

# Groundwater Contaminant Transport Modelling for Unsaturated Media using Numerical Methods (FEM, FDM)

Pappu kumar, Anshuman Singh

**Abstract:** This paper presents a numerical solution of the advection-dispersion equation (ADE) with the varying dispersion coefficient and velocity. Contamination of water decreases the quality and quantity of groundwater as well as surface water. Hydraulic conductivity and water retention property also affect the groundwater solute transport model in the vadose zone. The mass balance equation and governing equation of solute transport of advection-dispersion both are interrelated. The governing equation is formulated using Darcy's law and some basic concept of groundwater flow. In the governing differential equations solved numerically. The Equation containing all the parameter and it is a program on the MATLAB and the equation is applicable in the entire situation which is saturated or unsaturated zone. The model is generated for all condition and which tested for taking many sample problems. The numerical solution is more effective and more sensitive used for the to the groundwater solute-transport. The set of programs created and evaluated these solutions. This solution contains many structures that improve their precision, ease of use and its validity. the analysis of Advection-dispersion equation FDM MATLAB code is developed to obtain the numerical solution. Derivation of most of the solution, MATLAB codes for programs, and samples of program input and output are included. In the present paper, the finite difference model developed to asses for groundwater solute transport model for advection and diffusion in the saturated aquifer.

**Keyword:** Advection; Dispersion; Finite difference method; Groundwater; MATLAB.

## I. INTRODUCTION

In the case of groundwater aquifer Forecasting the hydraulic head, change in concentration and change in level aquifer protection strategies used to groundwater modeling tool [1]. Water is natural resources that can be used by plant animal and human being. It is necessary for all living and non-living like industry, power plant, and all factories. World bank state that 1.6 billion people currently face scarcity of water and the number is expected to rise 2.8 billion by 2025[2]. The groundwater aquifer by human activities, such as disposal of west water, industrial wastage and chemical[3]. due to climate change and groundwater depletion, water resources becoming

complex and challenging to policymakers [4]. Mathematical models' representation of predictive site-specific surface water and groundwater contaminant transport processes. The advection and dispersion equation is based on mass balance equation and Fick's law [5]. Mathematical Models can helpful tool for designing, testing and implementing soil, groundwater. Many research has been done on solute transport in groundwater system [6]. A large number of the numerical model and analytical solution now exist in analyzing the solute in groundwater [7]. Groundwater contamination is nonlinear and the analytical solution is for prediction of the contaminant is similar to the solution of two-dimensional unsaturated flow problem[8]. Numerical method and analysis of the stochastic process, mathematical differential equation offers a research power tool for evaluating uncertainty quantification of risk analysis [9]. In this solution of governing equation ADE of groundwater solute transport model, analytical and numerical (finite difference method) are used. Finite difference analysis of an engineering system should be used for designing the different varying boundary condition second-order statistical moments of the response process[10]. In the case of sand stratification, the flow direction is taken as pore water velocity [11]. Dispersive mixing resulting from complex theory three-dimensional porous media evaluate the dispersive flux and describing the hydraulic conductivity[12].

## Contaminant Transport Mechanisms

There are three main physical processes affecting groundwater contaminant transport namely advection, dispersion and diffusion. In addition, chemical processes that affect transport are decay and sorption. advection-dispersion equation is based on conservation of mass balance equation. Due to advection bulk of solute transported in groundwater. The driving force is the hydraulic gradient is effected by a soluble substance in the flowing fluid and variation of velocity shown analytically distribution of concentration [13]. In highly permeable materials such as sand and gravel, advection is the most important transport process, and each transport prediction will only be as accurate as of the flow description. Advective flow becomes more complex when the density and/or the viscosity of water change with solute concentration. Eulerian-Lagrangian methods (ELMs) have been developed to decrease spreading and alternations in numerically created advection-dominated transport solutions. ELMs Improvements in front propagation characteristics are increased, but new sources of errors are introduced. [14]. Numerical models of contaminant transport function as descriptive and predictive tools to support important policy decisions regarding the regulation and remediation of contamination in aquifers[15].

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solute concentration in the groundwater aquifer flows higher concentration region to lower concentration region. Diffusion over geological time, however, can have a significant impact. The effect of diffusion will normally be masked by the effect of advection in groundwater zones with high flow velocities. Dispersion occurs due to concentration gradient in groundwater. Dispersive spreading will lead to an increase in plume uniformity with travel distance. The combination of dispersion and diffusion termed as hydrodynamic dispersion. Degradation process also decreases the sources of contamination with time. The first order radioactive decay follows the first order degradation equation. Reactions of the first order are linear and do not change the characteristics of the transport equation. sorption is the process that the solute particle attached one molecule to the other molecules, this is also known as adsorption and desorption. Adsorption is the process the particle itself attached to the other particle of molecules. The degree of sorption depends on a number of factors, including the concentration and the characteristics of the contaminant, the soil type and its composition, the pH value of water, and the presence of other water solutes. These factors are in time and space, resulting in a variation of retardation in the natural environment. The rate of adsorption onto the solid material as related to the concentration in the groundwater is expressed by adsorption kinetics. The association between the concentration of a solute in the adsorbed phase and in the adjacent water phase at equilibrium is an adsorption isotherm[16]. It can be assumed that sandy materials which continue soil water contaminants transport while silts and clays, particularly those having significant amounts of organic substance, provide an ionic sorptive environment for all three types of contaminants. Even the most porous and highly industrious aquifers, composed of sands and gravels, usually have some fine-grained material, and which is increase the sorption properties of contaminant.

## II. GOVERNING EQUATION FOR CONTAMINANT TRANSPORT MODEL FORMULATION

The general mathematical formulation of a differential equation is based on the hypothesis of groundwater flow. The groundwater movement is based on molecular diffusion and flows velocity. The head of flow is important parameter due to head groundwater flow is occurs. Under the groundwater is flow due to convection. Finite difference scheme for producing a head distribution follows.it is the general equation of groundwater flow,

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = s \frac{\partial^2 h}{\partial t} + Q \quad (1)$$

Where,

$t = \text{time,}$

$k_x = \text{conductivity of water aquifer in the x direction,}$

$k_y = \text{conductivity of water aquifer in the x direction,}$

$k_z = \text{conductivity of water aquifer in the x direction, , s}$

= aquifer storage coefficient, h = head difference above the bottom of the aquifer layer, Q = source or sink functions expressed as net

flow rates per unit area. The equation is based on the theory of Fick's law, where c is the concentration of diffusant expressed in mass unit volume and d is the diffusion coefficient the two cross ponding equation in contaminant flow[17-19].

$$F = -k \frac{\partial c}{\partial x} \quad (2)$$

$$\frac{\partial c}{\partial t} = \frac{k}{\rho} \frac{\partial c}{\partial x} \quad (3)$$

Where K hydraulics conductivity,  $\rho$  is density, and c concentration. x is the spatial distance between difference concentration. F is the amount of concentration flowing in the direction of x increasing per unit time. Assumptions taken into considerations in model development are that the homogeneous and isotropic soil is taken, the soil porosity is constant throughout the soil layer, groundwater conductivity is constant in the saturated aquifer, and the velocity of one-dimensional flow is taken as constant. The one-dimensional advective-dispersive equations in an aquifer subject to nonlinear diffusion and advection. The one-dimensional advection-diffusion equation can

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - \frac{\partial c}{\partial t} \quad (4)$$

## Finite difference approximation

$$\frac{c_{i,j}^{n+1} - c_{i,j}^n}{\Delta t} = D \frac{c_{i+1,j}^n - 2c_{i,j}^n + c_{i-1,j}^n}{(\Delta x)^2} - v \frac{c_{i+1,j}^n - c_{i,j}^n}{\Delta x} \quad (5)$$

This equation is used for solving the spatial and temporal equation of concentration, and boundary condition and initial condition is used for obtaining the solution of the two-dimensional advection-diffusion equation.

## III. RESULT

Model Parameters Employed in the Numerical Simulation for advection and dispersion equation for groundwater solute transport model. Numerical solution for parameter presented in Table. 1 and the problem for finite difference method are used for this case. FEM is presented for modeling the two-dimensional contaminant transport in groundwater saturated media.

Table. 1- parameter of sample problem

Model parameters	Values	values
Diffusion coefficient		0.0005
Advection		0.0005
n is the row number which represents the time discretization		5.0
Concentration at the source boundary		1.0



Initial concentration	1.0
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The numerical solution approximation FDM for explicit for two-dimensional. In fig. 1 contaminant concentration is predicted for the five-time duration it is observed that time increases concentration decreases.

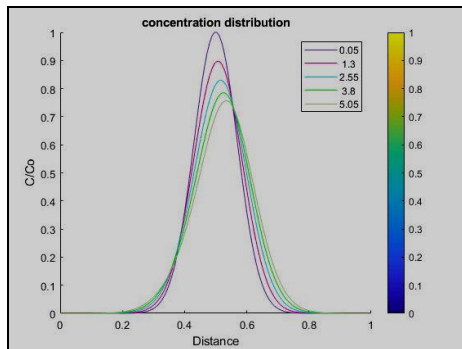


Fig.1 Solution of the one-dimensional diffusion equation

#### IV. CONCLUSION

In this paper, the application of FDM discretizing the model for obtaining the solution for water quality assessment. It is formulated in such a way that the use of prediction of solute concentration in the groundwater and the formulated numerical model found to be the analytical solution of advection-dispersion differential equation. The developed numerical model can be used for the predicting of contaminant dispersion under different diffusion condition. In this paper, the finite difference method used with implicit equation consideration and advection-dispersion solution is presented. The sorption parameter are taken different with a different condition. In this developed model groundwater velocity taken as a constant, we can develop another model using the same methodology for groundwater flow velocity. The methodology developed can easily be extended to two or three-dimensional problems.

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