Operations on Intuitionistic Anti Fuzzy Graphs

R. Muthuraj, S. Sujith, V.V. Vijesh

Abstract--- In the theory of intuitionistic fuzzy graphs, no theory on intuitionistic anti-fuzzy structures has been introduced. This motivated us to introduce the theory of intuitionistic anti-fuzzy graphs. In this paper, we define the notion of intuitionistic anti-fuzzy graphs and their special cases. Also, we introduce the concept of operations on intuitionistic anti-fuzzy graphs such as anti union and anti join of two intuitionistic anti-fuzzy graphs and prove some of their properties.

Keywords--- Intuitionistic Anti-fuzzy Graphs, Strong Intuitionistic Anti-Fuzzy Graphs, Regular Intuitionistic Anti-fuzzy Graph, Anti union and Anti Join of Two Intuitionistic Anti-fuzzy Graphs, Isomorphism between Two Intuitionistic Anti-fuzzy Graphs.

I. INTRODUCTION

The concept of fuzzy graph was introduced by Kaufmann [3] from the fuzzy relation introduced by Zedah [17]. The study of fuzzy graph theory started in the year 1975 after the phenomenal work published by Rosenfeld [12]. He has introduced another elaborated definition of fuzzy graphs and also proved many results on the fuzzy graph as an analog of graph theory. J. N. Moderson and P. S. Nair [5] introduced the concept of operations on fuzzy graphs, but this concept was extended by M. S. Sunitha and A. Vijayakumar [15]. Muhammad Akram [6] introduced the concept of connected anti fuzzy graphs, self centroid anti fuzzy graphs, constant and totally constant anti fuzzy graphs with some of their properties together with regularity and irregularity. R. Seethalakshmi and R. B. Gnanajothi [13]introduced the concept of anti fuzzy graph and discussed the concept of some operations such as anti union and anti join on anti fuzzy graphs.

Intuitionistic fuzzy sets are generalization of fuzzy sets [17]. Atanassov [1] introduced the concept of intuitionoistic fuzzy relation, which has both membership grades and non-membership grades.He has introduced different types of operations and their properties. He applied his ideas into expert systems, pattern recognition and mainly in decision making. A new emerging study of an intuitionistic fuzzy graph (IFG) has been addressed in [14]. The operations [11] and particular cases of intuitionistic fuzzy graphs [10] were done by Parvathy and Karunabigai. Whenever we discuss the intuitionistic fuzzy structures in any algebraic theory, analogously the notion of intuitionistic anti-fuzzy structures has been studied. However, in the theory of intuitionistic fuzzy graphs, no theory on intuitionistic anti-fuzzy structures has been introduced. This motivated us to introduce the theory of intuitionistic anti-fuzzy graphs. In this paper, we introduce some operations such as anti union andanti join of two intuitionistic anti-fuzzy graphs. We derived some theorems and results on them.

II. PRELIMINARIES

Definition 2.1: An intuitionistic anti-fuzzy graph is of the form $G = \langle V, E\rangle$ where (i) $V = \{v_1, v_2, \ldots\ldots v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ ………… (1) for every $v_i \in V$, (i = 1, 2, …n), (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \ge \max\{\mu_1(v_i), \mu_1(v_j)\} \ ,\ldots\ldots\ldots(2)$$

$$\gamma_2(v_i, v_j) \ge \min\{\gamma_1(v_i), \gamma_1(v_j)\} \ \ldots\ldots\ldots(3)$$

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ ………………(4)for every $(v_i, v_j) \in E$, (i, j = 1, 2, …n).

Note 2.1: If one of the inequalities (1) or (2) or (3) or (4) is not satisfied, then the graph G is not an intuitionistic anti-fuzzy graph.

Note 2.2: An intuitionistic anti-fuzzy graph $\langle V, E\rangle$ is denoted by $G_A\langle V, E\rangle$.

Definition 2.2: An intuitionistic anti-fuzzy graph $H_A(V', E')$ is an intuitionistic anti-fuzzy sub graph of $G_A\langle V, E\rangle$ if $V' \subseteq V, E' \subseteq E$ such that $\mu_{1i}{}' \le \mu_{1i}, \gamma_{1i}{}' \ge \gamma_{1i}$ and $\mu_{2ij}{}' \le \mu_{2ij}, \gamma_{2ij}{}' \ge \gamma_{2ij}$.

Definition 2.3: An intuitionistic anti-fuzzy sub graph $H_A(V', E')$ is called a spanning intuitionistic anti-fuzzy sub graph of $G_A\langle V, E\rangle$ if (i) $V' = V, E' = E$ and (ii) $\mu_{1i}{}' = \mu_{1i}, \gamma_{1i}{}' = \gamma_{1i}, \forall$ i, j.

Definition 2.4: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph. Then the vertex cardinality of V is defined by $|V| = \sum_{v_i \in V}\left(\frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right)$.

Definition 2.5: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph. Then the edge cardinality of Eis defined by $|E| = \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2(v_i,v_j)-\gamma_2(v_i,v_j)}{2}\right) = \sum_{e_i \in E}\left(\frac{1+\mu_2(e_i)-\gamma_2(e_i)}{2}\right)$.

Definition 2.6: Let $G_A = \langle V, E\rangle$ be an intuitionistic anti-fuzzy graph. Then the cardinality of G_A is defined by $|G_A| = \big||V| + |E|\big| = \left|\sum_{v_i \in V}\left(\frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right) + \right.$
(vi,vj)∈E1+μ2vi,vj−γ2(vi,vj)2.

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Definition 2.7: Let $G_A = \langle V, E \rangle$ be an intuitionistic anti-fuzzy graph. The degree of a vertex u is defined by a sum of weights of the edges that are incident at u and it is denoted by $d_{G_A}(u)$. *ie*, $d_{G_A}(u) = \left(d_\mu(u), d_\gamma(u)\right)$, where $d_\mu(u) = \sum_{v \neq u} \mu_2(u,v)$ and $d_\gamma(u) = \sum_{v \neq u} \gamma_2(u,v)$.

Definition 2.8: The minimum degree of an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is $\delta(G_A) = \left(\delta_\mu(G_A), \delta_\gamma(G_A)\right)$, where $\delta_\mu(G_A) = \min\{d_\mu(v) \,/\, v \in V\}$ and $\delta_\gamma(G_A) = \min\{d_\gamma(v) \,/\, v \in V\}$.

Definition 2.9: The maximum degree of an intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is $\Delta(G_A) = \left(\Delta_\mu(G_A), \Delta_\gamma(G_A)\right)$, where $\Delta_\mu(G_A) = \max\{d_\mu(v) \,/\, v \in V\}$ and $\Delta_\gamma(G_A) = \max\{d_\gamma(v) \,/\, v \in V\}$.

Definition 2.10: An edge $e = (u, v)$ of intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is said to be an effective edge if $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$

Definition 2.11: An intuitionistic anti-fuzzy graph $G_A = \langle V, E \rangle$ is said to be complete if $\mu_{2ij} = \max\left\{\mu_{1i}, \mu_{1j}\right\}$ and $\gamma_{2ij} = \min\{\gamma_{1i}, \gamma_{1j}\}, \forall v_i, v_j \in V$.

Note 2.3: The underlying graph of a complete intuitionistic anti-fuzzy graph is complete.

Definition 2.12: An intuitionistic anti-fuzzy graph $G_A\langle V, E \rangle$ is said to be strong if $\mu_{2ij} = \max\left\{\mu_{1i}, \mu_{1j}\right\}$ and $\gamma_{2ij} = \min\{\gamma_{1i}, \gamma_{1j}\}, \forall (v_i, v_j) \in E$.

Definition 2.13: An intuitionistic anti-fuzzy graph $G_A\langle V, E \rangle$ is said to be a (K_1, K_2)-regular if $d_{G_A}(v_i) = (K_1, K_2), \forall\, v_i \in V$ and also G_A is said to be a regular intuitionistic anti-fuzzy graph of degree (K_1, K_2). Here K_1 and K_2 are real constants.

III. SOME OPERATIONS ON INTUITIONISTIC ANTI-FUZZY GRAPHS

Definition 3.1: Let $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$ be two intuitionistic anti-fuzzy graphs. Then the antiunion of G_{A_1} and G_{A_2} is defined as $G_A = G_{A_1} \cup G_{A_2} = (V_1 \cup V_2, E_1 \cup E2$, such that

$$(\mu_1 \cup \mu_1')(v) = \begin{cases} \mu_1(v), \text{if } v \in V_1 - V_2 \\ \mu_1'(v), \text{if } v \in V_2 - V_1 \\ \min\{\mu_1(v), \mu_1'(v)\}, \text{if } v \in V_1 \cap V_2 \end{cases}$$

$$(\gamma_1 \cup \gamma_1')(v) = \begin{cases} \gamma_1(v), \text{if } v \in V_1 - V_2 \\ \gamma_1'(v), \text{if } v \in V_2 - V_1 \\ \min\{\gamma_1(v), \gamma_1'(v)\}, \text{if } v \in V_1 \cap V_2 \end{cases}$$

and

$$(\mu_2 \cup \mu_2')(e_{ij}) = \begin{cases} \mu_2(e_{ij}), \text{if } e_{ij} \in E_1 - E_2 \\ \mu_2'(e_{ij}), \text{if } e_{ij} \in E_2 - E_1 \\ \min\{\mu_2(e_{ij}), \mu_2'(e_{ij})\}, \text{if } e_{ij} \in E_1 \cap E_2 \end{cases}$$

$$(\gamma_2 \cup \gamma_2')(e_{ij}) = \begin{cases} \gamma_2(e_{ij}), \text{if } e_{ij} \in E_1 - E_2 \\ \gamma_2'(e_{ij}), \text{if } e_{ij} \in E_2 - E_1 \\ \min\{\gamma_2(e_{ij}), \gamma_2'(e_{ij})\}, \text{if } e_{ij} \in E_1 \cap E_2 \end{cases}$$

Where (μ_1, γ_1) and (μ_1', γ_1') refer the vertex membership and non-membership of G_{A_1} and G_{A_2} respectively, (μ_2, γ_2) and (μ_2', γ_2') refer the edge membership and non-membership of G_{A_1} and G_{A_2} respectively.

Example 3.1: Consider the following intuitionistic anti-fuzzy graphs:

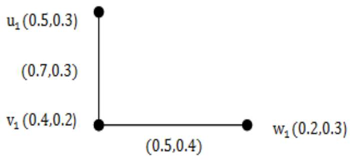

Fig. 1: G_{A_1}

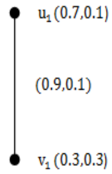

Fig. 2: G_{A_2}

Then

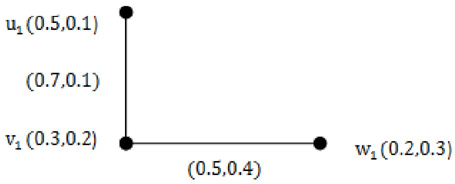

Fig. 3: $G_{A_1} \cup G_{A_2}$

Definition 3.2: Let $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$ be two intuitionistic anti-fuzzy graphs. Then the antijoin of G_{A_1} and G_{A_2} is an intuitionistic anti-fuzzy graph $G_A = G_{A_1} + G_{A_2} = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ defined by

$$(\mu_1 + \mu_1')(v) = (\mu_1 \cup \mu_1')(v) \text{, if } v \in V_1 \cup V_2$$

$$(\gamma_1 + \gamma_1')(v) = (\gamma_1 \cup \gamma_1')(v) \text{, if } v \in V_1 \cup V_2$$

$$(\mu_2 + \mu_2')(v_i, v_j) = \begin{cases} (\mu_2 \cup \mu_2')(v_i, v_j), \text{if } (v_i, v_j) \in E_1 \cup E_2 \\ \max\{\mu_1(v_i), \mu_1'(v_j)\}, \text{if } (v_i, v_j) \in E' \end{cases}$$

and

$$(\gamma_2 + \gamma_2')(v_i, v_j) = \begin{cases} (\gamma_2 \cup \gamma_2')(v_i, v_j), \text{if } (v_i, v_j) \in E_1 \cup E_2 \\ \min\{\gamma_1(v_i), \gamma_1'(v_j)\}, \text{if } (v_i, v_j) \in E' \end{cases}$$

Example 3.2: Consider the following intuitionistic anti-fuzzy graphs G_{A_1} and G_{A_2} with $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{v_1, v_2\}$

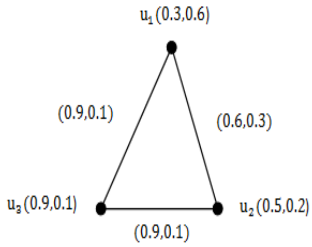

Fig. 4: G_{A_1}

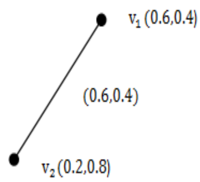

Fig. 5: G_{A_2}

Then

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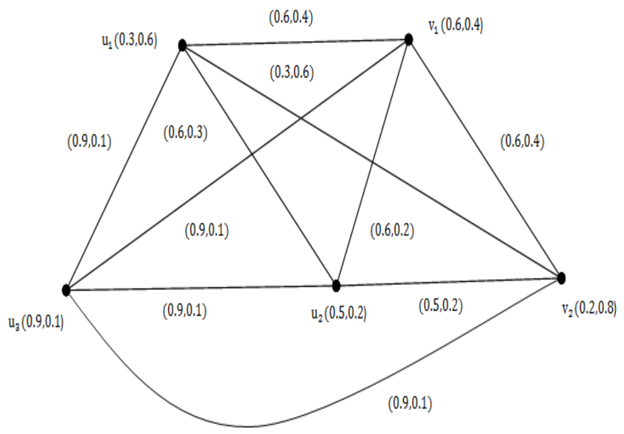

Fig. 6:$G_{A_1} + G_{A_2}$

Definition 3.3: Let $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$ be two intuitionistic anti-fuzzy graphs. An isomorphism between two intuitionistic anti-fuzzy graphs G_{A_1} and G_{A_2}, denoted by $G_{A_1} \cong G_{A_2}$, is a bijective function $h: V_1 \to V_2$ which satisfies

$\mu_1(v_i) = \mu_1'(h(v_i))$;

$$\gamma_1(v_i) = \gamma_1'(h(v_i))$$

and

$$\mu_2(v_i, v_j) = \mu_2'[h(v_i), h(v_j)];$$

$$\gamma_2(v_i, v_j) = \gamma_2'[h(v_i), h(v_j)], \forall\, v_i, v_j \in V_1$$

Theorem 3.1: The anti union of two intuitionistic anti-fuzzy graphs is again an intuitionistic anti-fuzzy graph.

Proof: Let $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$ be two intuitionistic anti-fuzzy graphs.

Let $G_A = G_{A_1} \cup G_{A_2} = (V_1 \cup V_2, E_1 \cup E_2) = \langle V, E\rangle$

Case I: Let $(u, v) \in E_1 - E_2$

Sub case (i): Let $u, v \in V_1 - V_2$

Then $(\mu_1 \cup \mu_1')(u) = \mu_1(u)$

$$(\mu_1 \cup \mu_1')(v) = \mu_1(v)$$

and

$$(\gamma_1 \cup \gamma_1')(u) = \gamma_1(u)$$
$$(\gamma_1 \cup \gamma_1')(v) = \gamma_1(v)$$

Now,

$$(\mu_2 \cup \mu_2')(u, v) = \mu_2(u, v), \text{since } (u, v) \in E_1 - E_2$$
$$\geq \max[\mu_1(u), \mu_1(v)]$$
$$= \max[(\mu_1 \cup \mu_1')(u), (\mu_1 \cup$$

μ1′v

Again,

$$(\gamma_2 \cup \gamma_2')(u, v) = \gamma_2(u, v)$$
$$\geq \min[\gamma_1(u), \gamma_1(v)]$$
$$= \min[(\gamma_1 \cup \gamma_1')(u), (\gamma_1 \cup$$

γ1′v

Sub case (ii): Let $u \in V_1 - V_2, v \in V_1 \cap V_2$

$$(\mu_1 \cup \mu_1')(u) = \mu_1(u)$$
$$(\mu_1 \cup \mu_1')(v) = \min[\mu_1(v), \mu_1'(v)]$$

and

$$(\gamma_1 \cup \gamma_1')(u) = \gamma_1(u)$$
$$(\gamma_1 \cup \gamma_1')(v) = \min[\gamma_1(v), \gamma_1'(v)]$$

Now, $(\mu_2 \cup \mu_2')(u, v) = \mu_2(u, v), \text{since } (u, v) \in$ $E_1 - E_2$

$$\geq \max[\mu_1(u), \mu_1(v)]$$
$$= \max[(\mu_1 \cup \mu_1')(u), (\mu_1 \cup \mu_1')(v)]$$

Again,

$$(\gamma_2 \cup \gamma_2')(u, v) = \gamma_2(u, v), \text{since } (u, v) \in E_1 - E_2$$
$$\geq \min[\gamma_1(u), \gamma_1(v)]$$
$$\geq \min[(\gamma_1 \cup \gamma_1')(u), (\gamma_1 \cup$$

γ1′v, since γ1v≥γ1∪γ1′v

Sub case (iii): Let $u, v \in V_1 \cap V_2$

$$\therefore (\mu_1 \cup \mu_1')(u) = \min[\mu_1(u), \mu_1'(u)]$$
$$(\mu_1 \cup \mu_1')(v) = \min[\mu_1(v), \mu_1'(v)]$$

and

$$(\gamma_1 \cup \gamma_1')(u) = \min[\gamma_1(u), \gamma_1'(u)]$$
$$(\gamma_1 \cup \gamma_1')(v) = \min[\gamma_1(v), \gamma_1'(v)]$$

Now, $(\mu_2 \cup \mu_2')(u, v) = \mu_2(u, v), \text{since } (u, v) \in$ $E_1 - E_2$

$$\geq \max[\mu_1(u), \mu_1(v)]$$
$$\geq \max[(\mu_1 \cup$$

μ1′u,μ1∪μ1′v

Again,

$$(\gamma_2 \cup \gamma_2')(u, v) = \gamma_2(u, v), \text{since } (u, v) \in E_1 - E_2$$
$$\geq \min[\gamma_1(u), \gamma_1(v)]$$
$$\geq \min[(\gamma_1 \cup \gamma_1')(u), (\gamma_1 \cup$$

γ1′v

Thus

$$(\mu_2 \cup \mu_2')(u, v) \geq \max[(\mu_1 \cup \mu_1')(u), (\mu_1 \cup$$

μ1′vand

$$(\gamma_2 \cup \gamma_2')(u, v) \geq \min[(\gamma_1 \cup$$

γ1′u,γ1∪γ1′v; ∀u,v∈E1−E2

Case II: Let $(u, v) \in E_2 - E_1$

As in case I, we arrive at

$$(\mu_2 \cup \mu_2')(u, v) \geq \max[(\mu_1 \cup$$

μ1′u,μ1∪μ1′vand

$$(\gamma_2 \cup \gamma_2')(u, v) \geq \min[(\gamma_1 \cup$$

γ1′u,γ1∪γ1′v; ∀u,v∈E2−E1

Case III: Let $(u, v) \in E_1 \cap E_2$

$$\therefore\ u, v \in V_1 \cap V_2$$
$$\therefore (\mu_1 \cup \mu_1')(u) = \min[\mu_1(u), \mu_1'(u)]$$
$$(\mu_1 \cup \mu_1')(v) = \min[\mu_1(v), \mu_1'(v)]$$

and

$$(\gamma_1 \cup \gamma_1')(u) = \min[\gamma_1(u), \gamma_1'(u)]$$
$$(\gamma_1 \cup \gamma_1')(v) = \min[\gamma_1(v), \gamma_1'(v)]$$

Then

$$(\mu_2 \cup \mu_2')(u, v) = \min[\mu_2(u, v), \mu_2'(u, v)]$$

$$\geq \min\{\max[\mu_1(u), \mu_1(v)], \max[\mu_1'(u), \mu_1'(v)]\}$$
$$\geq [\mu_1(u) \vee \mu_1(v)] \wedge$$

$[\mu_1'(u) \vee \mu_1'(v)]$

By the distributive law,

$(\sigma \vee \tau) \wedge (\xi \vee \varphi) = (\sigma \wedge \xi) \vee (\tau \wedge \varphi)$,

$$= (\mu_1 \wedge \mu_1')(u) \vee (\mu_1 \wedge \mu_1')(v)$$

$$= (\mu_1 \cup \mu_1')(u) \vee$$

$(\mu_1 \cup \mu_1')(v)$

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$$= \max[(\mu_1 \cup \mu_1{}')(u), (\mu_1 \cup \mu_1{}')(v)]$$

Again,

$$(\gamma_2 \cup \gamma_2{}')(u,v) = \min[\gamma_2(u,v), \gamma_2{}'(u,v)]$$

$$\geq \min\{\min[\gamma_1(u), \gamma_1(v)], \min[\gamma_1{}'(u), \gamma_1{}'(v)]\}$$

$$= \min[(\gamma_1 \cup$$

γ1′u,γ1∪γ1′v

Hence $G_A = G_{A_1} \cup G_{A_2} = (V_1 \cup V_2, E_1 \cup E_2) = \langle V, E \rangle$ is also an intuitionistic anti-fuzzy graph.

Theorem3.2: The anti union of two strong intuitionistic anti-fuzzy graphs G_{A_1} and G_{A_2} is again a strong intuitionistic anti-fuzzy graph, when $G_{A_1} \cap G_{A_2} = \varphi$.

Proof: Let $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$ be two disjoint strongintuitionistic anti-fuzzy graphs

Then $\mu_2(u,v) = \max[\mu_1(u), \mu_1(v)]$ and

$\gamma_2(u,v) = \min[\gamma_1(u), \gamma_1(v)], \forall (u,v) \in E_1$.

and

$$\mu_2{}'(u,v) = \max[\mu_1{}'(u), \mu_1{}'(v)]$$

$$\gamma_2{}'(u,v) = \max[\gamma_1{}'(u), \gamma_1{}'(v)], \forall (u,v) \in E_2$$

Let $G_A = G_{A_1} \cup G_{A_2} = (V_1 \cup V_2,\ E_1 \cup E_2) = \langle V, E \rangle$ be the anti union of G_{A_1} and G_{A_2}.

Let $u, v \in V$

Case I: Let$u, v \in V_1 - V_2$

Then$(\mu_1 \cup \mu_1{}')(u) = \mu_1(u)$

$$(\gamma_1 \cup \gamma_1{}')(u) = \gamma_1(u)$$

and

$$(\mu_1 \cup \mu_1{}')(v) = \mu_1(v)$$

$$(\gamma_1 \cup \gamma_1{}')(v) = \gamma_1(v)$$

Now,

$$(\mu_2 \cup \mu_2{}')(u,v) = \mu_2(u,v)$$

$=\max[\mu_1(u), \mu_1(v)]$, since G_{A_1} is strong.

$$= \max[(\mu_1 \cup \mu_1{}')(u), (\mu_1 \cup \mu_1{}')(v)]$$

Again,

$$(\gamma_2 \cup \gamma_2{}')(u,v) = \gamma_2(u,v)$$

$= \min[\gamma_1(u), \gamma_1(v)]$, since G_{A_1} is strong.

$$= \min[(\gamma_1 \cup \gamma_1{}')(u), (\gamma_1 \cup \gamma_1{}')(v)]$$

Case II: Let$u, v \in V_2 - V_1$

Then $(\mu_1 \cup \mu_1{}')(u) = \mu_1{}'(u)$

$$(\gamma_1 \cup \gamma_1{}')(u) = \gamma_1{}'(u)$$

and

$$(\mu_1 \cup \mu_1{}')(v) = \mu_1{}'(v)$$

$$(\gamma_1 \cup \gamma_1{}')(v) = \gamma_1{}'(v)$$

Therefore, $(\mu_2 \cup \mu_2{}')(u,v) = \mu_2{}'(u,v)$

$= \max[\mu_1{}'(u), \mu_1{}'(v)]$, since G_{A_2} is strong.

$$= \max[(\mu_1 \cup$$

μ1′u,μ1∪μ1′v

Again,

$$(\gamma_2 \cup \gamma_2{}')(u,v) = \gamma_2{}'(u,v)$$

$= \min[\gamma_1{}'(u), \gamma_1{}'(v)]$, since G_{A_2} is strong.

$$= \min[(\gamma_1 \cup \gamma_1{}')(u), (\gamma_1 \cup \gamma_1{}')(v)]$$

Hence $G_A = G_{A_1} \cup G_{A_2}$ is also a strong intuitionistic anti-fuzzy graph.

Theorem 3.3: The anti join of two intuitionistic anti-fuzzy graphs is an intuitionistic anti-fuzzy graph.

Proof: Let $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$ be two intuitionistic anti-fuzzy graphs.

Let $G_A = G_{A_1} + G_{A_2} = (V_1 \cup V_2, E_1 \cup E_2 \cup E') = \langle V, E \rangle$

Case I: Let $(u,v) \in E_1 \cup E_2$

Then $(\mu_2 + \mu_2{}')(u,v) \geq \max[(\mu_1 + \mu_1{}')(u), (\mu_1 +$

μ1′v

and $(\gamma_2 + \gamma_2{}')(u,v) \geq \min[(\gamma_1 + \gamma_1{}')(u), (\gamma_1 +$

γ1′v, follows from the theorem 3.1.

Case II: Let $(u,v) \in E'$

Then $(\mu_2 + \mu_2{}')(u,v) = \max[\mu_1(u), \mu_1{}'(v)]$, from the definition of antijoin.

$= \max[(\mu_1 \cup \mu_1{}')(u), (\mu_1 \cup \mu_1{}')(v)]$, since $u \in V_1, v \in V_2$

$$= \max[(\mu_1 + \mu_1{}')(u), (\mu_1 + \mu_1{}')(v)]$$

And $(\gamma_2 + \gamma_2{}')(u,v) = \min[\gamma_1(u), \mu_1{}'(v)]$, from the definition of antijoin

$= \min[(\gamma_1 \cup \gamma_1{}')(u), (\gamma_1 \cup \gamma_1{}')(v)]$, since $u \in V_1, v \in V_2$

$$= \min[(\gamma_1 + \gamma_1{}')(u), (\gamma_1 + \gamma_1{}')(v)]$$

Hence G_A is also an intuitionistic anti-fuzzy graph.

Theorem 3.4: If G_A is the anti union of two intuitionistic anti-fuzzy graphs $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$, then every intuitionistic anti-fuzzy sub graph of G_A is the anti-union of intuitionistic anti-fuzzy sub graphs of G_{A_1} and G_{A_2}.

Proof: Define $(\mu_1{}^1, \gamma_1{}^1), (\mu_2{}^1, \gamma_2{}^1), (\mu_1{}^2, \gamma_1{}^2), (\mu_2{}^2, \gamma_2{}^2)$ on V_1, E_1, V_2, E_2 respectively as follows:

$$\mu_1{}^i(u) = \mu_1(u)$$

$$\gamma_1{}^i(u) = \gamma_1(u), \forall\ u \in V_i, i = 1,2$$

and $\mu_2{}^i(u,v) = \mu_2(u,v)$

$$\gamma_2{}^i(u,v) = \gamma_2(u,v), \forall\ (u,v) \in E_i, i = 1,2$$

Then $\mu_2{}^i(u_i, v_i) = \mu_2(u_i, v_i)$

$$\geq \max[\mu_1(u_i), \mu_1(v_i)]$$

$$= \max[\mu_1{}^i(u_i), \mu_1{}^i(v_i)], \forall\ (u_i, v_i) \in E_i, i = 1,2$$

and $\gamma_2{}^i(u_i, v_i) = \gamma_2(u_i, v_i)$

$$\geq \min[\gamma_1(u_i), \gamma_1(v_i)]$$

$$= \min[\gamma_1{}^i(u_i), \gamma_1{}^i(v_i)], \forall\ (u_i, v_i) \in E_i, i = 1,2$$

Thus $[(\mu_1{}^i, \gamma_1{}^i), (\mu_2{}^i, \gamma_2{}^i)]$ is an intuitionistic anti-fuzzy sub graph of G_{A_i}, $i = 1,2$.

Clearly $\mu_1 = \mu_1{}^1 \cup \mu_1{}^2, \gamma_1 = \gamma_1{}^1 \cup \gamma_1{}^2$

and $\mu_2 = \mu_2{}^1 \cup \mu_2{}^2, \gamma_2 = \gamma_2{}^1 \cup \gamma_2{}^2$

Theorem 3.5: If G_A is the anti join of two intuitionistic anti-fuzzy graphs $G_{A_1} = (V_1, E_1)$ and $G_{A_2} = (V_2, E_2)$, then every strong intuitionistic anti-fuzzy sub graph of G_A is the anti join of a strong intuitionistic anti-fuzzy sub graph of G_{A_1} and a strong intuitionistic anti-fuzzy sub graph of G_{A_2}.

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Proof: Define $(\mu_1{}^1,\gamma_1{}^1),(\mu_2{}^1,\gamma_2{}^1),(\mu_1{}^2,\gamma_1{}^2),(\mu_2{}^2,\gamma_2{}^2)$on V_1,E_1,V_2,E_2 respectively as follows:

$$\mu_1{}^i(u)=\mu_1(u)$$
$$\gamma_1{}^i(u)=\gamma_1(u),\forall\, u\in V_i, i=1,2$$

and $\mu_2{}^i(u,v)=\mu_2(u,v)$

$\gamma_2{}^i(u,v)=\gamma_2(u,v),\forall\,(u,v)\in E_i, i=1,2$

Then $\mu_1=\mu_1{}^1\cup\mu_1{}^2, \gamma_1=\gamma_1{}^1\cup\gamma_1{}^2$ as in theorem 3.4.

Hence $\mu_1=\mu_1{}^1+\mu_1{}^2, \gamma_1=\gamma_1{}^1+\gamma_1{}^2$

If $(u,v)\in E_1\cup E_2$, then $\mu_2(u,v)=(\mu_2{}^1\cup\mu_2{}^2)(u,v)$
$=(\mu_2{}^1+$
μ22(u,v),from the definition of antijoin

and$\gamma_2(u,v)=(\gamma_2{}^1\cup\gamma_2{}^2)(u,v)$

$=(\gamma_2{}^1+\gamma_2{}^2)(u,v)$,from the definition of antijoin

If $(u,v)\in E'$, then$u\in V_1, v\in V_2$.

Now,$(\mu_2{}^1+\mu_2{}^2)(u,v)=\max[\mu_1{}^1(u),\mu_1{}^2(v)]$
$=\max[\mu_1(u),\mu_1(v)]$
$=\mu_2(u,v)$

and$(\gamma_2{}^1+\gamma_2{}^2)(u,v)=\min[\gamma_1{}^1(u),\gamma_1{}^2(v)]$
$=\min[\gamma_1(u),\gamma_1(v)]$
$=\gamma_2(u,v)$

Theorem 3.6: The anti join of two strong intuitionistic anti-fuzzy graphs is also a strong intuitionistic anti-fuzzy graph.

Proof: Let $G_{A_1}=(V_1,E_1)$ and $G_{A_2}=(V_2,E_2)$ be two strong intuitionistic anti-fuzzy graphs.

Let $G_A=G_{A_1}+G_{A_2}=\langle V,E\rangle$ be the anti join of $G_{A_1}$and G_{A_2}.

Case I: Let $(u,v)\in E_1\cup E_2$

Sub case (i): Let$(u,v)\in E_1-E_2$

$(\mu_2{}^1+\mu_2{}^2)(u,v)=(\mu_2{}^1\cup\mu_2{}^2)(u,v)$,fromthe definition of antijoin

$=\mu_2{}^1(u,v)$
$=\max[\mu_1{}^1(u),\mu_1{}^1(v)]$
$=\max[(\mu_1{}^1\cup\mu_1{}^2)(u),(\mu_1{}^1\cup\mu_1{}^2)(v)]$
$=\max[(\mu_1{}^1+\mu_1{}^2)(u),(\mu_1{}^1+\mu_1{}^2)(v)]$

and$(\gamma_2{}^1+\gamma_2{}^2)(u,v)=(\gamma_2{}^1\cup\gamma_2{}^2)(u,v)$
$=\gamma_2{}^1(u,v)$
$=\min[\gamma_1{}^1(u),\gamma_1{}^1(v)]$
$=\min[(\gamma_1{}^1\cup\gamma_1{}^2)(u),(\gamma_1{}^1\cup\gamma_1{}^2)(v)]$
$=\min[(\gamma_1{}^1+\gamma_1{}^2)(u),(\gamma_1{}^1+\gamma_1{}^2)(v)]$

Sub case (ii): Let$(u,v)\in E_2-E_1$

$(\mu_2{}^1+\mu_2{}^2)(u,v)=(\mu_2{}^1\cup\mu_2{}^2)(u,v)$
$=\mu_2{}^2(u,v)$
$=\max[\mu_1{}^2(u),\mu_1{}^2(v)]$
$=\max[(\mu_1{}^1\cup\mu_1{}^2)(u),(\mu_1{}^1\cup\mu_1{}^2)(v)]$
$=\max[(\mu_1{}^1+\mu_1{}^2)(u),(\mu_1{}^1+\mu_1{}^2)(v)]$

and$(\gamma_2{}^1+\gamma_2{}^2)(u,v)=(\gamma_2{}^1\cup\gamma_2{}^2)(u,v)$
$=\gamma_2{}^2(u,v)$
$=\min[\gamma_1{}^2(u),\gamma_1{}^2(v)]$
$=\min[(\gamma_1{}^1\cup\gamma_1{}^2)(u),(\gamma_1{}^1\cup\gamma_1{}^2)(v)]$
$=\min[(\gamma_1{}^1+\gamma_1{}^2)(u),(\gamma_1{}^1+\gamma_1{}^2)(v)]$

Case II: Let $(u,v)\in E'$

$(\mu_2{}^1+\mu_2{}^2)(u,v)=\max[\mu_1{}^1(u),\mu_1{}^2(v)]$, by the definition of antijoin.

$=\max[(\mu_1{}^1\cup\mu_1{}^2)(u),(\mu_1{}^1\cup\mu_1{}^2)(v)]$
$=\max[(\mu_1{}^1+\mu_1{}^2)(u),(\mu_1{}^1+\mu_1{}^2)(v)]$

and$(\gamma_2{}^1+\gamma_2{}^2)(u,v)=\min[\gamma_1{}^1(u),\gamma_1{}^2(v)]$
$=\min[(\gamma_1{}^1\cup\gamma_1{}^2)(u),(\gamma_1{}^1\cup\gamma_1{}^2)(v)]$
$=\min[(\gamma_1{}^1+\gamma_1{}^2)(u),(\gamma_1{}^1+\gamma_1{}^2)(v)]$

Hence $G_A=G_{A_1}+G_{A_2}$ is also a strong intuitionistic anti-fuzzy graph.

IV. CONCLUSION

This studies on different operations explicit various structures of intuitionistic anti-fuzzy graphs and their properties. Different operations likeanti union andanti join are applied on intuitionistic anti-fuzzy graphs. Some theorems and results are obtained on them. The relationship between the resulting graphsis discussed and derived some theorems on them. The theory of intuitionistic anti-fuzzy graphhas more applications in communication networks, information technology, pattern clustering, image retrieval and so on. In future, it is anticipated to do these perceptions on the other extension of intuitionistic anti-fuzzy graphs.

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