

# Reversible Thinking Ability in Calculus Learn-ing using Maple Software: A Case Study of Mathematics Education Students

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**Abstract:** Along with the rapid advancement of technology, the lecturers are demanded to be able to integrate technological developments in the teaching process. Calculus as a subject matter in mathematics education study program which is full of algebraic symbols and graph simulation requires visualization media. Maple software is one of the most appropriate technology tools which can be used in teaching Calculus for mathematics education students. The linkage of graphic visualization of derivative and anti-derivative functions can be understood through reversible thinking. Thus, this study aimed at identifying students' reversible thinking abilities of mathematics education study programs Calculus learning by using Maple. The research design used was a case study. The results of this study indicate that reversible thinking abilities can be identified when students review the results of calculations and graphs obtained using Maple. The results of this study have implications dealing with the importance of learning technology.

**Keywords:** calculus, derivative, anti-derivative, Maple, reversible thinking

## I. INTRODUCTION

Calculus is one of the compulsory subjects in both mathematics study program and mathematics education study program based on the curriculum of higher education in Indonesia. In several different study programs, Calculus is also an essential subject in the curriculum of higher education. This is proofed from the number of subsequent courses that use the concept of Calculus as a basis and tool to understand topics or break down certain problems in several courses in universities, such as physics-mathematics in the faculty of engineering [1], mathematics-economics in the faculty of economics, and several other fields.

The success of students in learning Calculus can be seen from their ability to understand the concepts of derivative and anti-derivative. The students' concept of derivative and anti-derivative can also be seen through their ability to convert a function into a derivative function and change the derivative function through an integral process so that the function returns to its initial form. This action requires the ability to review and make anticipation. The ability to review and make anticipation is reversible thinking ability [2].

It takes time to change a function into a derivative function and then is processed through an anti-derivative if

it is done manually. To overcome this, we need a technology tool that can facilitate and help students without manual calculations that are sometimes less accurate. One of the learning technologies that can be utilized is maple software [3]. Maple is software that can be used not only as a calculating tool but also as a tool for creating graphics, determining the derivative of a function and so on.

The use of learning technologies, such as Maple, is only a supporting tool for students to show the results obtained from manual calculation. The results obtained from manual calculation can be reviewed based on the results obtained from the Maple software [4]. Likewise, the results obtained from the Maple software can be reviewed using manual calculations. This method is certainly useful to strengthen the students' concept. Determining derivative and anti-derivative functions by using Maple is also helpful in strengthening the students' concept of derivative and anti-derivative Calculus. The graph of the anti-derivative function, if it is reviewed back to the graph of the initial function after going through the derivative process, the same graph will be obtained. Graphs obtained from the derivative process and proceed to the anti-derivative process for a function will return to the initial function graph. The results of this process will lead students' thinking to conduct a review.

The students' mental activity in conducting a review of the results obtained is an act of students' reversible thinking. Reversible thinking in learning derivative and anti-derivative Calculus is still lacked attention from lecturers. A study held by Hacıomeroglu, et al. [5] which involves reversibility in derivative and anti-derivative functions is only seen from manual calculations and without using computer assistance. Learning using computers can facilitate lecturers to train students to think reversibly. Through reversible thinking, students can find a relationship between the concept of derivative functions and anti-derivative. While Calculus learning using computers, especially Maple software, can help students work easily and quickly.

There are many studies which have analyzed Calculus learning by using Maple [1], [3], [4], [6]–[12]. However, until now, there are no studies that investigated students' reversible thinking when lecturers teaching Calculus by using Maple. Reversible thinking of students, in this case, is also related to reversibility as used and recommended by mathematics education researchers [2], [5], [13]–[19]. Therefore, the purpose of this study will be focused on

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teaching Calculus by using Maple in identifying reversible thinking of mathematics education students.

Research that provides insights into how students' understanding can be enriched by building reversible relationships between initial function graphs and derivative or anti-derivative graphs has shown the importance of reversibility of students' thinking [5]. But research from Hacımeroglu, et al. [5] does not describe how the reversible thinking of students in developing relationships between graph of functions and graph of derivative and anti-derivative. Meanwhile, research that identifies reversible thinking carried out by Mafulah et al. [15] is still limited to research at elementary school students. In addition, Hacımeroglu, et al. [5] did not involve the use of mathematical software in calculus learning activities in the study. Along with the rapid technological developments, the students are required to be able to keep up with technological developments, especially in the education world. Therefore, it is important to do research in identifying reversible thinking skills of students in calculus learning using Maple software. The benefits that can be obtained in identifying reversible thinking skills of students in learning calculus is one of which is to provide convenience for lecturers in knowing the students' understanding of derivative and anti-derivative concepts. In addition, students' reversible thinking ability can be used as a basis for evaluating students' calculus abilities, and then can be used as a basis for evaluating calculus learning for students.

#### a. Teaching Using Maple

The two most powerful and widely used computer algebra systems (CAS) are Maple and Mathematic [8]. Maple is a computer software or Computer Algebra System (CAS) which is able to solve the problem of equations in numerical and symbolic form. Maple is a very comprehensive program with many commands and options, one of which one learns throughout life [10]. Maple is also able to present symbolic processing and visualization. In general, Maple consists of the main menu, toolbar, and worksheet. The worksheet is a place to write Maple commands in mathematical calculations. Maple is equipped with 'Help' facilities on the main menu. In the main menu, users can select sub menus to see how to work using maple. Maple also provides users with the tools that are very easy to operate contained in the Palettes. Palettes are used to simplify writing in worksheets, and palettes are generally located on the left. Some types of palettes available are the symbol palette, expression palette, and matrix palette. Symbol palette is used to write mathematical symbols, expression palettes are used to facilitate writing mathematical expressions such as integrals, Sigma series, root forms, etc., while the matrix palette is used to facilitate users in writing a matrix.

With a variety of facilities and other advantages, Maple can be used as an alternative support in teaching Calculus through practical activities. Their use is very simple, really intuitive, students easily follow the pattern and create new (own) examples. The program allows us to work fluently with students of different computer skills [10]. Teaching Calculus using a variety of computer facilities and equipped with Maple software will facilitate lecturers to deliver

material quickly, and students can take lessons directly with practice.

Learning that is accompanied by computer practice using Maple will facilitate students to understand the concepts being studied, especially the concept of derivative functions and anti-derivative in Calculus courses. Students who are able to understand the concepts of derivative and anti-derivative of a function will involve reversible thinking.

## II. METHOD

The design used in this study was a case study. According to Creswell [20], case studies are inquiry strategies where researchers explore programs, events, activities, processes, or one or more individuals in depth. Therefore, this study will focus on a group of students who are taking Computer-Assisted Mathematics courses.

In this study, there were 30 students who participated in class were selected. The selection was based on the recommendation from the lecturer, and the students must be able to communicate their thoughts both in writing and speaking. From the 30 students, it was chosen 15 students who were willing and able to communicate their thoughts in writing and speaking. The 15 selected students were observed during the learning process. Some students were asked to explain their thoughts in writing. Meanwhile, in identifying students' reversible thinking abilities, observation data and written test results were collected before and during calculus learning process using Maple software. From the results of the written reason, the interview was conducted to validate the data that had been obtained and then analyzed. The written test results were reported and discussed.

Initially, the written test was given to all students who got involved in the lecture. 15 out of the 30 students who took part in the lecture were selected based on the criteria set out in this study. The initial test results before calculus learning activities using Maple software were analyzed. The analysis of the initial test results was focused on several components, i.e. (1) the students' ability to perform calculations when substituting several points on the coordinate axis in a given initial function; (2) the students' ability to chart the initial function; (3) the students' ability to determine derivative functions from the initial function; (4) the students' ability to chart the derivative function; (5) the students' ability to determine the anti-derivative functions of previous derivative functions; and (6) the students' ability to chart the anti-derivative functions previously obtained. The test results were used as the basis for identifying students' reversible thinking abilities in understanding the concepts of derivatives and anti-derivatives. The indicator of reversible thinking ability in this study is based on his opinion [2] which states that Piaget's reversible thinking is based on actions of anticipation and backward review. The students' ability to anticipate the results obtained based on the ability to review the process that occurred or review to the previous conditions.



The students' reversible thinking ability in calculus learning by using Maple software was also identified based on students' work processes in completing the tasks involving the concept of derivatives and anti-derivatives. The students' work process was in line with the initial test which is based on 6 components. Likewise, the students' reversible thinking abilities were also based on the process of anticipation and review.

To dig more detailed information about students' reversible thinking abilities in this study, 8 out of 15 participants were selected for in-depth interviews. Seven participants in this study were selected based on their works and as the representative of the members of the discussion during the learning activities. This step simply represented all students who took part in the lecture during the research activities. To guarantee the privacy of the respondents, the researchers used anonymized data by changing the names of participants with S1, S2, and so on. The data were analyzed descriptively based on the number of participants who succeeded in understanding and completing the task during the learning process. Meanwhile, the results of the interview were analyzed based on the description of the data obtained.

In this study, the data collection techniques were divided into 3 parts, i.e. tests, observations, and interviews. The test was conducted at each session of learning activities. The Subjects' activities during learning were observed. Interview activities were carried out after observations and at the end of learning activities. Unstructured interviews were used to obtain detailed and in-depth information from the subject. The results of the interviews were then recorded by a digital camera so that the research data, such as the activities and talks of the subject, were kept well; Learning activities and students' work were also documented. The validity of the data in this study was carried out through a triangulation process [21], [22]. The triangulation technique was done by comparing the data collected from several data collection techniques. If the results of the triangulation show that the data from several techniques used are consistent, then the data obtained is valid.

### III. RESULT AND DISCUSSION

Before applying Maple software in calculus learning, the students were first given the task to know their understanding of the concepts of derivatives and anti-derivatives. The 'function' task was given once. The students' task, in this case, are (1) to do calculations by substituting several points on the coordinate axis in a given initial function; (2) to chart the initial function; (3) to determine the derivative function of the initial function; (4) to chart the derivative functions; (5) to determine the anti-derivative function of the previous derivative function; and (6) to chart the anti-derivative functions that have been obtained. The results of the work of the fifteen students who participated in this study are presented in "Fig. 1" below.

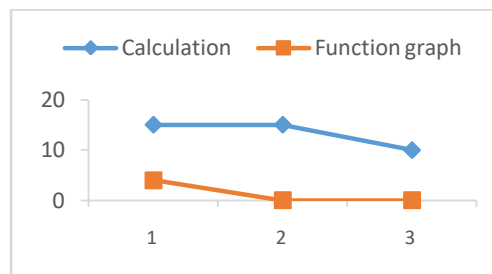


Fig 1: The initial tests

Based on the results of the initial tests in "Fig. 1", it shows that 15 students have the ability to calculate functions and determine the derivative function from the initial function in the given task, and 10 students can determine anti-derivative. This shows that anti-derivative is considered more difficult by some students because it is the opposite of derivative [23]. This result also shows that the problem of students lies in making a function graph. There were only 4 students who successfully made the initial function graph. This result affected the students' success in making graphs of subsequent functions, derivative and anti-derivative functions. These results indicate that the students' ability to create a function graph depends on the level of visual thinking they have. According to Presmeg[24], understanding mathematics is closely related to the ability to use visual and analytic thinking. The findings of the student problem in making a graph begin with the graphical function of the initial assignment given, and continue to the graph of the derivative and anti-derivative functions. This graph was made manually by using stationery, paper sheets, and notebooks owned by the students. These results have shown that students were not able to do anticipation because most of the students were not successful in forming the initial function graph. This will affect on the next functions of derivative and anti-derivative. This condition illustrates that students' reversible thinking ability in solving problems involving the concept of derivative and anti-derivative without using the help of Maple software did not go well.

The results of this initial test became the basis for the researcher to conduct derivative and anti-derivative *Calculus* teaching activities by using Maple. Mathematical understanding is very important for lecturers to ensure that their students can get a real understanding of completing mathematical tasks with technology [1]. Maple as a computer algebra system is one of the appropriate technologies by allowing students to concentrate on formulation solutions [25]. Therefore, the teaching in this study begins with introducing the components in the Maple software.

The components of Maple which were introduced to the students were the main menu, toolbar, and worksheet. The main menus introduced in this study were *File*, *Edit*, *View*, *Insert*, and *Format*. Each menu was described based on its function in solving mathematical problems. Considering that in this study Maple 11 software was used, the main menu display can be seen in "Fig. 2" below. Likewise, the worksheet that serves as a place to write Maple commands





in completing tasks related to the field of mathematics. Tools are also provided to facilitate operations and this can be found in the Palettes. To start typing in the worksheet, first, begin by typing the ">" or can press ">" on the toolbox. This is one of the requirements as the first step to work on the worksheet from Maple software.

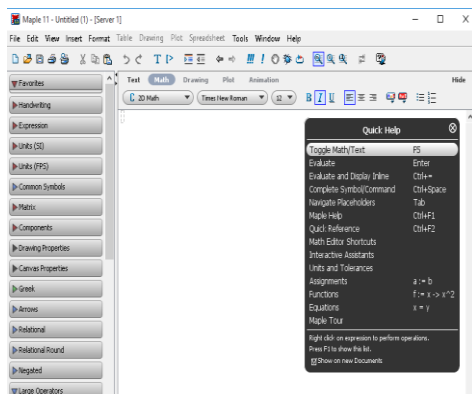


Fig 2: The display of Maple 11

Some functions used in calculus learning by Maple software, including function used as the initial test. The work of students in learning derivative and anti-derivative calculus using Maple software on cubed function can be seen in "Fig. 3", "Fig. 4", and "Fig. 5".

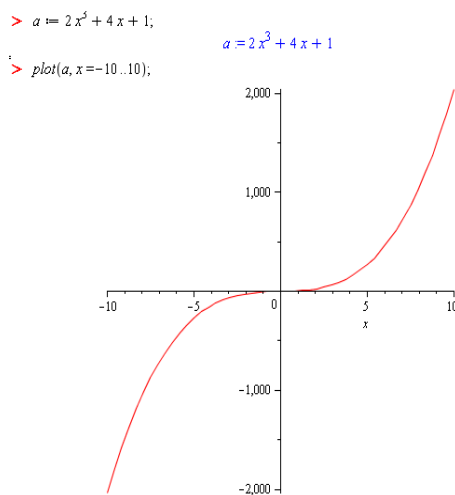


Fig 3: The first graph

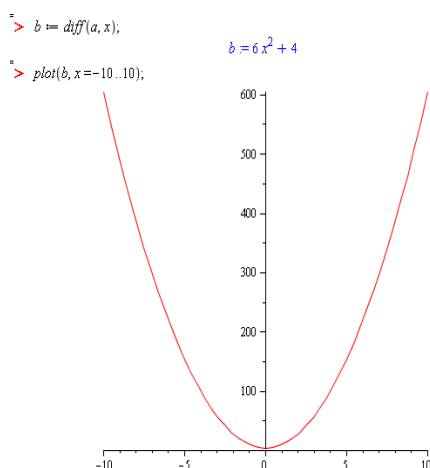


Fig 4: The derivative graph

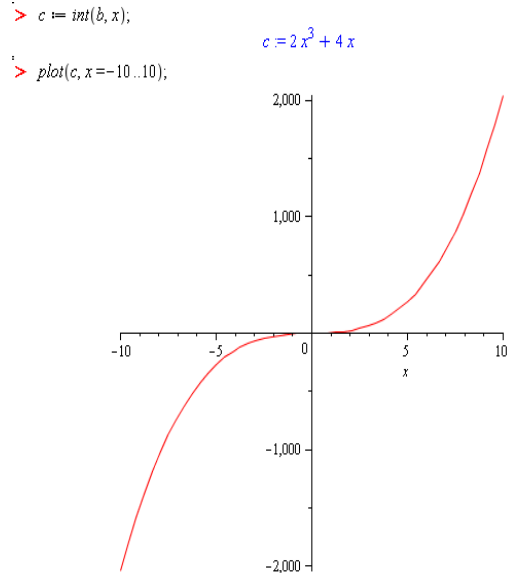


Fig 5: The anti-derivative graph

The results of students' work in determining derivative and anti-derivative functions, as well as graphs of initial functions, graphs of derivative functions, and graphs of anti-derivative functions simply indicate that this is very helpful for students in understanding the concepts of derivatives and anti-derivatives. This can be seen based on the results of interviews with the representatives of participants in this study.

Figure 5 is the anti-derivative of the derivative function of "Fig. 4" so that it becomes the original function (Fig. 2) of the function. The original function (S1) is different from the derivative function graph (Fig. 3). (S1)

Based on the graph of the initial function after going through the derivative process, the graph changes. After going through an anti-derivative process, the graph returns to its initial form. (S2)

From the 3 figures, "Fig.3" and "Fig.4" are totally different. Meanwhile, "Fig.3" and "Fig.5" are not too different. (S3)

The statements of S1, S2, and S3 are of the same meaning, that is, the participants used visual thinking in understanding derivative and anti-derivative differences. Interview results for S1, S2, and S3 are further emphasized by S4 & S5 statements as follows.

There is a difference between the main function graph and the derivative function graph, and there are similarities between the main function and the anti-derivative function graph. So a function is derived (through a derivative process), and if the function is integrated (anti-derivative) it will produce the original function. (S4)

The first function graph (Fig.3) and the third function graph (Fig.5) have the same shape, because it is an (anti-derivative) integral of  $f'(x)$  (derivative) in the second function graph, so the third function graph (Fig. 4) return as in the graph of the original function (Fig.3). While the second function graph (Fig.4) is different, because it (Fig.4) is a derivative of the first function. (S5)



A student who has a visual superiority will rely heavily on visual thinking and will have less dependent on analytic thinking [5]. The students' ability to conclude that the initial function graph is the same as the graph of the anti-derivative function is a reversible thinking ability. Reversible thinking is flexibility in revisiting and going forward, namely changing and building cognitive structures [2]. The students' reversible thinking, in this case, occurs when the students conducted a review process (graph of the initial function) based on the results obtained (graph of the anti-derivative function). The process of anticipation and review was shown based on participants S4 and S5 when they were interviewed. While S1, S2 and S3, although they can only show the difference between the initial function graph and the derivative function graph and the equation graph of the initial function with anti-derivative functions, it has given the view that the review process can be done by knowing the similarity of the graph of the initial function with the derivative function graph. This shows that the reversible thinking process that occurs in students in this study focuses more on changing the shape of the graph of the function. The students only see the shape and location of the points traversed by the lines that form the graph of the functions obtained by using Maple. If you pay attention to changes in the function graph in "Fig. 2", "Fig. 3", and "Fig. 4" it is quite clear that the first and third function graphs are the same when viewed from the location of the points on the function graph line by using Maple. Likewise for differences in graphs from the first and second functions both the shape and the location of the points on the line show the difference.

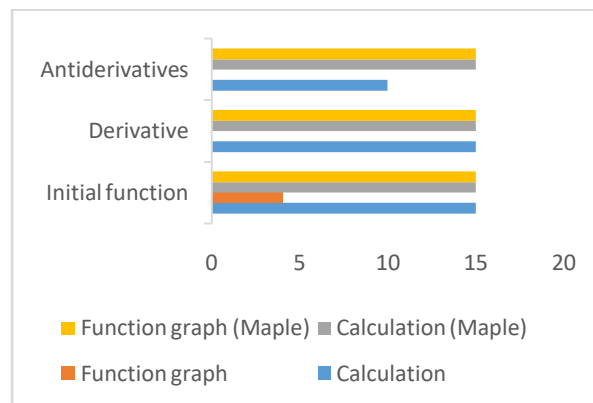
The changing graph in the initial function through the derivative process into a graph for derivative functions, and then returning to its initial form through the anti-derivative process in "Fig. 2", "Fig. 3", and "Fig. 4" shows that students have been able to solve the problem of derivative and anti-derivative *Calculus* by using Maple. These results provide new knowledge for students, that *Calculus* learning using Maple is very helpful in understanding the concepts of derivative and anti-derivative easily and quickly. This is based on the results of interviews with participants who stated, that

*If we do it (derivatives and anti-derivatives) manually, we need too many formulas. For example, we must determine the coordinate cut-off point, the appearance of the coordinating point, and so on. (S6)*

*Working with Maple software makes it easy to compare the results of manual work and by using mathematical applications. (S7)*

*Maple software can be used to make it easier to check the answers we produce. (S8)*

The ease of students in completing calculus assignments using Maple has shown that Maple software can provide benefits and convenience for students in understanding the concepts of derivatives and anti-derivatives. In addition, Maple software also provides faster solutions and good visualization [26]. If it is compared to the students' work data on the initial test without using Maple software, it gives significant difference results (see Fig. 6).



**Fig 6: Students' work on the conventional learning and learning using the Maple software**

The students' work based on "Fig. 6" shows a significant difference; the number of students who succeeded in completing the task of calculus learning using Maple software is much more than the number of students who successfully completed the task in the initial test. The findings of this study are in line with the results of the study [1], the results of work using Maple software are significantly better than those of students who do not use the software. This difference indicates that this system is very helpful for students in understanding and completing the tasks of derivative and anti-derivative functions. In addition, students can also develop their understanding through sensory-motor activities and visual features available in the system [1]. The use of Maple facilitates students to understand graph changes that occur between function graphs, derivative function graphs, and anti-derivative.

The great benefits of Maple for students when solving problems and understanding a concept cannot guarantee that students are also able to solve problems or understand concepts without using Maple. Besides, if a task requires a lot of calculation and does not need thinking too much, then students will become bored [11].

In this study, Maple is only a computer device used in work and teaching by a teacher and lecturer, and also as a tool to understand and solve a problem for students. The weakest and most curious students can explore Maple's 'Help' section to enrich vocabulary [10]. This ease, of course, must be supported by good mathematical concept mastery. A lecturer is needed in balancing between asking students to work what is routine and asking students to use computers as a tool so that they can carry out more in-depth analysis in order to gain knowledge and experience that they have not obtained before. This can be a further study. Balancing between routine work and work using a computer can be a solution to teaching, especially in *Calculus*, as well as a reflection for students in understanding concepts and solving problems. Reflection activity here is an activity in reviewing the results obtained between the results of calculations without computer assistance and computer-assisted results so that this reflection involves reversible thinking.

#### IV. CONCLUSION

Teaching derivative and anti-derivative Calculus by using Maple can provide convenience to Lecturers in providing an understanding of derivative and anti-derivative concepts. This ease is also felt by students who take part in the learning. Students become easier to understand the concept of derivative and anti-derivative functions (integrals). Using Maple to solve derivative and anti-derivative function problems also facilitates students to find results quickly and accurately. Graph changes in derivative and anti-derivative functions lead students to think reversibly. Reversible thinking can be identified when students review the results of calculations and graphs formed by using Maple. Students' reversible thinking ability is also influenced by visual thinking from students in understanding the derivative and anti-derivative concepts of a function.

This research is still limited to identifying the process of reversible thinking ability of students in understanding the concepts of derivative and anti-derivative in calculus learning using Maple. Identifying reversible thinking abilities of students in this study through changes in function graphs was obtained by using Maple software assistance. Meanwhile, students' reversible thinking regarding the concept of derivative and anti-derivative with manual calculations still needs to be studied further. In addition, the use of software as a tool used in reflecting the work of students and as a learning technology needs to be developed in different materials, especially in mathematical concepts.

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