The Use of Multiple Representation in Functional Thinking

Suci Yuniati, Toto Nusantara, Subanji, I Made Sulandra

Abstract: Functional thinking is focused on the relationship between two or more variant quantities which can be represented by using natural language, tables, graphs, and algebra. Representation is the way that students use to find solutions and express ideas or thought of problems encountered. There are several kinds of representations that can be used in functional thinking, for example verbal, algebraic, numerical, images, graphics, etc. The aim of this study is to describe the use of students’ representations in functional thinking. This study belongs to qualitative descriptive research. Research subjects amounted to 45 students of Senior High School. The instruments used in this research were problem solving and interview. The results showed that students use verbal, numerical, algebraic, and image representations in their functional thinking. Nevertheless, students are more dominant on using algebraic representations.

Keyword: multiple representation, function, functional thinking and mathematics.

I. INTRODUCTION

Functional thinking is an important aspect of mathematics activities at school [1]–[3], [3]–[8]. Functional thinking involves generalizing the relationship between the quantity of covariance which can be represented by natural language, algebraic notation (symbolic), tables and graphics [5]. Functional thinking is implemented when someone performs an activity of choosing in observing two or more variant quantities and then focusing on the relationship between the quantities. The focus is a relationship which centered on the concept of function [9]. Therefore, functional thinking is related to the concept of function.

Function is one of the mathematics materials taught in school [1]–[3], [3]–[8]. Function is not a concept that is likely understood by students. Students usually have misconceptions about the function and difficulty in expressing the use of algebraic notation [8], [12]. Difficulties and misconceptions of students about the function can be minimized by improving the development of functional thinking of students started from an early age. The development of functional thinking can be increased gradually and over time [1].

As a matter of fact, there have been lots of research carried out to improve the development of functional thinking [1]–[3], [3]–[8], [13]–[18], [18]–[30]. The findings of [3] suggest that functional thinking can be mastered by novice students in class earlier than what is expected. Besides, covariance thinking can be learnt by pre-kindergarten students, and quantity can be described by students in the beginning of first grade. [1] stated that functional thinking can be developed by elementary school students, verbally and symbolically. Furthermore, [8] discloses that generalizing correspondence relationships can be learnt by primary school students. [17] explored novice students in thinking functionally. Also, Blanton, et al [5] described the trajectory learning of 6-year-old children in generalizing functional relationship of algebraic material. [15] stated that the ability of generalizing and symbolizing the functional relationship between covariance quantities has been increased in Elementary School students. Moreover, [24] described pattern generalizations which is taught by Elementary School teachers. [14] developed students' ability in generalizing and representing functional relationships in early algebra interventions for 3-6 graders. [29] developed an assessment on the ability of Elementary School students in functional thinking. [30] conducted research on student behavior significant for developing an intervention. The above findings show that novice students are able to think of the relationship between two variant quantities and express the relationship to a more abstract form so they can provide insight about students’ thinking of function.

Based on the functional relationship shaped from mathematical perspectives, [9] proposed three kinds of functional thinking approach, namely: 1) recurrent patterns, which entail finding the variation or pattern of variation in a series of values for a variable in a way such that a specific value can be obtained based on the preceding value or values; 2) correspondence, the relationship between the pairs (a, f(a)) for the variable is stressed; and 3) covariation, which focuses on how the values of one variable is changed the values of another. The example of these three functional relationships mentioned above.
is explained in this Table 1 below.

**Table 1. Example of Functional Relationship**

<table>
<thead>
<tr>
<th>Correspondence. “The focus would be on the relation between x and y, which might be described as twice x plus on or algebraically as: 2x+6”.</th>
<th>Variation Between Two Quantities</th>
<th>Recurrent patterns. This involves looking for a relationship within a sequence of values. For instance, the focus is on the values of the first column (independent variable).</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td></td>
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<tr>
<td>...</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Based on Table 1, the researchers used several approaches in functional thinking namely 1) identifying and organizing data; 2) identifying recursive patterns; 3) identifying covariance relationships; and 4) identifying correspondence.

In functional thinking, students can express their ideas or thoughts by using multiple representations. [9] defines functional thinking as a thinking representation which is focused on the relationship of two or more variances, especially the specific relationship that is affected by particular type of thinking into a generalization of all sample relationships. Representation is the way that students use to discover solutions and express ideas or thoughts of a problem encountered [31]-[36]. There have been many studies confirming that representations can be used by students to express their ideas or thoughts in solving problems [10], [37]-[39], [33], [40]-[48][32][49]. For example, [32] suggests that prospective teachers use various representations to solve a given problem. The most widely used representation is verbal representation. At the planning stage, the representation used is algebraic representation. However, at problem understanding stage, the most widely used representation is verbal representation and graph. Meanwhile, at the solution planning stage, the most often used representation is verbal representation rather than graphical representation and algebra. According to [48] students solve problems more successfully using graphical representation than using symbolic representation.

According to [49] there are four kinds of representations: 1) verbal representation which is generally used in declaring problems at the beginning of the process and required to provide the final interpretation obtained in problem solving; 2) numerical representation is a representation which is introduced to students in the early stages of algebra learning. The use of numbers is important in gaining first understanding of the problem and in investigating certain cases; 3) graphical representation is an effective representation model which is used to describe the functional value of real variables, and 4) algebraic representation is a concise, general, and effective representation which is used to express mathematical patterns and models. According to [31] symbols, equations, words, images, tables, graphs, manipulative objects, and actions as well as mental, internal ways of thinking about mathematical ideas are categorized as representations. Several kinds of representations above can be used by students in solving mathematical problems including in functional thinking.

There have been some researches conducted about this issue, however, in the previous researches; there is no discussion about the use of representations in functional thinking, thus the researcher aimed to carry out a research on the use of representation in functional thinking in arithmetic sequence material. The quantity employed in this research was the quantity of the total of rows, tables, and chairs. Therefore, identification of covariance relationship in this research, particularly the correlation between the number of rows and tables; the correlation between the number of rows and chairs; and the correlation between the number of rows, tables chairs. The objective of this research is to describe the use of student’s representation in functional thinking. And the research question is “What representation used by students in functional thinking?"

**II. METHOD**

1. **Research Subject**

The subjects of this study were 45 high school students who participated in this research. The school is located in suburb area in Riau Province. Referring to the relevant literatures regarding on the importance of functional thinking in early classes [1], [3], [8], it is possible that high school students can perform functional thinking. Furthermore, the selected subjects had already got sequence material and they were selected based on the criteria of functional thinking approaches and representations used.

2. **Data Collection Process**

The instruments used in this research were problem solving and interview. The interviews used were unstructured interviews. The worksheet of problem solving was assigned to 45 students with the allocation time to solve the problem of 2 periods. The question items used were questions compiled based on a case or event to explore students’ functional thinking. Nevertheless, the level of difficulty of given problem is intermediate level. There was independent variable represented by "number of rows" and the dependent variable was represented by "number of tables and number of chairs" in the problem worksheet. The worksheets assigned to students were as follows:
Fig 1. Problem Solving Worksheet

3. Procedure

There were three stages in the procedure of this research namely: 1) stage of data collection; in this data collection, the researchers asked each student to complete the problem solving worksheet. During the process, the researchers did not intervene or involved in any interaction with the student; 2) the stage of analyzing the student’s work and conducting the interview, at this stage the researchers analyzed the student’s work and interviewed the students to explore further the use of representation in functional thinking; and 3) the last stage, the researchers examined and summarized the results of students’ work and the results of interview.

4. Data Analysis

In data analysis stage, the activities undertaken by the researchers were 1) collecting and preparing the data for analysis. In this case, the researchers collected all students’ answers sheets in completing the problem worksheet; 2) analyzing all student answer sheets in which the researchers analyzed all data obtained and distributed the use of representations that were adjusted to the criteria of the functional thinking approach. Should the student’s answer does not meet the criteria of the functional thinking approach, then the representation used is not included in the distribution of the use of student representation; and 3) conclusions, at this stage the researchers concluded the results of the study [50].

III. RESULT AND DISCUSSION

This research investigated the use of multiple representations in functional thinking. The approach used in functional thinking namely 1) identifying and organizing data; 2) identifying recursive patterns; 3) identifying stratified covariance relationships; and 4) identifying correspondence. Thus the findings obtained from the research results will be presented below:

1. Identifying and Organizing Data

Numerical representation is really known by students in their early time learning algebra. The use of number is considerably necessary in discovering students’ understanding in analyzing some particular cases. As it is shown in Figure 2, Nada identified and organized data using numerical representation in ordering the set of numbers.

Fig 2. Nada Used Numerical Representation

2. Identifying Recursive Patterns

In identifying recursive patterns, the students used verbal representations, algebraic representations, and numerical representations shown in the solutions below:

a. Verbal Representation

In Fig 3, Lina answered the questions by using verbal representation. Lina explained it by using her own language. She found out that the number of tables is multiple1 started from number 4.

Fig 3. Lina Used Verbal Representation

b. Algebraic Representation

Algebraic representation is Rozi’s choice in identifying a recursive pattern. Rozi tried to answer this question based on his learning experience in the classroom, where Rozi used formula Un = a+(n-1)b and Rozi wrote “b = the common difference between the sequenced terms”. We can see Rozi’s answer in this following Figure 4.

Fig 4. Rozi used Algebraic Representation

Problem 1

AdiBuana School Surabaya will hold students’ parents meeting in Multi Purpose Building. In this building, the tables and chairs will be arranged in rows with the front row is arranged 4 tables and 8 chairs, the second row is arranged 5 tables and 10 chairs, the third row is arranged 6 tables and 12 chairs, and so on.

a. Find out how to find the number of tables on the 100th row. Explain how you found it.
b. Find out how to find the number of chairs on the 100throw. Explain how you found it.
c. Find out the formula to find the number of tables on the n-th row. Explain how you found the formula.
d. Find out the formula to find the number of chairs on the n-th row. Explain how you found the formula.
e. Find out the formula that states the relationship between the number of tables and the number of chairs on the n-th row. Explain how you found the formula.

Fig 2. Nada Used Numerical Representation

2. Identifying Recursive Patterns

In identifying recursive patterns, the students used verbal representations, algebraic representations, and numerical representations shown in the solutions below:

a. Verbal Representation

In Fig 3, Lina answered the questions by using verbal representation. Lina explained it by using her own language. She found out that the number of tables is multiple1 started from number 4.

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Fig 4. Rozi used Algebraic Representation
c. Numerical Representation

Figure 5 showed that Sari answered the questions manually in identifying recursive patterns. Numerical representation was Sari choice of registering all the answers starting from “B.1 = 8” which means the first row is 8 chairs up to “B.100 = 206” which means the 100th row is 206 chairs. In this case, Sari could identify the pattern by adding 2 on each line.

In identifying covariance relationships, Julia used algebraic representations. Julia determined the number of tables by writing “b = 1” and the number of chairs by writing “b = 2” which means b is symbolized as a difference or change of each row. Julia’s work can be seen in Figure 7.

b. Verbal Representation

The solution below (Figure 6) shows that Sara could identify covariance relationships by using verbal representation. Sara explained that to find the number of tables in the 100th row, she simply counted the next number after the number 4 to the 100th row. In this case, Sara added 1 on each row. Meanwhile, to find the number of chairs in the 100th row, it was done by counting multiples of 2 starting from the number 8.

In identifying covariance relationships, Yani used algebraic representation (Figure 9). In this case, Yani found the formula on the n-th row in discovering the number of tables with the formula $3 + n$ and the formula on the n-th row in discovering the number of chairs by the formula $6 + 2n$. To discover the relationship formula between the row and the number of tables and chairs, Yani added $3 + n$ formula with $6 + 2n$, moreover Yani wrote "the number of tables and chairs are $(3 + n) + (6 + 2n) = 9 + 3n$ with a table of $3 + n$ and $6 + 2n$ chairs ".

3. Identifying Covariance Relationships

In identifying covariance relationships, students used verbal representation, algebraic representation, and image representation as it is shown in the solution below:

a. Verbal Representation

The solution below (Figure 6) shows that Sara could identify covariance relationships by using verbal representation. Sara explained that to find the number of tables in the 100th row, she simply counted the next number after the number 4 to the 100th row. In this case, Sara added 1 on each row. Meanwhile, to find the number of chairs in the 100th row, it was done by counting multiples of 2 starting from the number 8.

In identifying covariance relationships, Julia used algebraic representations. Julia determined the number of tables by writing “b = 1” and the number of chairs by writing “b = 2” which means b is symbolized as a difference or change of each row. Julia’s work can be seen in Figure 7.

4. Correspondence

In identifying correspondence, Yani used algebraic representation (Figure 9). In this case, Yani found the formula on the n-th row in discovering the number of tables with the formula $3 + n$ and the formula on the n-th row in discovering the number of chairs by the formula $6 + 2n$. To discover the relationship formula between the row and the number of tables and chairs, Yani added $3 + n$ formula with $6 + 2n$, moreover Yani wrote "the number of tables and chairs are $(3 + n) + (6 + 2n) = 9 + 3n$ with a table of $3 + n$ and $6 + 2n$ chairs ".
5. Covariance and Correspondence

In identifying covariance and correspondence, the students used verbal representation, algebraic representation, and image representation shown in the solution below:

a. Verbal Representation

Putri used verbal representation in identifying covariance and correspondence (Figure 10). Putri wrote "the formula is that the first row has 4 tables then the second row has 5 tables, the third row has 6 tables and the fourth row has 7 tables, so we can formulate that the first row of 4 tables is (1+3), as well as row 2 and 3 where it increases 3 until it reaches the 100th row i.e. 103 tables (100) ".

b. Algebraic Representation

Mahda’s work shows that Mahda used algebraic representation in identifying covariance and correspondence. Mahda wrote \(b = 1\) that describes the pattern or the difference in the number of tables as much as 1 and the formula \(U_n = n + 3\) to determine the number of the \(n\)-th table. Mahda also wrote \(b = 2\) that describes the pattern or the difference in the number of chairs as much as 2 and \(U_n = 2n + 6\) to determine the number of the \(n\)-th chairs. Mahda’s work can be discovered in Figure 11 as follows.

c. Verbal Representation and Image Representation

From Aldi’s work (Figure 12), we can identify that Aldi used verbal representation and image representation in identifying covariance and correspondence. Aldi wrote that in the first row there are 4 tables and 8 chairs and he concluded the difference of the rows is 1, the difference of the table is 1, and the difference of chairs is 2. Then Aldi described that there are 4 tables and 8 chairs. He also concluded that every 1 table is multiplied 2 to get the number of the chairs.
representations. In correlating the covariance, high school students tend to choose algebraic representation. It contrasts with the research of [8] explaining that beginner students use words and semi symbolic in correlating the covariance. Fourth, in identifying correspondence, students tend to use algebraic representations.

IV. CONCLUSION

Based on data analysis and research findings, algebraic representation is the most dominant representation used by students in functional thinking. Students are more dominant in choosing algebraic representation due to students' learning experience in the classroom in which teachers always teach the sequence using algebraic formula. In teaching-learning process, teachers need to teach about the types of representations that are useful in improving students' creativity in solving math problems. According to [31] the use of representation is expected to support students' understanding of mathematical concepts and their relationship in communicating mathematics, arguments, and understanding of one another, in recognizing the relationship between mathematical concepts. Therefore, the use of representation needs to be mastered by students, so that when students encounter non-routine questions, students can represent the problem in finding solutions to completion. Finally, the future researcher is expected to focus on how students who think functionally has a relationship of three variant quantities.

V. REFERENCES


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