L^\infty(C)$

Proof: Since we know that the solution of (3) is exists iff the solution of the integral equation

$F_e(s) = \int_0^1 I(t,s)K(s, F_e(s)ds)$ is exist and the same.

Define T : $L^\infty(C) \to L^\infty(C)$ as

$$TF_e(s) = \int_0^1 I(t,s)K(s, F_e(s)ds)$$

for all t ∈ E. Clearly $F_{e'} \in L^\infty$(C) that is a fixed point of T.

Let $\tilde{C} = r\,\tilde{I}_{\tilde{C}}$ then $\tilde{C} \in L(H)_+$ and $\|\tilde{C}\|$ = r < 1.

For any h ∈ H and let $F_{e_1}, F_{e_2} \in L^\infty$(C), such that $\theta(F_{e_1}(t), F_{e_2}(t)) \ge \tilde{0}$ for all t ∈ E.

From (a) we have

$$\widetilde{d_{C*}}(TF_{e_1} TF_{e_2}) = M_{\inf\{|\mu^t F_{e_1}(s) - \mu^t F_{e_2}(s)| / s \in C\}}$$

$$\|\widetilde{d_{C*}}(TF_{e_1}, TF_{e_2})\|$$

$$= \sup_{\|h\|=1} (M_{\inf\{|\mu^t F_{e_1}(s) - \mu^t F_{e_2}(s)| / s \in C\}} h, h)$$

$$\sup_{\|h\|=1} \int_0^1 [\inf\{\int_0^1 I(t,s)\left(K\left(s, F_{e_1}(s)\right) - K\left(s, F_{e_2}(s)\right)\right)| \ / s \in C \}ds\ ]h(t)\ \overline{h(t)}dt$$

$$\le \sup_{\|h\|=1} \int_0^1 [\inf\{\int_0^1 I(t,s)|\}\ r\Psi(\inf\{|F_{e_1}(s) - F_{e_2}(s)| \ / s \in C \})ds\ ]\ \|h(t)\|^2 dt$$

$$\le r \sup_{\|h\|=1} \int_0^1 [\inf\{|\int_0^1 I(t,s)|\}\ ds\ ]\ \|h(t)\|^2 dt. \ \|\Psi(\inf\{|F_{e_1}(s) - F_{e_2}(s)| \ / s \in C\})\|$$

$$\le r \sup_{\|h\|=1} \int_0^1 \left[\inf\{|\int_0^1 I(t,s)|\}ds\right] \sup_{\|h\|=1} \int_0^1 |h(t)|^2\ dt.\ \|\Psi(\inf\{|F_{e_1}(s) - F_{e_2}(s)|\})\|_\infty$$

$$\le r\|\Psi(\inf\{|F_{e_1}(s) - F_{e_2}(s)|\})\|_\infty$$

$$\le r \frac{\|\Psi(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\|}{\|\widetilde{d_{C*}}(F_{e_1}, F_{e_2})\|} \|\widetilde{d_{C*}}(F_{e_1}, F_{e_2})\|$$

$$\le \|\tilde{a}\| \|\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\|\ \|\widetilde{d_{C*}}(F_{e_1}, F_{e_2})\|$$

Thus we have

$\widetilde{d_{C*}}(F_{e_1}, F_{e_2}) \le \widetilde{a^*}\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\ \widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a}$

for all $F_{e_1}, F_{e_2} \in L^\infty(C)$,

Such that $\theta(F_{e_1}(t), F_{e_2}(t)) \ge \tilde{0}$ for all t ∈ E.

We define $\propto: L^\infty(C) \times L^\infty(C) \to \tilde{C}_+$ by $\propto(F_{e_1}, F_{e_2}) = \begin{cases} \tilde{I}_{\tilde{C}}, & if\ t \in E,\ \theta(F_{e_1}(t), F_{e_2}(t)) \ge \tilde{0}, \\ \tilde{o}_{\tilde{C}}, & Otherwise \end{cases}$

Then for all $F_{e_1}, F_{e_2} \in L^\infty(C)$, we have

$$(\propto(F_{e_1}, \mathrm{T}\ F_{e_1}) \propto (F_{e_2}, \mathrm{T}\ F_{e_2}))\ \widetilde{d_{C*}}(TF_{e_1}, \mathrm{T}F_{e_2}) \le \widetilde{a^*}\tau(\widetilde{d_{C*}}(F_{e_1}, F_{e_2}))\ \widetilde{d_{C*}}(F_{e_1}, F_{e_2})\tilde{a}$$

Obviously, T is triangular ∝ –admissible. From (b) there is $F_{e_1} \in L^\infty$(C), such that $\propto(F_{e_1}, \mathrm{T}\ F_{e_1}) = \tilde{I}_{\tilde{C}}$ and $F_{e_2} \in L^\infty$(C) such that $\propto(F_{e_2}, \mathrm{T}\ F_{e_2})) = \tilde{I}_{\tilde{C}}$

By (d), we have that for any cluster point $F_{e'}$ of a sequence $\{F_{e_n}\} \in L^\infty$(C) with $\propto(F_{e_n}, F_{e_{n+1}}) = \tilde{I}_{\tilde{C}}$ and $\lim_{n\to\infty} inf \propto (F_{e_n}, F_{e'}) = \tilde{I}_{\tilde{C}}$. By applying Theorem 3.3, T has fixed point in L^∞(C) . Also $F_{e'} \in L^\infty$(C) is a solution of BVP (3).

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III. CONCLUSION

In the paper we introduced new notion called triangular $\propto$ -Admissible mapping. Subsequently, we proved some Fixed point theorems and Examples.

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