International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-8, Issue-1, May 2019

Fuzzy Haar Hexagonal VIKOR with FCM Techniques to Analyze the Solid Waste Management

A. Selvaraj, Saroj Kumar Dash, N. Punithavelan

Abstract*: In this paper, a novel attempt is made to formulate a hybrid system by combining the Fuzzy Cognitive Map (FCM), the Delphi Method, and the VIKOR method through the Haar ranking of Hexagonal Fuzzy Number. To illustrate this technique, the solid waste management is taken. Solid Waste arises from human and animal activities can generate adverse effect on human health and affect the living environment drastically when proper solid waste management is not done efficiently. The proper disposal of solid waste management aids to diminish the horrible impacts on both human health and environment to sustain economic growth and better quality of life. Therefore, this present study analyses the Solid Waste Management through the novel Haar Hexagonal VIKOR with FCM techniques.

***Index Terms*: Fuzzy Logic, Hexagonal Fuzzy Number, Haar ranking, FCM, Delphi method, VIKOR.**

I. INTRODUCTION

Multiple Attribute Decision Making (MADM) systems are always used to answer various problems whenever decision making is involved [9]. There are well known effective tools for solving MADM problems such as AHP, DEMATEL, TOPSIS, VIKOR and etc., [5, 6, 25]. The VIKOR method is for establishing a compromise ranking list, a compromise solution [21, 22]. In 2009, Chen and Wang developed the Fuzzy-VIKOR method for the assessment and evaluation of potential suppliers/vendors to make efficient delivery approach. It provided a systematic method to develop the best choice and compromise result from all the selected criteria [3]. In 2010, Sanayei, et.al, developed the VIKOR technique in the supply chain system to deal with the trader selection problems. The solution was close to the ideal answer and the alternatives were determined by all the recognized criteria [24]. In 2011, Devi has extended VIKOR method for Intuitionistic fuzzy situation and solved multiple–criteria decision making problems to study the selection of robots in objects handling assignment [8]. In 2011, Opricovic has extended the fuzzy VIKOR method with a trade-offs analysis to illustrate water resources development, utilizing of the Mlava River and its tributaries for regional irrigation [20]. In 2014, Chang applied the VIKOR method to find out the importance of weights for evaluating criteria and to consolidate the quality performance ratings in service for feasible alternatives [2]. Failure mode and effects analysis (FMEA) is broadly used for assessing risks in tools and spot the failure parts during product design. In order to pick the serious failure modes, the VIKOR method is extended by Liu, et.al, in 2012 [18] to find out the influencing factors of the failure modes. A novel approach for FMEA is developed by the combination of the fuzzy VIKOR method, the fuzzy analytic hierarchy process (AHP) and the entropy method to verify the strength of the risk in ranking and a comparison study [19]. In 2013, Zhang and Wei have solved MCDM problems with uncertain fuzzy set information by combining the VIKOR method and the TOPSIS method. In which, the uncertain fuzzy set information and the equivalent concepts were explained to extend the VIKOR method [26]. In 2010, Kaya and Kahraman have integrated the VIKOR-AHP model for selecting the best energy strategy and manufacturing site [13]. Fuzzy Cognitive Map was extended by Kosco, B in 1986 [17] from the Cognitive maps. It is one of the simple techniques inspired by the cognition of human brain and it is a proficient system for decision making. Fuzzy Cognitive Map is a digraph connecting the concepts. The weight of the link depends on the strength of relationship between the two nodes. In order to represent the complexity of the connection strength, fuzzy weights were taken up and named as Fuzzy Cognitive Map (FCM). FCM is predominant method to capture the expert knowledge in a natural way. Simple FCM only considered the connection weights as 0, if there is no relation between the factors and when there is a relation between the factors, the weight is 1. It is also notable that the connection weights could be intervals, linguistic or any special case fuzzy numbers such as triangular, trapezoidal etc.

Revised Manuscript Received on 30 May 2019.
***** Correspondence Author
A. Selvaraj*, Mathematics Division, School of Advanced Sciences, VIT Chennai, Tamilnadu, India.
Saroj Kumar Dash, Mathematics Division, School of Advanced Sciences, VIT Chennai, Tamilnadu, India.
N. Punithavelan, Physics Division, School of Advanced Sciences, VIT Chennai, Tamilnadu, India.

Devadoss et.al., introduced nonagonal, decagonal and hendecagonal fuzzy numbers to represent linguistic terms as causal weights [7, 10, 11].

In 2010, the incorporation of qualitative scenarios in decision making model has been examined for the urbanization of the coastal town in Indonesia [14]. Factor prioritization is crucial for decision making and planning. Therefore, a study was conducted to obtain the sufficient number of criteria with a negligible loss of information from 48 criteria using Fuzzy Cognitive [1]. In 2012, Kontogianni and Papageorgiou, have proposed the extensions of cognitive maps to use for modeling complex chains of casual relationships [16]. In 2015, Chen, et.al, have found a new perspective to develop a Timed-Automata-Based Fuzzy Cognitive Maps [4]. In planning and policy making, deriving the decision is not an easy task. Due to lack of statistical data and inaccurate verbal terminologies, the construction of a quantitative mathematical form may be hard or unattainable. Therefore, Papageorgiou & Hatwágner, have focused on reducing complexity in the modeling process, which provides a more transparent and easy to use model for policy makers [23]. In 2017, Gutierrez, et.al, used FCM as an instrument for providing feasible decisions in fluvial management by anticipating responses to different policy schemes [12]. The FCM is applied to recognize the significant factors that will have an effect on the large-scale production of bio ethanol from bio waste and the results indicated that the different groups of stakeholders have a different perception and identify different factors as the driving forces of the project [15]. From this review, it is observed that a new hybrid technique can be proposed by integrating the salient features of the FCM and the TOPSIS through the Haar ranking of the Hexagonal Fuzzy Number to analyze the solid waste management.

II. PRELIMINARIES

2.1 Fuzzy set

Fuzzy set [27] is a mapping from the universal set to [0, 1] or is an ordered tuple $(x, \mu_{\tilde{A}}(x))$ where x is an element of universal set and $\mu_{\tilde{A}}(x)$ is a membership grade whose value lies in [0, 1] which gives the closeness of an element belonging to the fuzzy set. Fuzzy set holds significant properties like convex, piece wise continuous and normal.

2.2 Fuzzy Number

Fuzzy number is a fuzzy set in real numbers, a mapping $\mu:\mathbb{R}\to[0,1]$, must hold the following properties:

i. μ is convex, i.e.

$$\mu(x)\geq \min(\mu(y),\mu(z)), \text{ for } y\leq x\leq z$$

ii. μ is normal, i.e. there exists some x in $\mathbb{R}$ such that $\mu(x)=1$

iii. μ is piecewise continuous.

2.3 Alpha Cut

The alpha cut of the fuzzy set is a crisp set and is a subset of the universe of discourse with each element's membership value in the alpha cut is greater than or equal to $\alpha\in[0,1]$ and is defined as

$$\tilde{A}_{\alpha}=\{t\in X \,/\, \mu_{\tilde{A}}(t)\geq\alpha\}$$

2.4 Triangular Fuzzy Number

A fuzzy number, whose membership function curve forms a triangle shape. Triangular fuzzy number $\tilde{N}$ can be defined as a triplet (i, j, k) such that $i\leq j\leq k$ all real numbers whose membership function $\mu_{\tilde{N}}(x)$ is defined as

$$\mu_{\tilde{N}}(x)=\begin{cases} 0 & x<i \\ \left(\dfrac{x-i}{j-i}\right) & i\leq x\leq j \\ \left(\dfrac{k-x}{k-j}\right) & j\leq x\leq k \\ 0 & x>k \end{cases}$$

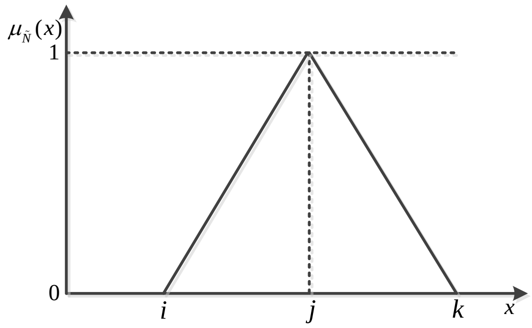

Figure 1: Triangular Membership Curve

2.5 Trapezoidal Fuzzy Number

A fuzzy number, whose membership function's curve forms a trapezoid shape and is a generalization of the triangular fuzzy number. A trapezoidal fuzzy number (a_1,a_2,a_3,a_4) all are real numbers such that $a_1\leq a_2\leq a_3\leq a_4$ whose membership function is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} \left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1\leq x\leq a_2 \\ 1 & a_2\leq x\leq a_3 \\ \left(\dfrac{a_4-x}{a_4-a_3}\right) & a_3\leq x\leq a_4 \\ 0 & a_1\leq 0\,\&\,a_4\geq 0 \end{cases}$$

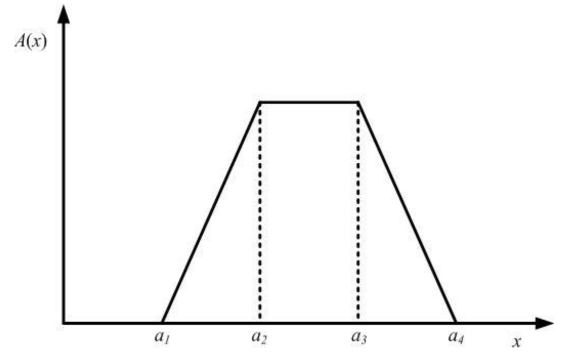

Figure 2: Trapezoidal Membership Curve

2.6 Linear hexagonal fuzzy number with symmetry (LHFNS)

A linear hexagonal fuzzy number is defined as $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5, a_6; h)$ whose membership function is given by

$$\mu_{\tilde{A}_{LS}}(t) = \begin{cases} h\left(\dfrac{t-a_1}{a_2-a_1}\right), & a_1 \le t \le a_2 \\ 1-(1-h)\left(\dfrac{t-a_2}{a_3-a_2}\right), & a_2 \le t \le a_3 \\ 1, & a_3 \le t \le a_4 \\ 1-(1-h)\left(\dfrac{a_5-t}{a_5-a_4}\right), & a_4 \le t \le a_5 \\ h\left(\dfrac{a_6-t}{a_6-a_5}\right), & a_5 \le t \le a_6 \\ 0, & t \le a_1 \ and \ t \ge a_6 \end{cases}$$

2.7 Alpha cut -Symmetry linear Hexagonal fuzzy number (SLHFN)

Given a fuzzy set A defined on X and any number $\alpha \in [0, 1]$, the $\alpha - cut$, then we define α_A the crisp sets as $\alpha_A = \{t \in X \,/\, \mu_{\tilde{A}}(t) \ge \alpha\}$

$$A_\alpha = \begin{cases} A_{1L}(\alpha) = a_1 + \dfrac{\alpha}{h}(a_2 - a_1) \ for \ \alpha \in [0,h] \\ A_{2L}(\alpha) = a_2 + \left(\dfrac{1-\alpha}{1-n}\right)(a_3 - a_2) \ for \ \alpha \in [h,1] \\ A_{2R}(\alpha) = a_5 - \left(\dfrac{1-\alpha}{1-h}\right)(a_5 - a_4) \ for \ \alpha \in [h,1] \\ A_{1R}(\alpha) = a_6 - \dfrac{\alpha}{h}(a_6 - a_5) \ for \ \alpha \in [0,h] \end{cases}$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ are increasing functions with respect to α and $A_{3R}(\alpha)$, $A_{2R}(\alpha)$ decreasing functions with respect to *α*.

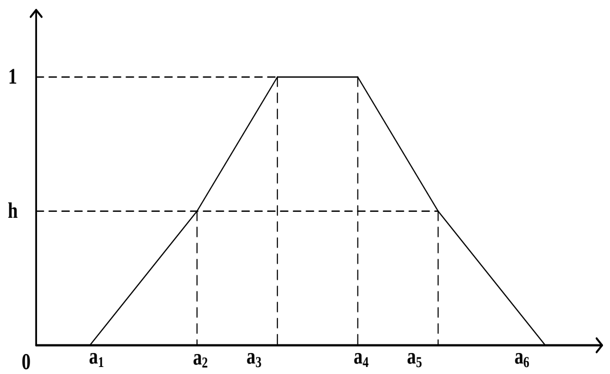

Figure 3: Alpha cut -Symmetry linear Hexagonal fuzzy number (SLHFN)

III. PROPOSED METHODOLOGY

This present study integrates the salient features of FCM and VIKOR technique to bring out the new hybrid technique Scenario FCM-VIKOR through Haar ranking of Hexagonal Fuzzy number. This technique consists of the following four stages.

1. FCM Process:

(i) Frame the initial linguistic uncertain direct relation matrix.

- Consider $G = \{G_1, G_2, G_3, ..., G_n\}$ to be a finite number of attributes and $E = \{E_1, E_2, E_3, ..., E_k\}$ to be the finite number of experts. Then, Experts are solicited to provide their views on correlating the influencing factors from the linguistic set L = {Very Low, Low, Medium, High, Very High}.
- The relation matrices $D_k = [a_{kij}]_{n \times n}$ are formed from the FCM for the attributes $G = \{G_1, G_2, G_3, ..., G_n\}$ where a_{kij} - causality connection weight between *i* and *j* based on k^{th} expert view.

(ii) Transform the initial linguistic uncertain relation matrix $\hat{U}_k = [\hat{u}_{kij}]_{n \times n}$ to hexagonal fuzzy matrix $\tilde{U}_k = [\tilde{u}_{kij}]_{n \times n}$ using the Hexagonal linguistic scale.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Alternative Assessment	Fuzzy Scale
No Influence (NI)	(0, 0, 0, 0.06, 0.12, 0.18)
Very Low (VL)	(0.06, 0.12, 0.18, 0.24, 0.3, 0.36)
Low (L)	(0.24, 0.3, 0.36, 0.42, 0.48, 0.54)
Medium (M)	(0.42, 0.48, 0.54, 0.6, 0.66, 0.72)
High (H)	(0.6, 0.66, 0.72, 0.78, 0.84, 0.9)
Very High (VH)	(0.78, 0.84, 0.9, 1, 1, 1)

Table: 1 Hexagonal Fuzzy Scale

(iii) Construct the total uncertain direct relation matrix $\tilde{U}_k = [\tilde{u}_{kij}]_{n\times n}$.

All the separate uncertain direct relation matrices $\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_m$ are then grouped to form a single total uncertain direct relation matrix $\tilde{U}_k = [\tilde{u}_{kij}]_{n\times n}$ and is given as

$\tilde{u}_{kij} = (u^1_{kij}, u^2_{kij}, u^3_{kij}, u^4_{kij}, u^5_{kij}, u^6_{kij})$ Where

$$u^1_{ij} = \frac{1}{m}\sum_{k=1}^{m} x^1_{kij},\ u^2_{ij} = \frac{1}{m}\sum_{k=1}^{m} x^2_{kij},\ u^3_{ij} = \frac{1}{m}\sum_{k=1}^{m} x^3_{kij},$$

$$u^4_{ij} = \frac{1}{m}\sum_{k=1}^{m} x^4_{kij},\ u^5_{ij} = \frac{1}{m}\sum_{k=1}^{m} x^5_{kij},\ u^6_{ij} = \frac{1}{m}\sum_{k=1}^{m} x^6_{kij},\ i, j = 1, 2, \ldots, n$$

2. Converting Hexagonal Fuzzy Number into Haar Hexagonal Fuzzy Number

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ be the Hexagonal fuzzy number. For the convenient of using Haar wavelet ranking, the Hexagonal fuzzy number is rewritten as $\tilde{\tilde{A}} = (a_1, a_2, a_3, a_3, a_4, a_4, a_5, a_6)$. The average and detailed coefficients namely the scaling and wavelet coefficients of Hexagonal fuzzy number is evaluated as given below.

(i) Group the Hexagonal fuzzy numbers in pairs.

$$[a_1, a_2], [a_3, a_3], [a_4, a_4], [a_5, a_6]$$

(ii) The first four elements of $\tilde{\tilde{A}}$ with the average of these pairs (approximation coefficients) and replace the last 4 four elements of $\tilde{\tilde{A}}$ with half of the difference of these pairs (detailed coefficients).

$$\alpha_1 = \left(\frac{a_1 + a_2}{2}\right), \alpha_2 = \left(\frac{a_3 + a_3}{2}\right), \alpha_3 = \left(\frac{a_4 + a_4}{2}\right), \alpha_4 = \left(\frac{a_5 + a_6}{2}\right);$$

$$\beta_1 = \left(\frac{a_1 - a_2}{2}\right), \beta_2 = \left(\frac{a_3 - a_3}{2}\right), \beta_3 = \left(\frac{a_4 - a_4}{2}\right), \beta_4 = \left(\frac{a_5 - a_6}{2}\right)$$

The $\tilde{\tilde{A}}_1$ changed into $\tilde{\tilde{A}}_1 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4)$

(iii) Group the pair of approximation coefficients of $\tilde{\tilde{A}}_1$. Then, find the new approximation coefficients and the detailed coefficients for the pair of approximation coefficient of $\tilde{\tilde{A}}_1$ $[\alpha_1, \alpha_2], [\alpha_2, \alpha_3]$

$$\gamma_1 = \left(\frac{\alpha_1 + \alpha_2}{2}\right), \gamma_2 = \left(\frac{\alpha_2 + \alpha_3}{2}\right);\ \eta_1 = \left(\frac{\alpha_1 - \alpha_2}{2}\right), \eta_2 = \left(\frac{\alpha_2 - \alpha_3}{2}\right)$$

The $\tilde{\tilde{A}}_1$ changed into $\tilde{\tilde{A}}_2 = (\gamma_1, \gamma_2, \eta_1, \eta_2, \beta_1, \beta_2, \beta_3, \beta_4)$

(iv) Find the pair of approximation coefficient in $\tilde{\tilde{A}}_2$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{\tilde{A}}_2$. $[\gamma_1, \gamma_2]$

$$\delta_1 = \left(\frac{\gamma_1 + \gamma_2}{2}\right), \delta_2 = \left(\frac{\gamma_1 - \gamma_2}{2}\right)$$

The $\tilde{\tilde{A}}_2$ changed into

$$H(\tilde{\tilde{A}}) = (\delta_1, \delta_2, \eta_1, \eta_2, \beta_1, \beta_2, \beta_3, \beta_4)$$

(v) Determine the sum of $H(\tilde{\tilde{A}})$. (i.e.)

$$D = Sum(H(\tilde{\tilde{A}})) = (\delta_1 + \delta_2 + \eta_1 + \eta_2 + \beta_1 + \beta_2 + \beta_3 + \beta_4)$$

3. Scenario Process:

i. Transform the total uncertain direct relation matrix $\tilde{U}_k = [\tilde{u}_{kij}]_{n\times n}$ into summed Haar Hexagonal Fuzzy Number matrix $D = [d_{ij}]_{n\times n}$.

$$D = Sum(H(\tilde{\tilde{A}})) = (\delta_1 + \delta_2 + \eta_1 + \eta_2 + \beta_1 + \beta_2 + \beta_3 + \beta_4)$$

ii. Design the total relation matrix $\tilde{T} = [\tilde{t}_{ij}]_{n\times n}$ using

$$[t^1_{ij}]_{n\times n} = D(I - D)^{-1},\ i, j = 1, 2, \ldots, n$$

iii. Take the different scenarios $S_i = \{s_1, s_2, s_3, \ldots, s_n\}$ which are taken as the input vector and passed through the dynamical system $\tilde{T}$ for identifying hidden pattern of the system. And adapted the Sigmoid function as the threshold function and is given by $f(x) = 1/(1 + e^{-\lambda x})$, $\lambda = 5$.

iv. The hidden pattern of the all scenario is formed as a matrix by $\tilde{H} = [h_{ij}]_{m\times n}$. By taking scenario output as rows , formulate matrix with n columns and m rows.

v. Normalize the matrix $N(\tilde{H}) = [h_{ij}]_{m\times n}$

4. VIKOR Method:

VIKOR method adopted to rank the scenario Haar FCM,

(i) Calculate the initial weight $\tilde{w}_i$ of attribute G_i.

(ii) Determine the Fuzzy Best Value $\tilde{f}^+_j$ (FBV) and Fuzzy

Worst Value $\tilde{f}_j^-$ (FWV) of all scenarios for a particular attribute $N(\tilde{H})$.

$$\tilde{f}_j^+ = \max_j \tilde{h}_{ij},$$

$$\tilde{f}_j^- = \min_j \tilde{h}_{ij}$$

(iii) Compute the index $\tilde{S}_i$ and $\tilde{R}_i$, the maximum majority rule and minimum regret rule of the opponent for the separation $\tilde{S}_i$ of G_i from $\tilde{f}_j^+$. likewise, the separation of $\tilde{R}_i$ of G_j from $\tilde{f}_j^-$ is also determined by

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{w}_j \otimes (\tilde{f}_j^+ - \tilde{h}_{ij}) / (\tilde{f}_j^+ - \tilde{f}_j^-),$$

$$\tilde{R}_i = \max_j [\tilde{w}_j \otimes (\tilde{f}_j^+ - \tilde{h}_{ij}) / (\tilde{f}_j^+ - \tilde{f}_j^-)]$$

(iv) Compute the index $\tilde{Q}_i$

$$\tilde{Q}_i = \frac{v(\tilde{S}_i - \tilde{S}_i^+)}{(\tilde{S}_i^- - \tilde{S}_i^+)} + \frac{(1-v)(\tilde{R}_i - \tilde{R}_i^+)}{(\tilde{R}_i^- - \tilde{R}_i^+)},$$ Where

$$\tilde{S}_i^+ = \min_i \tilde{S}_i, \tilde{S}_i^- = \max_i \tilde{S}_i$$

$$\tilde{R}_i^+ = \min_i \tilde{R}_i, \tilde{R}_i^- = \max_i \tilde{R}_i$$

The index $\min_i \tilde{S}_i$ is with the maximum majority rule and $\min_i \tilde{R}_i$ is with the minimum individual regret of opponent. v is taken as weight of the strategy of the maximum strategy of the utility. (1-v) represents the weight of individual majority rule (usually $v = 0.5$).

(v) Ranking of the attributes $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$ can be determined with respect to their values.

(vi) Propose a compromise solution,

If both of the following conditions are held good at the same time, then the system with least value of Q_i in ranking will be taken as the optimal compromise solution.

Condition 1:

The alternative $Q(G_1)$ has an acceptable advantage $Q(G_2) - Q(G_1) \geq \frac{1}{m-1}$, Where, G_1 ranked first, G_2 is in the second position in Q_i as alternative, m is the number of alternatives and $1/(m-1)$ is the threshold.

Condition 2:

The alternative $Q(G_1)$ is well built in the assessment process; that is to say, it has best ranking in S_i and R_i also.

(i) If the first condition is not fulfilled, which means $Q(G_2) - Q(G_1) < \frac{1}{m-1}$, then alternatives G_1, G_2, ... , G_m are the identical compromise solution, there is no relative advantage of G_1 from other alternatives. However, for the maximum value, the consequent alternative is taken as the closeness solution.

(ii) If the second condition is not fulfilled, the firmness in decision making is incomplete while G_1 has a relative benefit. So, G_1 and G_2 have the same compromise solution.

- Choose the best alternative as a closeness solution.

IV. ADAPTATION OF THE PROBLEM TO THE PROPOSED METHOD

To illustrate this model, the problem of solid waste management is taken. Solid waste refers to the variety of garbage generating from human and animal activities such as industrial, residential and commercial activities, etc., that are categorized into renewable and non renewable. The most important aim of solid waste management is to diminish the horrible impacts on both human health and environment to sustain economic growth and better quality of life. To identify the most important factor of solid waste management, the following factors are identified. G_1- Local resources consumption and reproducibility, G_2- Compatibility with environmental and geological characteristics, G_3- Atmospheric emissions, G_4- Water pollution and wastewater, G_5- Waste production, G_6- Land use and occupation, G_7- Fuel or non-renewable energy consumption, G_8- Water consumption, G_9- Non-renewable raw materials use, G_{10}- Respect for local culture, G_{11}- Percentage of collection and population served, G_{12}- Separated management of organic, hazardous or recyclable waste, G_{13}- Living conditions of local community would be a useful advance in the practice of solid waste management.

FCM Process:

The initial uncertain direct-relation in Table 2-3 were framed by two experts - educationist, scavenger from the municipal. They were requested to provide their opinions on correlating among the influencing factors from the linguistic set S= {No Influence, Very Low, Medium Low, Low, High, Medium High, Very High}. Matrices in the Table 2-3 are changed into hexagonal fuzzy numbers using Table 1.

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
G_1	VH	H	VL	VH	VH	M	H	VH	H	VH	L	VL	VL
G_2	VL	VL	H	H	M	VH	H	H	VL	L	VL	L	L
G_3	VH	H	VH	VH	VH	H	H	VH	M	VL	L	L	VL
G_4	L	H	VH	H	H	L	VL	VL	L	VL	VL	L	L
G_5	VL	L	L	H	VH	H	H	H	L	L	H	VL	H
G_6	L	VL	L	L	VL	VL	L	VH	L	H	VH	H	VH
G_7	VL	L	VL	L	L	VL	VL	H	H	L	H	VH	H
G_8	L	L	H	VL	L	H	L	VH	H	VL	L	VL	VL
G_9	VL	L	L	VH	VL	L	H	VL	L	VH	L	VL	L
G_{10}	L	VL	L	VL	VL	L	VL	VH	H	VH	L	VL	VL
G_{11}	VL	L	L	VL	VL	L	L	L	VL	VL	VH	M	H
G_{12}	L	VL	M	L	VL	M	VL	L	L	VL	VH	VL	VH
G_{13}	H	VH	H	L	M	VL	VH	H	H	VL	L	M	VH

Table 2: Initial uncertain judgment matrix by E_1

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
G_1	H	VH	L	H	H	VL	VH	H	VH	H	VL	M	L
G_2	L	VL	VH	VH	VL	H	VH	VH	M	L	L	VL	VL
G_3	H	VH	H	H	VH	H	VH	H	VL	L	VL	L	M
G_4	VL	VH	H	VH	H	L	M	M	VL	M	L	VL	L
G_5	M	VL	L	VH	H	VH	H	VH	VL	L	H	M	VH
G_6	VL	L	VL	L	M	L	VL	H	L	VH	H	VH	H
G_7	M	VL	L	VL	L	M	L	VH	H	VL	VH	H	VH
G_8	VL	L	VH	L	VL	H	VL	H	VH	M	L	M	L
G_9	M	VL	L	H	L	VL	VH	M	VL	H	VL	M	L
G_{10}	VL	M	L	M	L	VL	L	H	VH	H	VL	L	M
G_{11}	M	VL	L	M	L	VL	L	VL	M	L	H	M	VH
G_{12}	VL	M	VL	L	L	VL	L	VL	L	M	H	VL	H
G_{13}	VH	H	VH	VL	VL	M	H	VH	H	M	VL	VL	H

Table 3: Initial uncertain judgment matrix by E_2

Next, the direct relation matrix $\hat{U}_k = [\hat{u}_{kij}]_{n\times n}$ is changed into hexagonal fuzzy matrix as given in Table 4. After that, the group uncertain judgment matrix is designed by aggregating the two matrices.

	G_1	G_2	G_3
G_1	(0, 0,0, 0, 0, 0)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_2	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0, 0,0, 0, 0, 0)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_3	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0, 0,0, 0, 0, 0)
G_4	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_5	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_6	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_7	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_8	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_9	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{10}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{11}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{12}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{13}	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)

	G_4	G_5	G_6
G_1	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_2	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_3	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.781, 0.843, 0.903, 1, 1, 1)	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)
G_4	(0, 0,0, 0, 0, 0)	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_5	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0, 0,0, 0, 0, 0)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_6	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0, 0,0, 0, 0, 0)
G_7	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_8	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)
G_9	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_{10}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_{11}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_{12}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{13}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)

	G_7	G_8	G_9
G_1	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_2	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_3	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_4	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_5	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_6	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_7	(0, 0,0, 0, 0, 0)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)
G_8	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0, 0,0, 0, 0, 0)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_9	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0, 0,0, 0, 0, 0)
G_{10}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_{11}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{12}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{13}	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)

	G_{10}	G_{11}
G_1	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_2	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_3	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_4	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_5	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.601, 0.662, 0.723, 0.781, 0.842, 0.901)
G_6	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_7	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_8	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_9	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_{10}	(0, 0,0, 0, 0, 0)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_{11}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0, 0,0, 0, 0, 0)
G_{12}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_{13}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)

	G_{12}	G_{13}
G_1	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_2	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_3	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_4	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_5	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)

G_6	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_7	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_8	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)
G_9	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{10}	(0.152, 0.211, 0.273, 0.333, 0.391, 0.452)	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)
G_{11}	(0.421, 0.481, 0.542, 0.603, 0.661, 0.721)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_{12}	(0, 0,0, 0, 0, 0)	(0.691, 0.753, 0.812, 0.891, 0.923,0.951)
G_{13}	(0.241, 0.303, 0.362, 0.421, 0.481, 0.542)	(0, 0,0, 0, 0, 0)

Table 4: The total uncertain judgment matrix.

Process of Converting Hexagonal Fuzzy Number into Haar Hexagonal Fuzzy Number

Using the Haar ranking algorithm for Hexagonal fuzzy Number, the total uncertain direct relation matrix is changed in to Haar Hexagonal direct relation matrix as given in Table 5.

	G_1	G_2	G_3
G_1	(0, 0,0, 0, 0, 0, 0)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_2	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0, 0,0, 0, 0, 0, 0)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_3	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0, 0,0, 0, 0, 0, 0)
G_4	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_5	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_6	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_7	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_8	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_9	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{10}	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{11}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.04, -0.02, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{12}	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{13}	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)

	G_4	G_5	G_6
G_1	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_2	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_3	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.9275, -0.145, -0.045, 0, -0.03, 0, 0, 0)	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)
G_4	(0, 0,0, 0, 0, 0, 0)	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_5	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0, 0,0, 0, 0, 0, 0)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_6	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0, 0,0, 0, 0, 0, 0)
G_7	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)

G_8	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)
G_9	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_{10}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_{11}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_{12}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{13}	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)

	G_7	G_8	G_9
G_1	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_2	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_3	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_4	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_5	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_6	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_7	(0, 0,0, 0, 0, 0, 0)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)
G_8	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0, 0,0, 0, 0, 0, 0)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_9	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0, 0,0, 0, 0, 0, 0)
G_{10}	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_{11}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{12}	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{13}	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)

	G_{10}	G_{11}
G_1	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_2	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_3	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_4	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_5	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.75, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)
G_6	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_7	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_8	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_9	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_{10}	(0, 0,0, 0, 0, 0, 0)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_{11}	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0, 0,0, 0, 0, 0, 0)
G_{12}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_{13}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
	G_{12}	G_{13}

G_1	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_2	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_3	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_4	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_5	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_6	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_7	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_8	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.3, -0.15, -0.045, -0.0225, -0.03, 0, 0, -0.03)
G_9	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{10}	(0.15, 0.21, 0.27, 0.33,0.39, 0.45)	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)
G_{11}	(0.57, -0.15, -0.045, -0.045, -0.03, 0, 0, -0.03)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_{12}	(0, 0,0, 0, 0, 0, 0)	(0.839, -0.1475, -0.045, -0.0225, -0.03, 0, 0, -0.015)
G_{13}	(0.39, -0.15, -0.045, -0.045, -0.03, -0.03, -0.03, -0.03)	(0, 0,0, 0, 0, 0, 0)

Table 5: The Haar Hexagonal direct relation matrix.

Scenario Process: Next, Haar Hexagonal direct relation matrix $H(\tilde{\tilde{A}})=[\tilde{a}_{ij}]_{n\times n}$ is summed through

$$D = Sum(H(\tilde{\tilde{A}})) = (\delta_1+\delta_2+\eta_1+\eta_2+\beta_1+\beta_2+\beta_3+\beta_4)$$

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
G_1	0	0.5788	0.0225	0.5788	0.5788	0.03	0.5788	0.5788	0.5788	0.5788	0.0225	0.03	0.0225
G_2	0.0225	0	0.5788	0.5788	0.03	0.5788	0.5788	0.5788	0.03	0.03	0.0225	0.0225	0.0225
G_3	0.5788	0.5788	0	0.5788	0.7075	0.45	0.5788	0.5788	0.03	0.0225	0.0225	0.03	0.03
G_4	0.0225	0.5788	0.5788	0	0.45	0.03	0.03	0.03	0.0225	0.03	0.0225	0.0225	0.03
G_5	0.03	0.0225	0.03	0.5788	0	0.5788	0.45	0.5788	0.0225	0.03	0.45	0.03	0.5788
G_6	0.0225	0.0225	0.0225	0.03	0.03	0	0.0225	0.5788	0.03	0.5788	0.5788	0.5788	0.5788
G_7	0.03	0.0225	0.0225	0.0225	0.03	0.03	0	0.5788	0.45	0.0225	0.5788	0.5788	0.5788
G_8	0.03	0.03	0.5788	0.0225	0.0225	0.45	0.0225	0	0.5788	0.03	0.03	0.03	0.0225
G_9	0.032	0.0225	0.03	0.5788	0.0225	0.0225	0.5788	0.03	0	0.5788	0.0225	0.03	0.03
G_{10}	0.0225	0.03	0.03	0.03	0.0225	0.0225	0.0225	0.5788	0.5788	0	0.0225	0.0225	0.03
G_{11}	0.03	0.0225	0.03	0.03	0.0225	0.0225	0.03	0.0225	0.03	0.0225	0	0.27	0.5788
G_{12}	0.0225	0.03	0.03	0.03	0.0225	0.03	0.0225	0.0225	0.03	0.03	0.5788	0	0.5788
G_{13}	0.5788	0.5788	0.5788	0.0225	0.03	0.03	0.5788	0.5788	0.45	0.03	0.0225	0.03	0

Table 6: Summed Haar Hexagonal Fuzzy matrix $D=[d_{ij}]_{n\times n}$.

We constructed the total relation matrix $\tilde{T}=[\tilde{t}_{ij}]_{n\times n}$ using $[t^1_{ij}]_{n\times n} = D(I-D)^{-1}, i, j = 1, 2,..., n$

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
G_1	-0.457	-0.064	-0.322	0.138	0.112	-0.209	-0.110	-0.189	0.029	0.173	-0.293	-0.266	-0.436
G_2	-0.161	-0.330	0.039	-0.120	-0.137	0.212	-0.246	-0.144	-0.574	-0.314	-0.157	-0.094	-0.265
G_3	-0.004	-0.072	-0.543	-0.129	0.199	0.083	-0.346	-0.294	-0.722	-0.402	-0.246	-0.254	-0.359
G_4	-0.131	0.203	0.125	-0.264	0.285	0.137	-0.421	-0.347	-0.855	-0.492	-0.205	-0.246	-0.282
G_5	-0.089	-0.250	-0.282	-0.202	-0.370	-0.017	-0.190	-0.188	-0.332	-0.261	0.059	-0.123	0.164
G_6	-0.035	-0.252	-0.241	-0.374	-0.333	-0.443	-0.247	-0.005	0.152	0.365	0.142	0.203	0.176
G_7	-0.016	-0.202	-0.266	-0.313	-0.320	-0.477	-0.167	-0.201	0.240	-0.147	0.179	0.230	0.234
G_8	-0.101	-0.215	0.046	-0.112	-0.079	0.122	-0.230	-0.351	0.149	0.088	-0.141	-0.093	-0.230
G_9	-0.155	-0.114	-0.138	0.243	-0.072	-0.251	0.161	-0.235	-0.085	0.280	-0.112	-0.069	-0.171
G_{10}	-0.143	-0.185	-0.069	0.058	-0.086	-0.085	-0.066	0.184	0.544	0.166	-0.153	-0.099	-0.235
G_{11}	0.054	-0.019	-0.119	-0.180	-0.129	-0.233	-0.042	-0.241	-0.117	-0.167	-0.211	0.036	0.221
G_{12}	0.062	-0.020	-0.146	-0.225	-0.162	-0.282	-0.059	-0.297	-0.147	-0.201	0.193	-0.168	0.274
G_{13}	0.073	-0.001	-0.110	-0.211	-0.158	-0.265	-0.012	-0.232	-0.112	-0.205	-0.410	-0.287	-0.678

Table 7: The total relation matrix

Scenario FCM Process:

Scenario portrays the events and situations that would take place in the future. Decision makers use scenario method to predict the future events. Decision makers have a set of alternatives and outcomes associated with the possibility of occurrences using the results of scenario-based problem solving. Hence, the different solid waste management with regard to the situation and circumstances are taken as the input vectors. The different input vectors will pass through the system $\tilde{T}$ and with their resultant, scenario FCM matrix (H) is formed as shown in Table 8.

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
S_1	1.053	3.004	1.373	1.533	8.245	1.699	1.070	1.068	1.016	1.203	1.083	1.077	1.028
S_2	1.023	1.014	1.092	1.096	1.112	1.204	1.016	1.032	1.296	5.757	1.106	1.287	1.023
S_3	1.036	1.039	1.004	1.208	1.020	1.003	1.175	1.005	1.266	1.544	1.152	1.383	2.066
S_4	1.253	1.007	1.015	1.006	1.002	1.010	1.025	1.012	1.149	1.008	1.237	1.067	1.080
S_5	1.082	1.156	1.008	1.193	2.414	1.240	1.024	1.003	1.032	1.181	1.088	1.020	1.023
S_6	1.387	1.026	1.001	1.017	1.025	1.021	1.016	1.019	1.125	1.015	2.177	1.126	2.483
S_7	1.022	1.003	1.001	1.008	1.040	1.013	1.001	1.001	1.033	1.192	1.005	1.005	1.000

Table 8: Scenario FCM matrix (H)

Then, dynamical system $N(\tilde{H}) = [h_{ij}]_{m\times n}$ is constructed by dividing all the entries of scenario FCM matrix (H) by the sum of its corresponding column.

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
S_1	0.134	0.325	0.183	0.190	0.520	0.207	0.146	0.150	0.128	0.093	0.122	0.135	0.106
S_2	0.130	0.110	0.146	0.136	0.070	0.147	0.139	0.144	0.164	0.446	0.125	0.162	0.105
S_3	0.132	0.112	0.134	0.150	0.064	0.122	0.160	0.141	0.160	0.120	0.130	0.174	0.213
S_4	0.160	0.109	0.135	0.125	0.063	0.123	0.140	0.142	0.145	0.078	0.140	0.134	0.111
S_5	0.138	0.125	0.134	0.148	0.152	0.151	0.140	0.141	0.130	0.092	0.123	0.128	0.105
S_6	0.177	0.111	0.134	0.126	0.065	0.125	0.139	0.143	0.142	0.079	0.246	0.141	0.256
S_7	0.130	0.108	0.134	0.125	0.066	0.124	0.137	0.140	0.131	0.092	0.114	0.126	0.103

Table 9: Dynamical System $N(\tilde{H}) = [h_{ij}]_{m\times n}$

VIKOR Process:

With the aid of experts' opinions, the weight $\tilde{w}_i$ for the entire attribute G_i is assigned.

W	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
weight	0.5	0.6	0.7	0.8	0.4	0.6	0.3	0.6	0.5	0.4	0.6	0.5	0.6

Table 10: The initial weight $\tilde{w}_i$ of attribute G_i

Next, the Fuzzy Best Value $\tilde{f}_j^+$ (FBV) and Fuzzy Worst Value $\tilde{f}_j^-$ (FWV) of all the dynamical system $N(\tilde{H})$ are determined by column wise

f^+	0.177	0.325	0.183	0.190	0.520	0.207	0.160	0.150	0.164	0.446	0.246	0.174	0.256
f^-	0.130	0.108	0.134	0.125	0.063	0.122	0.137	0.140	0.128	0.078	0.114	0.126	0.103

Table 11: The Fuzzy Best Value $\tilde{f}_j^+$ (FBV) and Fuzzy Worst Value $\tilde{f}_j^-$ (FWV)

Then, we computed the index $\min_i \tilde{S}_i$ is with the maximum majority rule for the separation $\tilde{S}_i$ of G_i from $\tilde{f}_j^+$ and $\min_i \tilde{R}_i$ is with the minimum individual regret of opponent for the separation of $\tilde{R}_i$ of G_j from $\tilde{f}_j^-$ respectively. Finally $\tilde{Q}_i$ is derived

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
$\tilde{S}_i$	2.535	3.532	3.985	4.054	2.312	3.191	1.553	3.016	2.062	2.308	3.272	2.271	3.107
$\tilde{R}_i$	0.500	0.600	0.700	0.800	0.400	0.600	0.300	0.600	0.500	0.400	0.600	0.500	0.600

$\tilde{Q}_i$	0.021	0.283	0.436	0.513	-0.086	0.215	-0.300	0.180	-0.073	-0.087	0.231	-0.032	0.198

Table 12: The values of $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$

The ranking of the factors are given below based on the indices $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$

$$\tilde{S}_i = G_7 > G_9 > G_{12} > G_{10} > G_5 > G_1 > G_8 > G_{13} > G_6 > G_{11} > G_2 > G_3 > G_4$$
$$\tilde{R}_i = G_7 > G_{10} > G_{12} > G_9 > G_5 > G_1 > G_{13} > G_{11} > G_8 > G_6 > G_2 > G_3 > G_4$$
$$\tilde{Q}_i = G_7 > G_9 > G_{12} > G_{10} > G_5 > G_1 > G_8 > G_{13} > G_6 > G_{11} > G_2 > G_3 > G_4$$

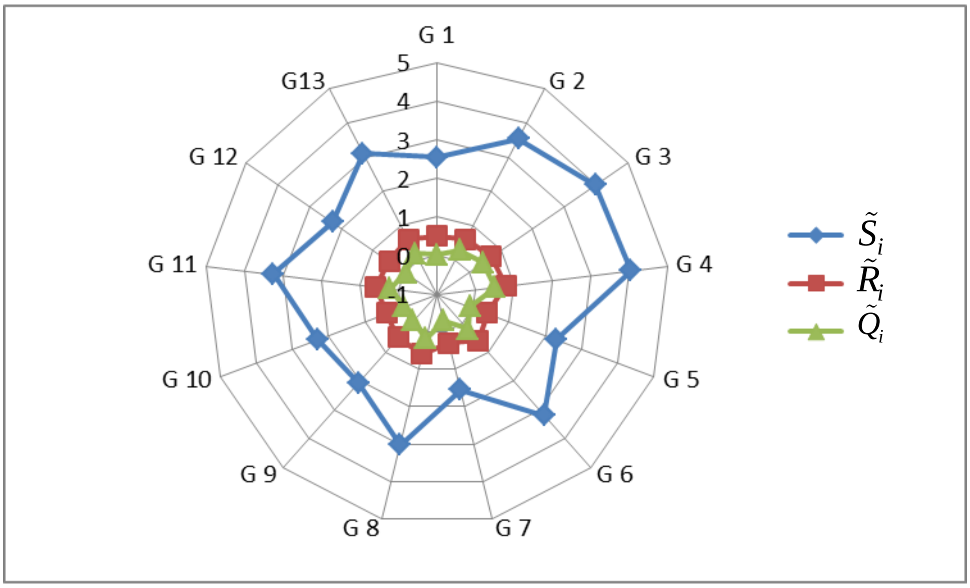

Figure 4: Graphical representation of ranking with respect to $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$

V. CONCLUSION

In this present investigation, a novel hybrid FCM and VIKOR method through Haar ranking of Hexagonal Fuzzy Number is proposed to determine the major influencing factor. This technique is an essential tool for solving any decision making problems with full of uncertainty and vagueness. To illustrate this technique, the solid waste management is studied. By using this technique, the attributes related solid waste management are ranked as follows $G_7 > G_9 > G_{12} > G_{10} > G_5 > G_1 > G_8 > G_{13} > G_6 > G_{11} > G_2 > G_3 > G_4$. It is obtained that G_7- Fuel or non-renewable energy consumption and G_9- Non-renewable raw materials used are the major influencing factors for Solid Waste Management. Henceforth, we should concentrate more on Fuel or non-renewable energy consumption and Non-renewable raw materials to reduce and to eliminate unpleasant impacts of these waste materials on the environment. Furthermore, the proposed technique can be useful to obtain the most feasible solution in all the MCDM problems with full of uncertainty.

REFERENCES

1. Altay, A., Kayakutlu, G., Fuzzy cognitive mapping in factor elimination: A case study for innovative power and risks, Procedia Computer Science 3, 1111-1119, 2011.
2. Chang, T.H., Fuzzy VIKOR method: A case study of the hospital service evaluation in Taiwan, Information Science., 271, 196-212, 2014.
3. Chen, L.Y., Wang, T.V., Optimizing partners choice in IS/IT outsourcing projects: The strategic decision of fuzzy VIKOR, International Journal of Production Economics, 120(1)233-242, 2009.
4. Chen, Z., Subramanian, R., Chung, L., Cooper, K.M.L., Exploring a Timed-Automata Fuzzy Cognitive Maps based approach for modeling Systems of Systems, Procedia Manufacturing, 3,1910-1917,2015.
5. Devadoss, A.V. & Felix, A. A new Fuzzy DEMATEL method in an Uncertain Linguistic Environment, Advances in Fuzzy Sets and Systems, 16(2), (2013), 93-123.
6. Devadoss, A.V., & Felix, A. A Fuzzy DEMATEL approach to Study cause and effect relationship of Youth Violence, International Journal of Computing Algorithm, 2, (2013), 363-372.
7. Devadoss V.A., A.D.Dhivya, Felix, A., A Hendecagonal Fuzzy Number and Its Vertex Method, International Journal of Mathematics And its Applications, 4(1B), 87- 98, 2016.
8. Devi K, Extension of VIKOR method in intuitionistic fuzzy environment for robot selection. Expert systems with Applications.,38(10), 14163-14168.2011.
9. Dhivya, A.D., Felix, A., A Fuzzy Rule based Expert System for T2DM Diagnosis, International Journal of Engineering & Technology, 7 (4.10), 432-435, 2018.
10. Felix, A., S. Christopher, Devadoss V.A., “A Nonagonal Fuzzy Number and its Arithmetic Operations”, International Journal of Mathematics and Its Applications, 3(2), 185-195, 2015.
11. Felix, A., Devadoss V.A., “A new Decagonal Fuzzy Number under uncertain linguistic environment”, International Journal of Mathematics and Its Applications, 3(1), 89-97, 2015.
12. Gutiérrez, J.S., Rincón,G., Alonso, C., Jalón, D.G.D., Using fuzzy cognitive maps for predicting river management responses : A case study of the Esla River basin, Spain, Ecological Modelling, 360(24)260-269, 2017.

13. Kaya, T., Kahraman, C., Multicriteria renewable energy planning using an integrated fuzzy VIKOR&AHP methodology; the case of Istanbul, Energy.,35(6), 2517-2527.
14. Kok, J.L., Titus, M., Application of fuzzy sets and cognitive maps to incorporate social science scenarios in integrated assessment models, Integrated Assessment, 3(1),177-188,2000.
15. Konti, A., Damigos, D., Exploring strengths and weaknesses of bioethanol production from biowaste in Greece using Fuzzy Cognitive Maps, energy policy,112, 4-11, 2018.
16. Kontogianni., Papageorgiou, E., Risks for the Black Sea marine environment as perceived by Ukrainian stakeholders :A fuzzy cognitive mapping application, Ocean & Coastal Management, 62, 34-42, 2012.
17. Kosko, B., Fuzzy cognitive maps. International journal of man-machine studies, 24(1), (1986), 65-75.
18. Liu, H.C., Liu, L., Liu, N., Mao, L.X., Risk evaluation in failure mode and effects analysis with extended VIKOR method under fuzzy environment, Expert systems with Applications 39(17),12926-12934, 2012.
19. Liu, H.C., You, J.X., You, X.Y., Shan, M.M, A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method, Applied soft computing, 28,579-588, 2015.
20. Opricovic S., Fuzzy VIKOR with an application to water resources planning, Expert systems with Applications.,38(10) 12983-12990.2011.
21. Opricovic, S., Multi-criteria optimization of civil engineering systems, Faculty of Civil Engineering, Belgrade, (1998).
22. Opricovic, S., Tzeng, G.-H. Extended VIKOR method in comparison with outranking methods, European Journal of Operational Research 178(2), (2007), 514–529.
23. Papageorgiou, E.I., Hatwágner, M.F, A concept reduction approach for fuzzy cognitive map models in decision making and management, Neuro Computing 232,16-33, 2017.
24. Sanayei, A., Mousavi, S.F., Yazdankhah, A, Group decision making process for supplier selection with VIKOR under fuzzy environment, Expert systems with Applications., 37(1)24-30, 2010.
25. Sankar R.J, Felix, A., An Improved Fuzzy DEMATEL Technique And Its Applications, International Journal of Pharmacy and technology, Volume-8(3), 19122-19134, 2016.
26. Zhang, N., Guiwu, Extension of VIKOR method for decision making problem based on hesitant fuzzy set, Applied Mathematical Modeling., 37(7), 4938-4947, 2013.
27. Zadeh, L. A. (1965). Fuzzy Sets, Information and Control, 8(3), 338-353.

AUTHORS PROFILE

Mr. A. Selvaraj has completed his B.Sc Mathematics and M.Sc Mathematics in University of Madras, Tamilnadu, India. He is currently doing Ph.D. in mathematics division, School of Advanced Sciences, VIT Chennai. His research interests are Fuzzy Decision-Making System, Fuzzy Inference System and Fuzzy Numbers.

Dr. Saroj Kumar Dash is working in VIT Chennai since 2011. Before joining this Institution he was doing his Post-Doctoral work in IIT Madras during 2009 to 2011 by NBHM Post-Doctoral fellowship. He finished his Ph.D. work from the Utkal University, India. His main areas of interests are: Random Fourier series, Control and Game Theory, Fuzzy logic, Web-page ranking algorithm.

Dr. N. Punithavelan joined in VIT Chennai in 2011, as an Assistant Professor. He received his B.Sc in Physics from Madurai Kamarajar University, M.Sc in Materials Science from Bharathiar University, India and Ph.D. in MEMS from GIST, South Korea. His current research interests are theoretical and experimental investigations in Thin Film Technology, Solid State Physics, and Fuzzy Logic.