Einstein Operations of Intuitionistic Fuzzy Matrices

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Abstract— In this paper, the authors defined the Einstein operations of fuzzy matrices, intuitionistic fuzzy matrices and proved several properties of them, particularly those involving the intuitionistic fuzzy implication with other operations.

Keywords:Fuzzy Matrix, Intuitionistic Fuzzy Matrix, Einstein Sum and Einstein Product.

I. INTRODUCTION

It is well known that matrices play major role in various areas such as Mathematics, Physics, Statistics, Engineering, Social sciences and many others. However, we cannot successfully use classical matrices because of various types of uncertainties present in real world situations. Now a days probability, Fuzzy sets, Intuitionistic fuzzy sets, Vague sets are used as Mathematical tools for dealing uncertainties. Fuzzy matrices arise in many applications, one of which is as adjacency matrices of fuzzy relations. A fuzzy matrix is a matrix over the fuzzy algebra 3=[0,1] under the fuzzy operations formulated by Zadeh in 1965 [8]. Several authors presented a number of results on fuzzy matrices.In 1977, Thomoson [6] studied the behaviour of powers of fuzzy matrices using max-min composition. The theory of fuzzy matrices was systematically developed by Kim and Roush in [2]. Ragab and Emam [4] studied some properties of the min-max compositions of fuzzy matrices; it can be regarded as the dual of max-min composition of fuzzy matrices .Among the well known operations which can be performed on fuzzy matrices are the operators V, ∧ and complementation. In addition to these operations, the operations ∠ and ⊙ are introduced by Shyamaland Pal [6]. Also several properties on ∠ and ⊙, some results on existing operators along with these operations are studied. Wang and Liu[8] introduced some Einstein operations of intuitionistic fuzzy sets and analyze some desirable properties of the proposed operations. In [5] Selvarajan et.al studied Einstein operations to fuzzy matricesand proved several properties of them. In this paper, we extend the Einstein operations to intuitionistic fuzzy matrices and proved several properties of them.

II. FUZZY MATRICES

Definition: If A and B are two fuzzy matrices of same size, where \( A = [a_{ij}] \) and \( B = [b_{ij}] \) then

(i) The Einstein sum of \( A \) and \( B \) is defined by \( A \oplus B = \left[ \frac{a_{ij} + b_{ij}}{1 + a_{ij} blow_{ij}} \right] \).

(ii) The Einstein product of \( A \) and \( B \) is defined by

\[ A \otimes B = \left[ \frac{a_{ij} b_{ij}}{1 + a_{ij} blow_{ij}} \right] \]

Definition:If \( A = [a_{ij}] \) and \( B = [b_{ij}] \) are two intuitionistic fuzzy matrices (IFMs) of same size, then

(i) The Einstein sum of \( A \) and \( B \) is defined by

\[ A \oplus B = \left[ \frac{a_{ij} + b_{ij}}{1 + a_{ij} blow_{ij}} \right] \]

(ii) The Einstein product of \( A \) and \( B \) is defined by

\[ A \otimes B = \left[ \frac{a_{ij} b_{ij}}{1 + a_{ij} blow_{ij}} \right] \]

III. NECESSITY AND POSSIBILITY OPERATORS

The necessity and possibility operators for an IFM are defined by Pal [14]. In this section, the algebraic properties of necessity and possibility operators of IFMs with respect to Einstein operations are discussed.

Definition

Let \( A \) be an IFM of size \( m \times n \). Then the necessity operator \( \square \) of \( A \) is defined by \( \square A = \{ (a_{ij}, 1 - a_{ij}) \} \), the possibility operator \( \diamond \) of \( A \) is defined by \( \diamond A = \{ (1 - a_{ij}, a_{ij}) \} \).

Theorem 1: For any two IFMs \( A \) and \( B \) of same size, the following results are hold hood:

(i) \( \square (A \oplus B) = A \oplus B \)

(ii) \( \diamond (A \otimes B) = A \otimes B \).

Proof

(i) \( \square (A \oplus B) = \left[ \frac{a_{ij} + b_{ij}}{1 + a_{ij} blow_{ij}} \right] \)

(ii) \( \diamond (A \otimes B) = \left[ \frac{a_{ij} b_{ij}}{1 + a_{ij} blow_{ij}} \right] \)

Theorem 2: For any two IFMs \( A \) and \( B \) of same size, the following results are hold hood:

(i) \( \square (A \otimes B) = A \otimes B \)

(ii) \( \diamond (A \oplus B) = A \oplus B \).

Theorem 3: For any two IFMs \( A \) and \( B \) of same size, the following results are hold hood:

(i) \( \diamond (A \otimes B)^c = \diamond A \otimes B \)

(ii) \( \square (A \oplus B)^c = \square A \oplus B \)

Proof

(i) \( \diamond (A \otimes B)^c = \left[ \frac{a_{ij} b_{ij}}{1 + a_{ij} blow_{ij}} \right] \)

\[
\begin{aligned}
\left( A^C \otimes B^C \right)^C &= \left( \frac{1 - a'_i (1 - b'_{ij})}{1 + a'_i b'_{ij}} \right) \\
&= \emptyset A \otimes \emptyset B
\end{aligned}
\]

Theorem 4: For any two IFMs \( A \) and \( B \) of same size, the following results are hold hold:

(i) \( (\emptyset (A^C \oplus B^C))^C = \emptyset A \oplus \emptyset B \)

(ii) \( (\emptyset (A^C \oplus B^C))^C = \emptyset A \oplus \emptyset B \)

Define \( nA \) and \( A^n \) for any positive integer \( n \geq 0 \) with respect to the Einstein operations, by using these definitions we have the following theorems.

Definition: For an IFM \( A \), an integer \( n \geq 0 \),

\[
\begin{aligned}
nA &= \left( (1 + a_{ij}) - (1 + a_{ij})^n \right) \\
A^n &= \left( (1 + a_{ij})^n + (1 + a_{ij})^{n-1} (1 - a_{ij})^n \right)
\end{aligned}
\]

Theorem 5: For an IFM \( A \) and for any positive integer \( n \geq 0 \),

\[
\begin{aligned}
(i) \circ (nA) &= n \circ A \\
(ii) \emptyset (nA) &= n \emptyset A
\end{aligned}
\]

Proof: Similar to Theorem 7.

Theorem 6: For an IFM \( A \) and for any positive integer \( n \geq 0 \),

\[
\begin{aligned}
(i) \circ (A^C) &= (\circ n)A \\
(ii) \emptyset (A^C) &= (\emptyset n)A
\end{aligned}
\]

Proof: Similar to Theorem 5.

Theorem 7: For an IFM \( A \) and for any two positive integers \( m > n \geq 0, \text{ if } m > n \text{ then } ma > na.\)

Proof: Let \( A = (a_{ij}, a'_{ij}) \) be an intuitionistic fuzzy matrix. Let \( A = (c_{ij}, c'_{ij}) \), \( nA = (d_{ij}, d'_{ij}) \). Let \( f(x) = (1 + a_{ij})^x \\
\]

\[ g(x) = \frac{2^x (1 + a_{ij})^x}{2^x (2 - 1) + b^2} \]

Then the function is increasing and decreasing functions respectively. So if \( m > n \) then \( f(m) > f(n) \) and \( g(m) < g(n) \), i.e., \( c_{ij} > d_{ij} \).

Theorem 8: For an IFM \( A \) and for any two positive integers \( m > n \geq 0, \text{ if } m > n \text{ then } A^m > A^n.\)

Proof: Similar to Theorem 7.

Definition: The concentration of an IFM \( A \) is denoted by \( \text{CON}(A) \) and is defined by \( \text{CON}(A) = \{(b_{ij}, b'_{ij})\} \) where

\[
b_{ij} = \frac{2(a_{ij})^2}{(1 + a_{ij})^2 + b^2}, b'_{ij} = \frac{(1 + a_{ij})^2}{(1 + b^2)}
\]

In otherwords, concentration of an IFM is defined by \( \text{CON}(A) = A^2. \)

Theorem 9: For an IFM \( A, \text{CON}(A) \leq A. \)

Theorem 10: Let \( A = (a_{ij}, a'_{ij}), B = (b_{ij}, b'_{ij}) \), \( C = (c_{ij}, c'_{ij}) \) and \( D = (d_{ij}, d'_{ij}) \) be IFMs of same size, if \( A \geq C, B \geq D \) then \( A \oplus C \geq B \oplus D, A \otimes C \geq B \otimes D. \)

Proof: Let \( A \geq C, B \geq D \text{ i.e., } a_{ij} \geq c_{ij}, a'_{ij} \leq c'_{ij}, b_{ij} \geq d_{ij} \text{ and } b'_{ij} \leq d'_{ij}. \)

Then \( (a_{ij} - c_{ij})(1 - b_{ij} + d_{ij}) \geq (1 - a_{ij} c_{ij}) \geq 0 \)

\[ a_{ij} + b_{ij} \geq 1 + c_{ij} d_{ij} \]

The following theorem is now straight forward.

Theorem 11: Let \( A = (a_{ij}, a'_{ij}) \text{ and } B = (b_{ij}, b'_{ij}) \) be any two intuitionistic fuzzy matrices (IFMs) of same order then

\[
(\circ (A^C \oplus B^C)) \leq (\circ A^C) \oplus (\circ B^C)
\]

By definition we have, \( A \oplus C \geq B \oplus D \). Similarly we have \( (A \otimes C) \geq (B \otimes D). \)

Theorem 12: Let \( A = (a_{ij}, a'_{ij}) \text{ and } B = (b_{ij}, b'_{ij}) \) be any two intuitionistic fuzzy matrices (IFMs) of same order then

\[
(i) (A^C \rightarrow B^C) \oplus e (A \rightarrow B^C) = A \oplus B \text{ Band}
(ii) (A^C \rightarrow B^C) \oplus (A \rightarrow B^C) = A \oplus B.
\]

Proof: Let \( A = (a_{ij}, a'_{ij}) \text{ and } B = (b_{ij}, b'_{ij}) \) be any two intuitionistic fuzzy matrices of same order. Also \( A^C = (a'_{ij}, a_{ij}) \) and \( B^C = (b'_{ij}, b_{ij}). \)

\[
(A^C \rightarrow B^C) = \left( \text{con}(a_{ij}, b_{ij}) \right) \text{ and } \text{con}(a'_{ij}, b'_{ij}) \text{ and } (A \rightarrow B^C) = \left( \text{con}(a'_{ij}, b_{ij}) \right)
\]

\[
(A \rightarrow B^C) = \left( \text{con}(a_{ij}, b_{ij}) \right) \text{ and } \text{con}(a_{ij}, b'_{ij}) \text{ and } (A \rightarrow B^C) = \left( \text{con}(a_{ij}, b_{ij}) \right)
\]

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\[
\begin{align*}
    \max(a_{ij}, b_{ij}) & + \min(a_{ij}, b_{ij}) \\
    1 + \max(a_{ij}, b_{ij}) \min(a_{ij}, b_{ij}) & - \max(a_{ij}, b_{ij}) \min(a_{ij}, b_{ij}) \\
    1 + \max(a_{ij}, b_{ij}) & \\
    1 + \max(a_{ij}, b_{ij}) & \\
    1 + \max(a_{ij}, b_{ij}) & \\
    1 + \max(a_{ij}, b_{ij}) & \\
\end{align*}
\]

Also for any two real numbers c and d, we have max(c, d) + min(c, d) = c + d and max(c, d) min(c, d) = cd.

\[
\begin{align*}
    a_{ij} + b_{ij} & = 1 + (1 - \max(a_{ij}, b_{ij}) - \min(a_{ij}, b_{ij}) + \max(a_{ij}, b_{ij}) \min(a_{ij}, b_{ij})) \\
    a_{ij} + b_{ij} & = 1 + (1 - a_{ij} + b_{ij}) + a_{ij} b_{ij} \\
    a_{ij} + b_{ij} & = 1 + (1 - a_{ij} + b_{ij}) + a_{ij} b_{ij} \\
    a_{ij} b_{ij} & = 1 + a_{ij} b_{ij} \\
    a_{ij} b_{ij} & = 1 + a_{ij} b_{ij} \\
\end{align*}
\]

\[A \oplus B = (A^T \rightarrow B) \otimes (A \rightarrow B)^T\]

\[
\begin{align*}
    \max(a_{ij}, b_{ij}) & + \min(a_{ij}, b_{ij}) \\
    1 + \max(a_{ij}, b_{ij}) & \\min(a_{ij}, b_{ij}) \\
    \max(a_{ij}, b_{ij}) & + \min(a_{ij}, b_{ij}) \\
    \max(a_{ij}, b_{ij}) & \\min(a_{ij}, b_{ij}) \\
    \max(a_{ij}, b_{ij}) & \\min(a_{ij}, b_{ij}) \\
    \max(a_{ij}, b_{ij}) & \\min(a_{ij}, b_{ij}) \\
\end{align*}
\]

\[= A \otimes B \]

\[\text{IV. CONCLUSION}\]

In this article, the Einstein operations on intuitionistic fuzzy matrices have been defined and various properties are presented. Further, we proved De Morgan’s laws for these operations over complement and necessity and possibility operators.

\[\text{REFERENCES}\]


