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Complete Cototal Domination Number of Certain Graphs

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Abstract--- The main purpose of this paper is to investigate Complete Cototal Domination Number of Certain Graphs. In addition its some parameters are found.

Keywords--- Domination Number, Total Domination Number, CototalDomination Number, CompleteCototalDomination Number.

I. INTRODUCTION

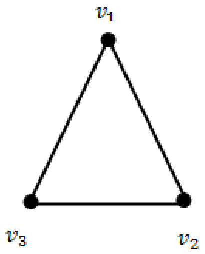
A set $D \subseteq V$of a graph is called a total dominating set if the induced subgraph$\langle D \rangle$ has no isolated vertices. The total domination number$\gamma_{td}(G)$ of$G$ is the minimum cardinality of a total dominating set of$G$ [2,3,4,5]. A dominating set$D$is said to be a cototal dominating set if the induced subgraph$\langle V - D \rangle$ has no isolated vertices. The cototal domination number$\gamma_{ctd}(G)$ of$G$is the minimum cardinality of a cototal dominating set of G[6].

II. COMPLETE COTOTAL DOMINATION NUMBER

Definition: 2.1

A total dominating set D is said to be a complete cototal dominating setif the induced subgraph$\langle V - D \rangle$ has no isolated vertices. The complete cototal domination number $\gamma_{cc}(G)$of $G$is the minimum cardinality of a complete cototal dominating set of G[1,6].

Example:2.2



$\gamma_{td}(G) = \{v_1, v_2\}$
$\gamma_{cctd}(G) = \{v_1, v_2\} \cup \{v_3\}$
$\gamma_{cc}(G) = 3.$

Theorem:2.3 For a Friendship graph F_n,$\gamma_{cc}(F_n) = 3, n \geq 2$.

Proof. TheFriendship graph F_n has $(2n+1)$ nodes $v_1, v_2, \dots, v_{2n}, u$ and $3n$ edges $v_1v_2, v_3v_4, \dots, v_{2n-1}v_{2n}, v_iu, 1 \leq i \leq 2n$.Here$u$is the middle node. Let us consider the total dominating set $\gamma_{td}(F_n) = \{u, x\}$ where x is any one of the node v_i. Minimal cototaldominating set is obtained by $(G - \{u, x\} \cap \{y\})$ where y is an isolated node.

Hence$\gamma_{cctd}(F_n) = \{u, x\} \cup \{y\}$
Therefore $\gamma_{cc}(F_n) = 3$.

Theorem:2.4 For a Complete bipartite graph $K_{m,n}$,$\gamma_{cc}(K_{m,n}) = 2, m, n \geq 2$.

Proof. The Complete bipartite graph $K_{m,n}$ has $(m+n)$ nodes $v_1, v_2, \dots, v_m, u_1, u_2, \dots, u_nandmn$edges $v_iu_j, 1 \leq i \leq m, 1 \leq j \leq n$. Let us consider the total dominating set

$\gamma_{td}(K_{m,n}) = \{v_1, u_1\}$. Minimal cototaldominating set is obtained by $G - \{v_1, u_1\}$.

Hence$\gamma_{cctd}(K_{m,n}) = \{v_1, u_1\}$.

Therefore $\gamma_{cc}(K_{m,n}) = 2$.

Theorem:2.5 For a Fan graph f_n,$\gamma_{cc}(f_n) = 2, n \geq 3$.

Proof. The Fan graph f_n has $(n+1)$ nodes $u, v_1, v_2, \dots, v_n$ and $(2n-1)$ edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_iu, 1 \leq i \leq n$.Here$u$ be the middle node. Let us consider the total dominating set$\gamma_{td}(f_n) = \{u, v_1\}$. Minimal cototaldominating set is obtained by$G - \{u, v_1\}$.

So $\gamma_{cctd}(f_n) = \{u, v_1\}$. Hence $\gamma_{cc}(f_n) = 2$.

Theorem:2.6 For a Helm graph H_n,$\gamma_{cc}(H_n) = n + 1$.

Proof. The Helm graph $H_n$has $(2n+1)$ nodes $u, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ and $3n$edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_iu_i, 1 \leq i \leq n$and $uv_i, 1 \leq i \leq n$.Let us consider the total dominating set$\gamma_{td}(H_n) = \{u_1, u_2, \dots, u_n, x\}$ where x is any one of the node of cycle C_n. Minimal cototaldominating set is obtained by $G - \{u_1, u_2, \dots, u_n, x\}$.

$\therefore \gamma_{cctd}(H_n) = \{u_1, u_2, \dots, u_n, x\}$so that $\gamma_{cc}(H_n) = n + 1$.

Theorem:2.7 For a Comb graph $P_n \odot K_1$,$\gamma_{cc}(P_n \odot K_1) = 2n, n \geq 2$.

Proof. The Comb graph $P_n \odot K_1$ has $2n$ nodes $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ and $(2n-1)$ edges $v_iv_{i+1}, 1 \leq i \leq n-1$ and $v_iu_i, 1 \leq i \leq n$. Let $v_1, v_2, \dots, v_n$ be the nodes of P_n and $u_1, u_2, \dots, u_n$ be the pendant nodes. Let us consider the total dominating set $\gamma_{td}(P_n \odot K_1) = \{v_1, v_2, \dots, v_n\}$where y is anisolated node. Minimal cototaldominating set is obtained by $(G - \{v_1, v_2, \dots, v_n\}) \cap \{y\}$ where y is an isolated node so that $\gamma_{cctd}(P_n \odot K_1) = \{v_1, v_2, \dots, v_n\} \cup \{y\}$.Therefore $\gamma_{cc}(P_n \odot K_1) = 2n$.

Theorem:2.8 For a Ladder graph L_n,$\gamma_{cc}(L_n) = n, n \geq 2$.

Proof. The Ladder graph $L_n$has $2n$ nodes $x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n$ and $(3n-2)$ edges $x_iy_i, x_ix_{i+1}, y_iy_{i+1}, 1 \leq i \leq n$.Let us consider the total dominating set$\gamma_{td}(L_n) = \{x_1, x_2, \dots, x_n\}$. Minimal cototaldominating set is obtained by$G - \{x_1, x_2, \dots, x_n\}$

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So $\gamma_{cctd}(L_n) = \{x_1, x_2, \dots, x_n\}$. Hence $\gamma_{cc}(L_n) = n$.

Theorem:2.9 For a n-sunlet graph, $\gamma_{cc}(n - \text{sunlet}) = n + 1, n \geq 3$.

Proof. The n-sunletgraphhas $2n$ nodes $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ and

$2n$edges$v_1v_2, v_3v_4, \dots, v_{n-1}v_n$, and$u_iv_i$ $1 \leq i \leq n-1$ and $v_iu_i, 1 \leq i \leq n$.Let us consider the total dominating set$\gamma_{td}(n - \text{sunlet}) = \{u_1, u_2, \dots, u_n, x\}$ where x beany one of the node of cycle C_n.Minimal cototaldominating set is obtained by $G - \{u_1, u_2, \dots, u_n, x\}$.

$\therefore \gamma_{cctd}(n - \text{sunlet}) = \{u_1, u_2, \dots, u_n, x\}$so that $\gamma_{cc}(n -$ sunlet$=n+1$.

Theorem:2.10 For a Bistar graph $B_{m,n}$,$\gamma_{cc}(B_{m,n}) = m + n + 2$.

Proof. The Bistar graph $B_{m,n}$ has $(m + n + 2)$ nodes $u, u_1, u_2, \dots, u_m, v, v_1, v_2, \dots, v_n$, and$(m + n + 1)$ edges uv, uu_iandvv_j, $1 \leq i \leq m, 1 \leq j \leq n$.Let us consider the total dominating set$\gamma_{td}(B_{m,n}) = \{u, v\}$. Minimal cototaldominating set is obtained by $(G - \{u, v\}) \cap \{y\}$ where y is an isolated node so that$\gamma_{cctd}(B_{m,n}) = \{u, v\} \cup \{y\}$. Hence $\gamma_{cc}(B_{m,n}) = m + n + 2$.

Theorem:2.11 For a Corona graph $P_n \odot K_2$,$\gamma_{cc}(P_n \odot K_2) = n, n \geq 2$.

Proof. The Corona graph $P_n \odot K_2$has $3n$ nodes $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_{2n}$ and $(4n - 1)$ edges.Let$v_1, v_2, \dots, v_n$ be the nodes of P_n and $u_1, u_2, \dots, u_{2n}$ be the nodes of triangles.Let us consider the total dominating set $\gamma_{td}(P_n \odot K_2) = \{v_1, v_2, \dots, v_n\}$. Minimal cototaldominating set is obtained by$G - \{v_1, v_2, \dots, v_n\}$.$\therefore$ $\gamma_{cctd}(P_n \odot K_2) = \{v_1, v_2, \dots, v_n\}$. Hence$\gamma_{cc}(P_n \odot K_2) = 2n$.

Theorem:2.12 For a Coconut tree graph (m, n),$\gamma_{cc}(CT(m, n)) = m + n$where$m, n \geq 2$.

Proof. The Coconut tree graph $CT(m, n)$has $(m + n)$ nodes $u, u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_{n-1}$and $(m + n - 1$edges uui, $uv1$and$vjvj+1$, $1\leq i\leq m, 1\leq j\leq n$.Let$u1, u2, \dots, um$ be the pendant nodes of star $K_{1,m}$and $u, v_1, v_2, \dots, v_{n-1}$ be the nodes of path P_n with u as a common node. Let us consider the total dominating set$\gamma_{td}(CT(m, n)) = \{u, v_1, x\}$where$x = v_i$,

$i = 3,5,7, \dots$.Minimal cototaldominating set is obtained by $(G - \{u, v_1, x\}) \cap \{y\}$ where y is an isolated node. Hence $\gamma_{cctd}(CT(m, n)) = \{u, v_1, x\} \cup \{y\}$.Therefore $\gamma_{cc}(CT(m, n)) = m + n$.

Observation:2.13 For a Pan graph $T_{n,1}$, $\gamma_{cc}(T_{n,1}) = n - 1$, where $n \geq 3$.

III. BOUNDS FOR $\gamma_{cc}(G)$

Theorem:3.1 Let G be a connected graph, then$\gamma_{cc}(G) > \left\lceil \frac{n}{\Delta(G)} \right\rceil$.

Proof: Let S ⊆ V (G) be a completecototal dominating set in G. Every node in S dominates

at most $\Delta(G) - 1$ vertices of$V(G) - S$and dominate at least one of the vertices in S. Hence, $|S|(\Delta(G) - 1) + |S| > n$. Since, S is an arbitrary completecototal dominating set, then$\gamma_{cc}(G) > \left\lceil \frac{n}{\Delta(G)} \right\rceil$.

Theorem:3.2 IfG is a connected graph with the girth of length $g(G) \geq 3$ and $\delta(G) \geq 2$, then $\gamma_{cc}(G) > n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Proof: Let Gbe a connected graph with $g(G) \geq 3$ and let Cbe a cycle of length $g(G)$. Remove $C$from $G$to form a graph G'. Suppose an arbitrary node$v \in V(G')$.Since$\delta(G) \geq 2$, $v$has at least two neighbors say $x$and y. Let $x, y \in C$. If $d(x, y) \geq 3$, then replacing the path from $x$to$y$on $C$with the path x, v, y, whichreduces the girth of G, a contradiction. If $d(x, y) \leq 2$, then $x, y, v$are on either C_3or C_4in G, contradicting the hypothesis that$g(G) \geq 3$. Hence, no node in G'has two or more neighbours on C. Since $\delta(G) \geq 2$, the graph G'has minimum degree at least $\delta(G) - 1 \geq 1$. Then Ghas no isolated node. Now letS'be a γ_{cc} –set for$C$. Then $S = S' \cup V(G')$ is acompletecototal dominating set for G. Hence,$\gamma_{cc}(G) > n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Theorem:3.3 If$G$is a Complete bipartite graph with the girth of length $g(G) \geq 4$ and $\delta(G) \geq 2$, then $\gamma_{cc}(G) < n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Proof: Let Gbe a Complete bipartite graph with $g(G) \geq 4$ and let Cbe a cycle of length $g(G)$. Remove $C$from $G$to form a graph G'. Suppose an arbitrary node$v \in V(G')$. Since $\delta(G) \geq 2$, then$v$has at least two neighbors say $x$and y. Let $x, y \in C$. If $d(x, y) \geq 2$, then replacing the path from $x$to$y$on $C$with the path x, v, ywhich reduces the girth of G, a contradiction. If $d(x, y) \leq 2$, then $x, y, v$are on C_4in G, contradicting the hypothesis that $g(G) \geq 4$. Hence, no node in G'has two or more neighbours on C. Since $\delta(G) \geq 2$, the graph G'has minimum degree at least $\delta(G) - 1 \geq 1$. Then Ghas no isolated node. Now letS'be a γ_{cc} –set for C. Then $S = S' \cup V(G')$ is a completecototal dominating set for G. Hence,$\gamma_{cc}(G) < n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Theorem:3.4 If$G$is a Fan graph with the girth of length $g(G) \geq 3$ and $\delta(G) \geq 2$, then $\gamma_{cc}(G) < n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Proof: Let Gbe a Fan graph with $g(G) \geq 3$ and let Cbe a cycle of length $g(G)$. Remove $C$from $G$to form a graph G'. Suppose an arbitrary node $v \in V(G')$.Since$\delta(G) \geq 2$, $v$has at- least two neighbors say $x$and y. Let $x, y \in C$. If $d(x, y) \geq 3$, then replacing the path from $x$to$y$on $C$with the path x, v, ywhichreduces the girth of G, a contradiction. If $d(x, y) \leq 2$, then $x, y, v$are on either C_3or C_4in G, contradicting the hypothesis that$g(G) \geq 3$. Hence, no node in G'has two or more neighbours on C. Since $\delta(G) \geq 2$, the graph G'has minimum degree at least $\delta(G) - 1 \geq 1$. Then Ghas no isolated node. Now letS'be a γ_{cc} –set for C. Then $S = S' \cup V(G')$ is a completecototal dominating set for G. Hence,$\gamma_{cc}(G) < n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Result: 3.5 The above bound is sharp for f_3 since $\gamma_{cc}(f_3) = 2$.

Theorem:3.6 If Gbe a Pan graph with the girth of length $g(G) \geq 3$and $\delta(G) \geq 2$, then $\gamma_{cc}(G) < n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

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Proof: Let Gbe a connected graph and a Pan graph with $g(G) \geq 3$ and let Cbe a cycle of length $g(G)$. Remove $C$from $G$to form a graph G'. Suppose an arbitrary node $v \in V(G^{'})$.Since$\delta(G) \geq 2$, v has at least two neighbors say $x$and y. Let $x, y \in C$. If $d(x, y) \geq 3$, then replacing the path from $x$to$y$on $C$with the path x, v, ywhich reduces the girth of G, a contradiction. If $d(x, y) \leq 2$, then $x, y, v$are on either C_3or C_4in G, contradicting the hypothesis that$g(G) \geq 3$. Hence, no node in G'has two or more neighbours on C. Since $\delta(G) \geq 2$, the graph G'has minimum degree at least $\delta(G) - 1 \geq 1$. Then Ghas no isolated node. Now let$S^{'}$be a γ_{cc} $-$set for C. Then $S = S' \cup V(G^{'})$ is a completecototal dominating set for G. Hence,$\gamma_{cc}(G) < n - \left\lceil \frac{g(G)}{2} \right\rceil + 1$.

Result:3.7 The above bound is sharp for $3 - pan$and $4 - pan$ since$\gamma_{cc}(3 - pan) = 2$ and $\gamma_{cc}(4 - pan) = 3$.

Theorem:3.8 Let G be a graph with $\text{diam}(G) \geq 1$, then $\gamma_{cc}(G) \geq \delta(G) + 1$.

Proof: Let$x \in V(G)$ and $\deg(x) = \delta(G)$. Since $\text{diam}(G) \geq 1$, then $N(x)$ is a total dominating set for G. Now $S = N(x) \cup \{x\}$ is a complete cototal dominating set for G and $|S| = \delta(G) + 1$. Hence, $\gamma_{cc}(G) \geq \delta(G) + 1$.

Result:3.9 The above bound is sharp for F_n,since $\gamma_{cc}(F_n) = 3$ and also $\delta(F_n) = 2$.

Theorem:3.10 Let G be a graph with $\text{diam}(G) \geq 1$, then $\gamma_{cc}(G) < \delta(G) + 1$, if$G$ is eitherComplete bipartite graph or Fan graph or ladder graph or corona graph.

Proof : Let$x \in V(G)$ and $\deg(x) = \delta(G)$. Since $\text{diam}(G) \geq 1$, then $N(x)$ is a total dominating set for G. Now $S = N(x) \cup \{x\}$ is a complete cototal dominating set for G and $|S| = \delta(G) + 1$. Hence, $\gamma_{cc}(G) < \delta(G) + 1$.

IV. CONCLUSION

In this paper, Complete Cototal Domination Number of Certain Graphs and some bounds were studied.

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