Cubic Sp-line Collocation Enhancement for Modeling of MOS Transient

Ahmed A. Afifi, Mohamed B. El_Mashade, Tayel E. Dabbous

Abstract: ***A symmetrical telescopic modification of the cubic sp-line collocation method with an application in modeling the transient response of MOS transistors is proposed. The novel methodology represents an extremely accurate spatial piecewise solution of the continuity equation in such a way that it transiently models the MOS device in terms of the normalized channel charge density. Based on this treatment, this paper discusses the full detailed derivation of optimum inversion channel segmentation in order to solve the partial differential form of continuity equation which is convenient to model the transient mode of MOS transistors. Additionally, the boundary as well as the initial conditions of the MOS transient equation are determined by means of the compact charge based of EKV model. The ability of such model to represent a unique formula of the continuity equation for all regions of operation of the MOS transistor is the reason of choosing it in our processing. Moreover, our proposal makes use of the physical reality of the commonness of spatial natural conditions along the inversion channel that distinguishes the transient propagation of the surface channel charges from the source to the drain and vice versa. Due to the symmetry of such physical meaning, the symmetrical telescopic framing of the cubic sp-line collocation method is declared. The suggested approach proves an accurate modeling up to 97% of the channel length. This approach is tested with transient ramp slope of*** 10^{10} ***V/sec. It has been applied to the standard CMOS 0.18µm technology for NMOS transistors. The PMOS transistors are modeled in the standard CMOS 0.15µm technology. The novel model is validated using MATLAB R2014a. The obtained results demonstrate that the introduced procedure has an astonishing impact on different electronic components that expert transient events such as low dropout regulators and switched capacitor circuits.***

Index Terms: ***Continuity equation, EKV MOS model, Sp-line collocation method, Symmetrical telescopic modification.***

I. INTRODUCTION

On the scope of the transient MOS model, the continuity equation represents the mathematical structure that describes the transient events of the charge carrier transportations. This equation is a one dimensional partial differential equation in terms of time as well as the position along the channel length. It is characterized by the separation of the partial temporal and spatial derivatives in such a way that its temporal side is assigned to the spatial one. The resultant physical parameter of solving such equation is the MOS device inversion channel charge density. Additionally, the transient terminal currents of the MOS transistor are calculated by the spatial integration of the temporal side of that equation [1]. As a result of this, the continuity equation formulation along with its boundary as well as initial conditions represents the first step in computing the MOS transient terminal currents. In order to achieve such objective, the large signal static MOS model along with its other physical phenomena must be mathematically described through a charge based compact model as EKV [6]. From the solution point of view, it is of importance to note that the continuity equation realization is usually proposed through explicitly or implicitly summation of multiple functions. Each one of the terms of this summation consists of a well known spatial function and an unknown temporal one. These temporal functions can be obtained through the solution of a temporal set, single type or hybrid, of ordinary differential equations or algebraic, explicit or implicit, ones. The basic principle of such set is that it is formulated in such a way that the substitution of the proposed solution in the temporal side closes the spatial one [2]-[5]. In this regard, the Gelrkine's approach represents one of the fundamental techniques of solution. However, the main drawback in this approach is the choice of the set of spatial orthogonal functions. Consequently, there is no obvious standard measurement for the trade-off between the model efficiency and its complexity. Amongst the simplest ones of these interesting approaches is that based on partitioning the channel length into very short equal segments what is called channel segmentation [7]. This concept can be validated via the calculation of the channel charge density using static condition formulas for each segment individually provided that the continuity between segments is satisfied. The accuracy of this algorithm can be improved by increasing the number of segments. However, this will result in increasing the complexity of the model. Another less complex as well as more efficient piecewise approach is to uniformly divide the channel length into limited and well informed number of segments. The strategy of this technique is to model explicitly each segment solution as a sum of products of distinct functions of time and position. The parameters of this model are dependent upon the normalized channel charge density at the collocation points and channel bounds. In this situation, the modeled solution intersects with the real one at the collocation points. This in turn avails an efficient mathematical interpolation between these points. Consequently, it is necessary for such piecewise model to keep spatial continuity up to the second derivative. Additionally, the continuity equation based physical condition must be taken into account. These constraints are mathematically represented through the nullifying of the spatial second derivative at the channel bounds.

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As a consequence of this, the model parameters need a set of algebraic equations of four times the number of segments.

Next, a set of ordinary differential equations of the same number of collocation points must be solved. The solution of these equations designates the model intersection with exact solution. Actually, the segment length is chosen in such a way that it allows the sensation of the transient event. Accordingly, this length is empirically defined as the distance traveled by an uniformly moving electron along the channel due to the influence of the drain-source voltage during tenth of the transient event time.

Referring to the solution of the continuity equation at the collocation points, it is evident that the normalized channel charge density is gradually decreasing towards the drain. Thus, the polynomial spatial interpolation for each segment is a convenient representation of the solution of that equation. Based on this notation, the cubic polynomial is the most suitable one due to the equality of its number of coefficients with the number of model parameters specified for each segment. Such interpolation is called cubic sp-line collocation technique [8]. Unfortunately, the performance of this method is poor, as it is applied to a physical based MOS model such as EKV. This is because it ignores the realization of natural conditions for any of its segments across the channel length. This ignorance means that the spatial first partial derivative of the normalized channel charge density is graded along the channel and consequently, the accuracy of the model will be degraded. This degradation can't be alleviated even the degree of continuity of the segments is raised. In addition, the estimation of the number of partitions cannot be achieved for small values of drain-source voltage. This is due to the mismatching of such estimation with the high segmentation sensitivity in that case. This problem arises because the stated number of segments excludes the modulation effect of the channel conductivity which is excited by the vertical electric field.

In spite of the drawbacks of the cubic sp-line collocation technique, it enjoys some merits that make it favorable to be developed. Among the remedy methods, there exists an explicit estimation of the interpolative formula for the solution of the continuity equation. Another one is the capability of mathematical representation of the transistor terminal currents as a polynomial function of the normalized charge densities at the collocation points along with the channel bounds. As a consequence of this, it is reasonable to motivate such approach in order to satisfy the need of an accurate model for MOS transient. In accordance with this, the paper is organized as follows. The symmetrical telescopic modification of the cubic sp-line collocation method is discussed in section II. Section III is concerned with the optimization of the proposed model segmentation. The simulation results are displayed in section IV. Section V is devoted to our concluded remarks.

II. SYMMETRICAL TELESCOPIC MODIFICATION

The verification of the natural conditions for each segment while keeping the effect of the model spatial continuity up to the second derivative represents the main challenges in face of developing the cubic sp-line collocation method. From this point of view, it is noticeable that the estimation of the normalized charge density for some segments has the superiority of accuracy in accordance with their closure to the source. This is due to their being near or away from the realization of both the natural conditions and the spatial continuity criteria at the first segment. The same thing is true for the last one; the model parameters of which have a little reciprocal proportionality in comparison with the first segment. In this situation, the usual cubic sp-line collocation doesn't efficiently make use of the advantage of solving the continuity equation at the collocation points. As s result of this, the misestimating of the channel normalized charge density, $r_s(\xi, t)$, will cause incorrect calculation of the transient terminal currents. The telescopic cubic sp-line collocation algorithm represents a weighted approach to deal with the underlined problem. This procedure applies cubic sp-line collocation on a sub-channel with a length starting from one of the segment ends to one of the channel bounds. Of course, the collocation points nearby the source will have sp-line collocation with respect to drain and vice versa. This processing will be taken into account in order to include more collocation points. So, half of the collocation points will be referred to the source and the other will be belong to the drain and hence the telescopic modelling will be symmetrical. For the number of segments of such model, it is necessary to be even in order to keep model equality at the point positioned on the half of the channel length. The only available odd case is that associated with single segment which is solely based on the boundary conditions of the continuity equation. As a consequence of this, the spatial continuity up to the second order is kept for each sub channel collocation points. However, this continuity is not realized in view of the overall channel. Accordingly, the natural conditions will be verified for each of the channel collocation points. For this reason, we are going to mathematically handle this problem.

The n^{th} segment symmetrical telescopic cubic sp-line collocation model can be formulated as:

$$\begin{aligned} r_{sc}(\xi,t) = {} & r_s\left(\frac{n-\delta}{N},t\right) + N_n\left(\left[\chi\left\{r_s\left(\frac{n}{N},t\right) - \right.\right.\right. \\ & \left.\left.r_s\left(\frac{n-1}{N},t\right)\right\} + a_{0_n}\right]\Theta_n + \left(a_{1_n} - N_n a_{0_n}\right)\Theta_n^{\,2} \\ & \left. - N_n a_{1_n}\Theta_n^{\,3}\right) \qquad \mathit{for} \quad \frac{n-1}{N} \le \xi \le \frac{n}{N} \\ \& \quad & \Theta_n = \chi\frac{N}{N_n}\left(\xi - \frac{n-\delta}{N}\right) \end{aligned} \tag{1}$$

where

$$\delta = \begin{cases} 1 & \mathit{for} \quad \xi \le \dfrac{1}{2} \quad \underrightarrow{\mathit{which\ means}} \quad n \le \dfrac{N}{2} \\ 0 & \mathit{otherwise} \end{cases} \tag{2}$$

$$\chi = \begin{cases} 1 & \mathit{for} \quad \xi \le \dfrac{1}{2} \quad \underrightarrow{\mathit{which\ means}} \quad n \le \dfrac{N}{2} \\ -1 & \mathit{otherwise} \end{cases} \tag{3}$$

and

$$N_n = \delta(N+1) - \chi n \qquad (4)$$

In the above formulas, $r_{sc}(\xi,\ t)$, denotes the approximate normalized inversion of surface charge density as a function of the normalized, with respect to channel length (ξ), channel position and the time (t), N represents the total number of segments and N_n refers to the total number of segments for the n^{th} segment sub channel.

Additionally, the constraint of continuity for the m^{th} collocation point, relative to the sub-channel and associated with the n^{th} segment, can be formulated by:

$$-(2m-1)a_{2(m-1)_n} - \frac{2m^2}{N_n} a_{2m-1_n} + a_{2m_n} = -(m-1)$$
$$(2r_w - r_{w-1} - r_{w+1})\ \mathit{for}\ \ r_w \triangleq r_s\left(\frac{w}{N}, t\right) \quad \&$$
$$w = n + \chi m - \delta \quad \forall \quad m \qquad (5)$$

$$2N_n a_{2(m-1)_n} + (2m+1)a_{2m-1_n} + a_{2m+1_n} =$$
$$N_n(2r_w - r_{w-1} - r_{w+1}) \qquad \forall \quad m \quad (6)$$

Furthermore, with the help of the following relations, the natural conditions at the sub-channel bounds can be evaluated. Thus,

$$a_{1_n} - N_n a_{0_n} = 0 \qquad (7-a)$$

$$(N_n + 1)\ a_{(2N_n-1)_n} + N_n a_{2(N_n-1)_n} = 0 \qquad (7-b)$$

By solving the set of algebraic equations, Eqs.(5-7), the n^{th} segment sub-channel model parameters a_{0_n} to $a_{(2N_n-1)_n}$ can be obtained. Consequently, the coefficients a_{0_n} and a_{1_n} can be determined. These coefficients are represented as a weighted sum of the normalized channel charge density at the channel bounds and the collocation points, r_w, with constant weights denoted by c_{0n_w} and c_{1n_w}. Thus,

$$a_{0_n} = \begin{cases} \sum_{w=n-1}^{N} c_{0n_w} r_w & \mathit{for} \quad n \le \frac{N}{2} \\ \sum_{w=0}^{n} c_{0n_w} r_w & \mathit{otherwise} \end{cases} \qquad (8)$$

$$a_{1_n} = \begin{cases} \sum_{w=n-1}^{N} c_{1n_w} r_w & \mathit{for} \quad n \le \frac{N}{2} \\ \sum_{w=0}^{n} c_{1n_w} r_w & \mathit{otherwise} \end{cases} \qquad (9)$$

Referring to Eqs.(8-9), the weights c_{0n_w} and c_{1n_w} are the values of a_{0_n} and a_{1_n} , respectively, at $r_w=1$ wherever the other densities are null.

The set of values of the normalized channel charge density at the collocation points, r_w, is simply the solution of the continuity equation associated with the EKV model parameters. The resulting continuity equation has a form:

$$\frac{\partial r_s(\xi,t)}{\partial t} = \frac{\mu_0}{L^2} V_T \frac{K_1}{K_2}\left(\frac{\partial^2 r_s(\xi,t)}{\partial \xi^2}\left\{1 + \left[\theta - \frac{1}{K_2}\right] r_s(\xi,t) \right.\right.$$
$$\left.\left. - \frac{\theta}{K_2} r_s^2(\xi,t)\right\} + \left(\frac{\partial r_s(\xi,t)}{\partial \xi}\right)^2 \left[\theta - \frac{1}{K_2} - \frac{2\theta}{K_2} r_s(\xi,t)\right] \right)$$
$$- \frac{1}{N_\rho}\left\{\frac{dN_\rho}{dt}\right\} r_s(\xi,t) \qquad (10)$$

In the above mathematical expression, μ_0 is the low field mobility, L is the channel length, V_T is the thermal electro-dynamic voltage, K_1 & K_2 are terminal voltage dependent parameters that model the mobility reduction effect, θ denotes a dimensionless factor which models the quantum non-idealistic effects resulted from band gap widening, and N_ρ is the charge slope factor. Substitution of the modified collocation formula for the collocation points into the continuity equation results in a set of N - 1 ordinary differential equations of the form given by:

$$\frac{dr_w}{dt} = \frac{\mu_0}{L^2} V_T \frac{K_1}{K_2}\left\{\left[N_{w+1-\delta}\, a_{0_{w+1-\delta}} + 2a_{1_{w+1-\delta}}\right]\left[1 + \left(\theta - \frac{1}{K_2}\right.\right.\right.$$
$$\left.\left.\right) r_w - \frac{\theta}{K_2} r_w^2\right] + \left\{N_{w+1-\delta}(r_w - r_{w-\chi} - a_{0_{w+1-\delta}}) - \right.$$
$$\left.\left. a_{1_{w+1-\delta}}\right\}^2 \left(\theta - \frac{1}{K_2} - \frac{2\theta}{K_2} r_w\right)\right\} - \frac{1}{N_\rho} r_w \frac{dN_\rho}{dt} \qquad (11)$$

On the other hand, the transient terminal currents of MOS transistor in terms of EKV model are formulated as [1], [5]:

$$I_S(t) = I_{avr}(t) + \theta C_{ox} V_T W L \frac{d}{dt}\left(N_\rho \int_0^1 (1-\xi) r_s(\xi,t) \right.$$
$$\left. d\xi \right) \qquad (12)$$

$$I_{avr}(t) = -\theta \mu_0 \frac{K_1}{K_2} V_T^2 N_\rho C_{ox} \frac{W}{L}\left\{ r_s(1,t) - r_s(0,t) \right.$$
$$+ \frac{1}{2}\left(\theta - \frac{1}{K_2}\right)\{r_s^2(1,t) - r_s^2(0,t)\} - \frac{\theta}{3K_2}$$
$$\left. [r_s^3(1,t) - r_s^3(0,t)] \right\} \qquad (13)$$

$$I_D(t) = I_{avr}(t) - \theta N_\rho C_{ox} V_T W L \frac{d}{dt}\int_0^1 \xi\, r_s(\xi,t) d\xi \qquad (14)$$

$$I_G(t) = W L \left(\frac{\kappa\ K_G\ \theta\ N_\rho\ V_T^2}{t_{ox}^2} \int_0^1 r_s(\xi,t)\ v_{ox}\ P_{tun}(v_{ox})\ d\xi + \right.$$
$$C_{ox}\frac{d}{dt}\left[\kappa\ \theta\ V_T \int_0^1 r_s(\xi,t)\ d\xi - \kappa\left|\psi_p\right| - \{V_G(t) - V_B(t)\} \right.$$
$$\left.\left. \right] \right) \quad \& \quad \kappa \triangleq \begin{cases} -1 & \mathit{for}\ \ NMOS \\ 1 & \mathit{for}\ \ PMOS \end{cases} \qquad (15)$$

$$v_{ox} \triangleq 2N_\rho\left(r_s(\xi,t) + r_{fc}\right) + \gamma_b \sqrt{\frac{\left|\psi_P\right|}{V_T} - \theta\, r_s(\xi,t)} \qquad (16)$$

$$P_{tun}(v_{ox}) \triangleq \begin{cases} \exp\left\{ -\frac{E_B\, t_{ox}}{v_{ox} V_T} \left[1 - \left(1 - \frac{v_{ox} V_T}{X_B} \right)^{3/2} \right] \right\} & for \quad v_{ox} \le \frac{X_B}{V_T} \\ \exp\left(-\frac{E_B\, t_{ox}}{v_{ox} V_T} \right) & otherwise \end{cases} \quad (17)$$

In this set of formulas, I_s is the dynamic source current, I_{avr} is the spatial average current, C_{ox} is the oxide capacitance per unit channel area, W is the channel width, I_D is the dynamic drain current, I_G is the dynamic gate current, κ is an indicator of the type of MOS transistor, K_G is the gate tunneling current mobility multiplied by C_{ox}, t_{ox} is the oxide thickness, v_{ox} is the normalized oxide voltage, P_{tun} stands for the tunneling probability, ψ_p is the pinch-off surface potential, V_G is the gate voltage, V_B is the bulk voltage, r_{fc} is the fixed normalized charge density, γ_b is the normalized body effect coefficient, E_B denotes the characteristic electric field, and X_B is the oxide-channel voltage barrier.

Finally, the application of the charge conservation principle makes the dynamic bulk current, I_B, to become:

$$I_B(t) = -\left[I_S(t) + I_D(t) + I_G(t) \right] \quad (18)$$

III. OPTIMUM ORDER INVESTIGATION

The convergence of our interpolative model to the exact solution among the collocation points indicates the closure of the modulus values of the differences between them without restrictive tracking. To satisfy this, the square-root of the difference (R), between both sides of the continuity equation represented as a difference equation and characterized by $r_{sc}(\xi, t)$ is formulated as:

$$\sqrt{R} = -\frac{\mu_0}{L^2} V_T \frac{K_1}{K_2} \Bigg(\{ r_{sc}(\xi + 2\Delta\xi, t) - 2\, r_{sc}(\xi + \Delta\xi, t) + r_{sc}(\xi, t) \} \left[1 + \left\{ \theta - \frac{1}{K_2} \right\} r_{sc}(\xi, t) - \frac{\theta}{K_2} r_{sc}^{\,2}(\xi, t) \right] + \left\{ \theta - \frac{1}{K_2} - \frac{2\theta}{K_2} r_{sc}(\xi, t) \right\} \left[r_{sc}(\xi + \Delta\xi, t) - r_{sc}(\xi, t) \right]^2 \Bigg) + r_{sc}(\xi + \Delta\xi, t) - r_{sc}(\xi, t) \quad for \quad \Delta t \ \underline{\Delta}\ 5 \times 10^{-12}\ \text{sec} \quad \& \quad \Delta\xi \ \underline{\Delta}\ 10^{-3} \quad (19)$$

In addition, at very large values of $r_s(\xi, t)$, the linear component dominates the logarithmic component in such a way that:

$$r_s \rightarrow v/\theta \quad as \quad v \rightarrow \infty \quad (20)$$

The application of the suggested collocation method is simulated with a number of partitions equal to 40, which is obtained after several trials, that makes very accurate estimation up to 95% of the channel length. The calculations are achieved using MATLAB R2014a for NMOS transistor, of $0.18\mu m$ CMOS technology [9]-[11]. The channel length is assumed to be $2\mu m$ as an intermediate length relative to the previously stated technology. The drain-source voltage is taken as 1V and the gate is assumed to be excited with an upward ramps of slope 10^{10} V/sec ranging from 0-1V. The variation, with time, of the slope factor is nullified as the source and bulk voltages are of the same value. The difference, R, is computed for the underlined transistor at the relative channel position of 6/25 and the results are shown in Fig.(1). This value of ξ is chosen to illustrate the values of R at the start of the channel. In other words, the stated transition position is a successful case for approximating the continuity equation by the collocation method. In order to confirm the concept of difference equation representation of the continuity equation, the time and relative position steps, Δt & $\Delta\xi$, are assigned to values of about thousandth of the full simulation time and relative position. In this situation, the null value indicates complete correlation between the continuity equation and the proposed model. This will be shown in our obtained numerical results. On the other hand, the occurrence of the bell shape refers to mismatch of them. As Fig.(1) demonstrates, the difference (R) has its maximum during the transition time which is the most critical character for estimation of how the model represents the continuity equation. However, the displayed results indicate that the model closes to the solution of the continuity equation very well. This is an evident that the good interpolation doesn't necessarily require a full simulation of the continuity equation.

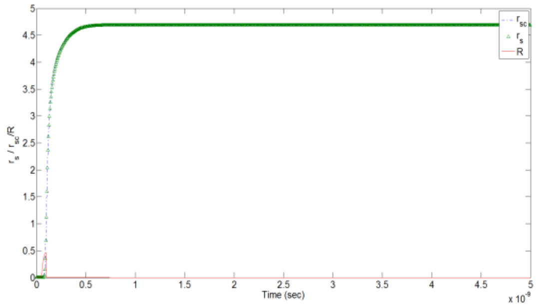

Fig.(1) $r_s(6/25,t)$, $r_{sc}(6/25,t)$& $R(6/25,t)$*temporal variations* of (NMOS 0.18μm technology) when $L=2\mu m$, $\Delta t=5\times10^{-12}$*sec*, $\Delta\xi=10^3$ and an upward ramp input of 10^{10}*V/sec slope*.

As a result of this behaviour, a simple but careful treatment of the continuity equation is required. Nearly, all the transient events of the transistor terminals are included within the boundary conditions other than the continuity equation itself. The reason behind this is that the dependence of both the mobility reduction coefficients and the threshold voltage on the MOS terminal voltages is modest in comparison with the normalized charge density at the channel bounds. Consequently, the terminal voltages and the transition ramp slope have no strong effect on the choice of the number of partitions. The initial conditions, on the other hand, are controlled by the initial biasing voltages. These voltages are involved, within the initial state of the interpolated data, by the proposed collocation method. The relationship between the relative channel position and the initial normalized charge density is cube root which varies slowly with such values of relative channel positions. The most important parameter amongst the coefficients of the continuity equation is the proportionality factor of the spatial and the temporal sides of the continuity equation which will be denoted by *f*. Such factor is defined as:

$$f = \frac{\mu_0}{L^2} V_T \frac{K_1}{K_2} \tag{21}$$

Regarding the continuity equation, it is noted that as *f* tends to zero, the solution of the partial differential equation will be closer to the initial condition. As a result, the collocation model that depends on time won't be able to track the real solution due to the inclusion of the transient effect represented by boundary conditions. In addition, the number of partitions will tend to infinity. So, it is obvious that the number of partitions is inversely proportional to *f*. From the mathematical point of view, the number of partitions tends to be of large value as *f* decreases that can't be applicable for nowadays technologies. For such technologies, the low field mobility increases due to the decreasing of effective masses of charge carriers. Accordingly, it sounds good to assign a unique value for the number of partitions that is realized when *f*→∞. The proper number of partitions is that suitable for tracking the static normalized channel charge density formula. However, the channel bound values included in such formula are of dynamic state which is denoted by quasi static modelling. Regarding to Eq.(21), the channel length acts as a modulation or scaling factor for the parameter *f* in such a way that decreasing the channel length will significantly increase the value of *f*. As a result, the convenient number of segments will be decreased much more for short channel lengths than that of long ones and vice versa. Under this operation condition, the quasi static normalized inversion charge density, $r_{s_{qs}}(\xi,t)$, can be obtained by solving the following cubic algebraic equation [5-6]

$$\xi\left\{r_s(1,t) + \left[\theta - \frac{1}{K_2}\right]\frac{r_s^{\,2}(1,t)}{2} - \frac{\theta r_s^{\,3}(1,t)}{3K_2}\right\} + (1-\xi)$$
$$\left\{r_s(0,t) + \left[\theta - \frac{1}{K_2}\right]\frac{r_s^{\,2}(0,t)}{2} - \frac{\theta r_s^{\,3}(0,t)}{3K_2}\right\} = r_{s_{qs}}(\xi,t) +$$
$$\left[\theta - \frac{1}{K_2}\right]\frac{r_{s_{qs}}^{\,2}(\xi,t)}{2} - \frac{\theta r_{s_{qs}}^{\,3}(\xi,t)}{3K_2} \tag{22}$$

$r_s(0, t)$ & $r_s(1, t)$ are the boundary conditions of the continuity equation at the channel's source and drain, respectively. These conditions are mathematically described by means of EKV charge-voltage formula which is [5]:

$$\frac{1}{V_T}\left\{\frac{|V_G(t) - V_B(t)| - |V_{th}|}{N_V} - \left[\,|V_S(t) - V_B(t)|\,\right]\right\} = \ln r_s(0,t) + \theta r_s(0,t) \tag{23-a}$$

$$\frac{1}{V_T}\left\{\frac{|V_G(t) - V_B(t)| - |V_{th}|}{N_V} - \left[\,|V_D(t) - V_B(t)|\,\right]\right\} = \ln r_s(1,t) + \theta r_s(1,t) \tag{23-b}$$

Referring to the above formulas, V_{th} is the threshold voltage, N_V is the voltage slope factor, V_S is the source voltage, and V_D is the drain voltage. Accordingly, the optimum number of partitions, N_{opt}, is that realizes the least square of the difference between the quasi static normalized charge density and its approximation by the proposed collocation method, $r_{sc_{qs}}(\xi,t)$. Thus,

$$G(N_{opt}) = \min\left[G(N)\right] \;\&\; G(N) = \int_0^1 \left\{r_{s_{qs}}(\xi,t) - r_{sc_{qs}}(\xi,t)\right\}^2 d\xi$$
$$where \quad r_{sc_{qs}}(\xi,t) = r_{sc}(\xi,t)\Big|_{r_s\left(\frac{n}{N},t\right) = r_{s_{qs}}\left(\frac{n}{N},t\right)} \tag{24}$$

Briefly, our procedure can be summarized in few steps as:

Step 1: Determining of the quasi static normalized charge density inversion in terms of the boundary conditions of continuity equation by symbolically solving Eq.(22).

Step 2: Completing the previous step in order to express the normalized channel charge density in terms of the transistor terminal voltages by calculating the inverse function of Eq.(23).

Step 3: Formulating the symmetrical telescopic cubic spline collocation modelling for the quasi static normalized charge density inversion that is depicted from the last two steps via the substitution of the symbolic solution of Eq.(22) at the collocation points into Eqs.(1 - 9).

Step 4: Optimizing the number of segments via calculating the output of Eq.(24).

Step 5: Establishing a set of non-linear ordinary temporal differential equations in terms of the optimized number of segments by means of Eq.(11).

Step 6: Solving the obtained set of ordinary differential equations to compute the normalized channel charge density at the collocation points.

Step 7: Solving the continuity equation via compensating of the normalized charge density inversion in the collocation points at Eqs.(1 - 9).

Step 8: Calculating of the transient terminal currents by substituting the modelled normalized channel charge density which is depicted from the last step into Eqs.(12 - 18).

IV. SIMULATION SETTINGS AND NUMERICAL RESULTS

In this section, we are dealing with developing some simulation results that demonstrate the superiority of our proposed technique over the previously published ones. This object is achieved by displaying some numerical plots that illustrate the closes of the results of our model to the exact solution of the continuity equation. Using MATLAB R2014a, our proposed model is applied to NMOS of CMOS *0.18μm* and *0.15μm* PMOS [9]-[11]. The relative position sweep of the whole of the curves of our results is taken at $\xi=\{55/60, 56/60, 58/60, 59/60\}$ in order to realize the validation of the suggested investigation at the most critical points of relative channel length near the drain. The channel lengths are taken as 2μm to represent the intermediate channel lengths. Moreover, the short channel lengths don't require interpolation and the quasi static approximation by means of the boundary conditions is sufficient [7]. On the other hand, the peak to peak value of the ramping event, which is denoted by V_{PP}, and the drain source voltage, V_{DS}, are assigned to 1V. The nominal value of the transient ramping slope is 10^{10}V/sec.

For the sake of simplicity, N_{opt} has been experimentally determined. Accordingly, in order to make $f\rightarrow\infty$ along with providing the availability of controlling the terminal voltages and channel length, the low field mobility has been set to a virtual value of $10^{10}/C_{ox}\ cm^2/V.sec$. The approximate, $r_{sc}(\xi, t)$, and the exact, $r_s(\xi, t)$, responses to the previously stated NMOS and PMOS transistors under the impact of an upward ramping gate voltage are shown in Figs.(2-3) and Figs.(5-6), respectively. Figs(2&5) illustrate these responses in case of hypothetical value of f. The same simulation is represented, for actual settings of f, by Figs(3&6). On the other hand, Figs.(4&7) illustrate the approximation using the ordinary cubic collocation method, $r_{soc}(\xi, t)$, as compared to $r_s(\xi, t)$ for the same points and under the same conditions as the previously mentioned plots for NMOS and PMOS, respectively. Referring to these figures, it is obvious that our modified cubic sp-line collocation model is extremely accurate for relative channel length up to 97%. On contrary, the ordinary collocation method is not accurate at all for region near to the drain as Figs(4 & 7) demonstrate. Again, this ensures what we are stressed on in sections (I & II) that excluding the natural conditions at each collocation point will have a catastrophic effect on the modeling solution of the continuity equation. Moreover, the usual collocation approach has a fake estimation that makes all relating calculations highly deviate from their exact values. Furthermore, our investigation of the optimum number of segments has been proven as the same accuracy is realized by using the proposed method for both the hypothetical and actual values of *f*.

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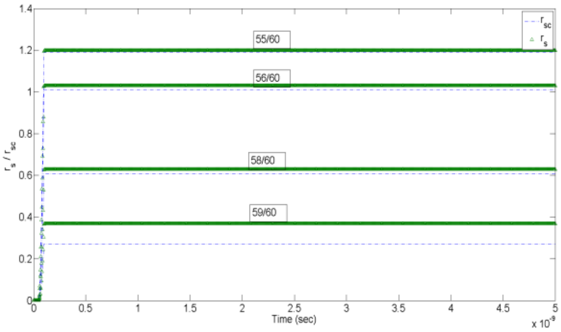

Fig.(2) $r_s(t)$ & $r_{sc}(t)$ temporal variations of NMOS at ξ={55/60,56/60,58/60,59/60} when $f=\infty$, L=2μm, N=40, V_{DS}= V_{PP}=1V and an upward ramp input of 10^{10}V/sec slope.

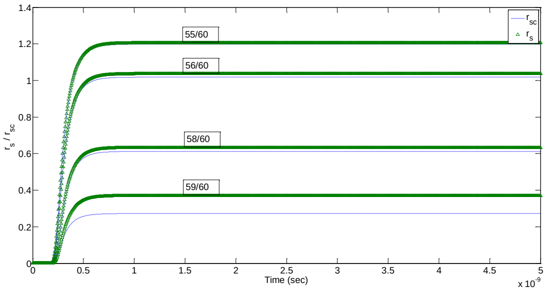

Fig.(3) $r_s(t)$ & $r_{sc}(t)$temporal variations of NMOS at ξ={55/60,56/60,58/60,59/60}when L=2μm, N=40, V_{DS}= V_{PP}=1V and an upward ramp input of 10^{10}V/sec slope.

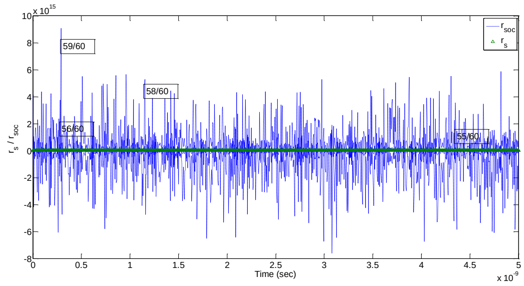

Fig.(4) $r_s(t)$ & $r_{soc}(t)$ temporal variations of NMOS at ξ={55/60,56/60,58/60,59/60} when L=2μm, N=40, V_{DS}= V_{PP}=1V and an upward ramp input of 10^{10}V/sec slope.

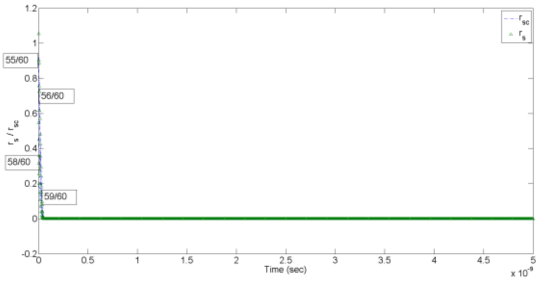

Fig.(5) $r_s(t)$ & $r_{sc}(t)$ temporal variations of PMOS at ξ={55/60,56/60,58/60,59/60} when f=∞, L=2μm, N=40, V_{DS}= V_{PP}=1V and an upward ramp input of 10^{10}V/sec slope.

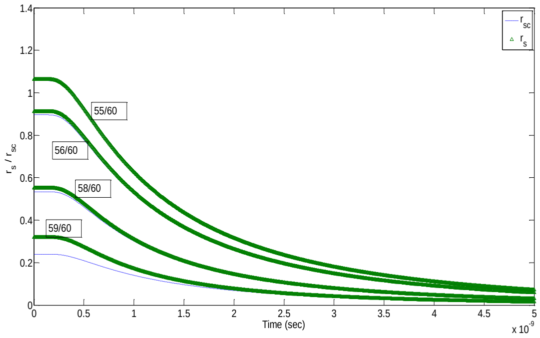

Fig.(6) $r_s(t)$ & $r_{sc}(t)$ temporal variations of PMOS at ξ={55/60,56/60,58/60,59/60} when L=2μm, N=40, V_{DS}= V_{PP}=1V and an upward ramp input of 10^{10}V/sec slope.

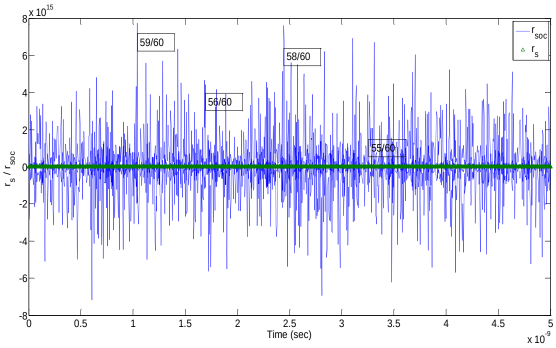

Fig.(7) $r_s(t)$ & $r_{soc}(t)$ temporal variations of PMOS at ξ={55/60,56/60,58/60,59/60} when L=2μm, N=40, V_{DS}= V_{PP}=1V and an upward ramp input of 10^{10}V/sec slope.

V. CONCLUSION

The typicality between the physical propagation of the channel surface charges and its mathematical representation via the symmetrical commonness of the natural conditions along the channel length can't be ignored. To take this circumstance into consideration, the application of sp-line collocation methods to the continuity equation requires realization of natural conditions for any segment along with preserving the interpolation under the influence of spatial continuity up to the second order derivative. Actually, the matching between segmental natural and continuity conditions is achieved through the application of our symmetrical telescopic modification of the cubic sp-line collocation method. Additionally, the channel charge continuity states that the flowing rate of the normalized surface inversion charge is the thermodynamic frequency of the charge carrier particles streaming along the process of modulation by their spatial gradient along the channel length. As a consequence of this, the proportionality constant between the spatial and temporal sides of the continuity equation is the main parameter in defining the proper number of partitions using sp-line collocation methods. Due to high mobility values of scaled down technologies, the future of CMOS technologies will require small number of segments to represent the continuity equation. As a result of this, the proper number of partitions, under some transient impact, is that matches the proposed collocation modeling to the quasi static approximation of the normalized inversion charge density.

REFERENCES

1. S.Y. Oh, D. Ward, and R. Dutton, Transient Analysis of MOS Transistors, IEEE Journal of Solid-State Circuits, SC-15, pp. 636-643, 1980.
2. Lei Zhang , Hernan Rueda , Kevin Kim & Peter Aaen, " Non-Quasi-Static Large-Signal Model for RF LDMOS Power Transistors ", 2018 IEEE/MTT-S International Microwave Symposium IMS, pp. 548 -550.
3. Antonio-Valletta, Matteo-Rapisarda, Sabrina-Calvi, Luigi-Mariucci & Guglielmo-Fortunato,"A large signal non quasi static model of printed organic TFTs and simulation of CMOS circuits ", 2017 European Conference on Circuit Theory and Design (ECCTD), pp. 1 - 4.
4. Amrita-Mandal , Dippiya-Saha , Drimit-Ghosal , Saheli-Bhattacharya Atanu-Kundu , & Mousiki-Kar, "Symmetric/asymmetric underlap DG-MOSFET: A study of analog/RF performance using non-quasi static approach ", 2017 Devices for Integrated Circuit (DevIC), pp. 837 - 840.
5. A. S. Roy, J. M. Vasi, and M. B. Patil, “A new approach to model nonquasi-static (NQS) effects for MOSFETs—Part I: Large-signal analysis, ”IEEE Trans. Electron Devices, vol. 50, no. 12, pp. 2393–2400, Dec. 2003.
6. CHRISTIAN C.ENZ and ERIC.A.VITTOZ, Charge-Based MOS Transistor Modeling, The EKV Model for Low-Power and RFIC Design, John Willey and Sons, 2006.
7. A. J. Scholten, L. F. Tiemeijer, P.W. H. de Vreede, and D. B.M. Klaassen,“A large signal non-quasi-static MOS model for RF circuit simulation,” in IEDM Tech. Dig, pp. 163–166, 1999.
8. H. Wang, T.-L. Chen, and G. Gildenblat, “Quasi-static and non-quasistatic compact MOSFET models based on symmetric linearization of the bulk and inversion charges,” IEEE Trans. Electron Devices, Vol. 50, No. 11, pp. 2262–2272, Nov. 2003.
9. VIRTUOSO Simulator Circuit Components and Devices Models Manual, CADANCE, Product Version 7.2, May 2010.
10. Antonios Bazigos and Matthias Bucher, EKV3.0 Model Code and Parameter Extraction, EKV Users Meeting/Workshop, EPFL, Lausanne, Switzerland, 2004.
11. http://ekv.epfl.ch/page-44591-en.html.

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