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Certain Product Set Labeling of Graphs and Their Cardinality

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Abstract*: *A product set-labeling of a graph G is an injective function f : V (G) →P(N) such that the induced edge function f : E(G) →P(N)defined by $f^*(uv) = f(u)*f(v)$ is injective . A product set labeling of a graph G is a geometric product set labeling if the set labels of all its elements , that is vertices and edges with respect to the function f are geometric progressions .The number of elements in the set label of a vertex or edge of a graph G is called its cardinality .In this paper , we have found alabeling in which all the edges of a graph G are in geometric progressions even though the set labels of one of its vertex is not a geometric progression. Also the edge cardinalty of such graphs

Index Terms*: *Edges, Geometric progressions, product set-labeling, Vertices .

I. INTRODUCTION

Graph Labeling was introduced in the year 1934 by Rosa . Graph labeling has various applications in coding theory , stronomy , radar etc. Here ,we discuss about the set labeling on Graphs .There are two main types of labeling on graphs (1) Edge set labeling (2) Vertex set labeling .Edge set labeling refers to the assignment of sets to the edges of a graph G with respect to particular conditions and Vertex set labeling refers to the assignment of sets to the vertices of a graph G with respect to particular conditions .The product set of any two finite set of integers A and B is denoted as A*B, and is defined as{A*B = ab: a ϵ A , b ϵ B}. [1] B D Acharya introduced the notion of set -valuation on graphs .Different kinds of set alued (set labeled) graphs and their properties has already been in existence. Here, we discuss the various properties of graphs admitting a geometric product set labeling and the cardinality of its elements (edges and vertices).The number of elements in a set which is assigned to an edge or vertex of a graph G is called the label size of the edge or label size of the vertex of the respective graph G .[5]If the label size of all the edges of the graph G is same ,such a graph is said to admit a uniform product set labeling under the injective function f : V (G) →P(N).[5] A product set labeling of a graph G is calledan Isogeometric product set labeling if the common ratio of set labels of all its elements withthe respect to the product set labeling are same.Finite simple connected graphs admitting a setlabeling and having no isolated vertices has unconsidered throughout this paper .Also thenotation GP is used for eometric progressions .

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II CERTAIN PRODUCT SET- LABELING OF GRAPHS

Definition 2.1-:

A Graph G is said to be a set labeled graph if there exist an injective function f: V(G) → P(N) such that the induced edge function f^+ :E(G) → P(N) is defined by $f^+(uv) = f(u) + f(v)$,where u and v are vertices of the graph G and uv denotes the edge connecting u and v .

Definition 2.2 -:

An injective set valued function f : V(G) → P(N) is called a product set labeling of a graph G ,if the induced edge function f*: E(G)→ P(N) is defined by f*(ij) = f(i) * f(j) , i ,j ϵ E(G) , where f(i) and f(j) are the sets assigned to the vertices i and j under the injective function f . A product set labeled graph is a graph that admits a product set labeling.

Theorem 1 -:

The edge set labels of a product set labeled graph G are geometric progressions even if one of the vertex have set labels r, r^2 r^4,where r ϵ N , and all other vertices having set labels have commonratio r .

Proof -: Let G(V,E) be a product set labeledgraph with |v| = 3 and f be its product setlabeling under which the set labels of every vertexexcept one are in GP. Let this unique vertex edenoted as u and all other vertices be denoted asv, w .... consider the two vertices u and v .Let thecardinality of all the vertices be 3. Let the setlabels of u under f be denoted as f(u) , similarly f(v). Let

$f(u) = \{ar, ar^2, ar^4\}$

$f(v) = \{br; br^2; br^3\}$

where a ,b ϵ N . We are proceeding with theassumption that there is an adjacency betweenboth the vertices u and v in the graph G . And letthe edge connecting u and v be denoted as uv

$A_1 = f(u) *\{br\} = \{ar ,ar^2,ar^4\} *\{br\}$

$=\{abr^2,abr^3,abr^5\}$

$A_2= f(u) * \{br^2\} =\{ar; ar^2; ar^4\}* \{br^2\}$

$= \{abr^3,abr^4, abr^6\}$

A_3 = f(u) * {br^3} ={ar , ar^2,ar^4} * {br^3}
= {abr^4,abr^5, abr^7}
[Taking each element from f(v) and multiplying with all the elements in f(u)] Union of the three

sets A_1, A_2 , A_3 gives the set labels of the edge uv or the elements of f(uv) , thusA_1UA_2UA_3= {abr^2, abr^3,abr^4, abr^5, abr^6, abr^7}which is clearly a GP , similarly computing for the remaining edges, we get the set labels as GP.Since all other vertices except u has set labels asGP .

Corollary 1.1 -:

Let G be a product set labeled graph in which thelabels of all the vertices are in GP with commonratio r, except one vertex with label r ,r^2, r^4, r^7……The i^{th} element of this unique vertex is obtainedby multiplying (i - 1)th element with $r^{(i-1)}$,(i > 1) then the cardinality of those edges incident on the vertices having set labels as GP is (2n-1) and the cardinality of those edges incident with the unique vertex is (m+n) -1, where m is the power of the n^{th} element of the unique vertex

Proof -:

Let G be a product set labeled graph and f be itsproduct set labeling under which the set labels ofall vertices except one are in GP. Let this uniquevertex be denoted as u and all other vertices bedenoted as v, w.... .And the cardinality of all the vertices is assumed to be same. Let

f(u) ={ar, ar^2, ar^4,ar^7,……..ar^m}
f(v) ={br ,br^2 ,br^3,br^4………br^n}
f(w)={cr, cr^2,cr^3, cr^4, ………cr^n}
f(z)={dr, dr^2 ,dr^3, dr^4, ………dr^n}

On computing for the edges uv , uw , uz ,vw ,vz ,wz, we get

f(uv) = {abr^2,abr^3,abr^4,abr^5,abr^6,abr^7,abr^8, abr^9,abr^{10},abr^{11},...abr^{m+1}, abr^{m+2},abr^{m+3}, abr^{m+4}...abr^{m+n}}
|uv| = (m + n) - 1
f(uw) = {acr^2,acr^3,acr^4,acr^5,acr^6,acr^7,acr^8, acr^9,acr^{10},acr^{11},...acr^{m+1}, acr^{m+2},acr^{m+3}, acr^{m+4}...acr^{m+n}}
|uw| = (m + n) - 1
f(uz) = {adr^2,adr^3,adr^4,adr^5,adr^6,adr^7,adr^8, adr^9,adr^{10},adr^{11},...adr^{m+1}, acr^{m+2},acr^{m+3}, adr^{m+4}...adr^{m+n}}
|uz| = (m + n) – 1
f(vw) = {bcr^2,bcr^3, bcr^4,bcr^5,bcr^6, bcr^7,bcr^8….bcr^m, bcr^{m+1}, bcr^{m+2}……bcr^{2n}}
|vw| = 2n – 1
f(vz) = {bdr^2,bdr^3, bdr^4,bdr^5,bdr^6, bdr^7,bdr^8….bdr^m, bdr^{m+1}, bdr^{m+2}……bdr^{2n}}
|vz| = 2n -1
f(wz)= {cdr^2,cdr^3, cdr^4,cdr^5,cdr^6, cdr^7,cdr^8….cdr^m, cdr^{m+1}, cdr^{m+2}……cdr^{2n}}
|wz| = 2n - 1 .

Illustration-:

f(u) ={ar, ar^2 ,ar^4 ,ar^7 , ar^{11}}
f(v) ={br, br^2, br^3, br^4, br^5}
f(w)={cr, cr^2, cr^3, cr^4, cr^5}
f(z)={dr, dr^2, dr^3, dr^4, dr^5}

On computing for the edges uv ,uw ,uz ,vw ,vz,wz

f(uv) ={abr^2, abr^3, abr^4, abr^5, abr^6, abr^7, abr^8, abr^9,abr^{10},abr^{11},abr^{12}, abr^{13} ,abr^{14} ,abr^{15}, abr^{16}}
|uv| = 15
f(uw) ={acr^2, acr^3, acr^4, acr^5,acr^6, acr^7, acr^8, acr^9, acr^{10}, acr^{11},acr^{12},acr^{13}, acr^{14} ,$abcr^{15}$, acr^{16}}
|uw| = 15
f(uz) ={adr^2, adr^3, adr^4, adr^5,adr^6, adr^7, adr^8, adr^9, adr^{10}, adr^{11},adr^{12},adr^{13}, adr^{14} ,adr^{15}, adr^{16}}
|uz| = 15
f(vw) = {bcr^2, bcr^3,bcr^4, bcr^5, bcr^6, bcr^7,bcr8}
|vw| = 7
f(vz) = {bdr^2, bdr^3,bdr^4, bdr^5, bdr^6, bdr^7,bdr^8}|vw| = 7
f(wz) = {cdr^2, cdr^3,cdr^4, cdr^5, cdr^6, cdr^7,cdr^8}|wz| = 7

Similarly, we can increase the cardinality of the vertices and thereby compute for the edges

Corollary 1.2-:

A product set labeled graph G in which the labelsof all the vertices are in GP except one vertexwith set labels r , r^2 , r^4(where r ϵN) and allother vertices with common ratio r and label size 3 ,the cardinality of edges of graph G is(a) 2n for those edges incident on the uniquevertex(b)2n-1 for the emaining edges

Proof -:

let G be a product set labeled graph and f be theinjective function under which the graph G admitsa product set labeling and all the vertices of graphG has same label size , let us hoose the label sizeas 3 that is , n=3 . Let us denote the uniquevertex as u and all other vertices as v, w...andsuppose that there is an adjacency between thevertices u and v .

f(u) = {ar ,ar^2,ar^4}
f(v) = {br ,br^2,br^3}
f(w) = {cr,cr^2,cr^3}
f(z) = {dr, dr^2, dr^3}

on computing for the edges uv , uw , uz , vw ,wz we get as follows -:

f(uv) = {abr^2,abr^3,abr^4,abr^5,abr^6,abr^7}
|uv| = 6
f(uw) = {acr^2, acr^3, acr^4, acr^5,acr^6,acr^7}
|uw| = 6
f(uz) = {adr^2,adr^3,adr^4 ,adr^5,adr^6,adr7}
|uz| = 6
f(vw) = {bcr^2, bcr^3 ,bcr^4,bcr^5, bcr^6}
|vw| = 5
f(wz) = {cdr^2 ,cdr^3, cdr^4,cdr^5, cdr^6}
|wz| = 5
f(vz) = {bdr^2; bdr^3; bdr^4; bdr^5; bdr^6}
|vz| = 5

Thus , cardinality of edge set labels of graph G is 6 and 5 , that is 2n and 2n-1 .

Theorem 2 -:

The edge set labels of a product set labeled graph are in GP even if one of the vertex with set labels r , r^2, r^5(where r ϵN) is not a geometric progression and all other vertices have set labels with common ratio r .

proof -: Let G be a product set labeled graph and f be its product set labeling under which the set labels of all vertices except one are geometric progressions. Let this unique vertex be denoted as u and all other vertices be denoted as v, w ... .Let us consider the two vertices u and v . Let the set labels of u under f be denoted as f(u) , where

f(u) = {ar, ar^2, ar^5}
f(v) = {br, br^2, br^3}

where a and b are any two positive integers . we are proceeding with the assumption that there is an adjacency between both the vertices u and v .

A_1 = f(u) * {br} = {ar; ar^2; ar^5}
* {br} =
{abr^2,abr^3, abr^6}

$A_2 = f(u) * \{br^2\} = \{ar; ar^2, ar^5\} * \{br^2\} = \{abr^3, abr^4, abr^7\}$
$A3 = f(u) * \{br^3\} = \{ar, ar^2, ar^5\} * \{br^3\} = \{abr^4, abr^5, abr^8\}$
[Taking each element from f(u) and multiplying with all the elements in f(v)] Union of the three sets A_1, A_2, A_3 gives the set labels of the edge uv or the elements of f(uv) , thus $A_1UA_2UA_3 = \{abr^2, abr^3, abr^4, abr^5, abr^6, abr^7, abr^8\}$ Which is also a GP,similarly computing for other edges we get the set labels as GP .

Corollary 2.1-:

In a product set labeled graph G in which thelabels of all the vertices are in GP except onevertex with set labels r, ,r^5(where r ϵN) and allother vertices with common ratio r and label size3 , then the label size of edges of graph G is(a) 2n+1 for those edges incident on the uniqueVertex(b)2n-1 for the remaining edges

Proof -:

let G be a product set labeled graph and f be the injective function under which the graph G admitsa product set labeling and all the vertices of graphG has same label size , let us hoose the label sizeas 3 that is , n=3 and the ratio r as 2 . Let us denote the unique vertex as u and all othervertices as v, w....And suppose that there is anadjacency between the vertices u and v

(a,b,c ϵ d_N)

$f(v) = \{br, br^2, br^5\}$
$f(w) = \{cr, cr^2, cr^3\}$
$f(z) = \{dr, dr^2, dr^3\}$

on computing for the edges uv , uw , uz , vw ,wz we get as follows -:

$f(uv) = \{abr^2, abr^3, abr^4, abr^5, abr^6, abr^7, abr^8\}$
$|uv| = 7$
$f(uw) = \{acr^2, acr^3, acr^4, acr^5, acr^6, acr^7, acr^8\}$
$|uw| = 7$
$f(uz) = \{adr^2, adr^3, adr^4, adr^5, adr^6, adr^7, adr^8\}$
$|uz| = 7$
$f(vw) = \{bcr^2, bcr^3, bcr^4, bcr^5, bcr^6\}$
$|vw| = 5$
$f(wz) = \{cdr^2, cdr^3, cdr^4, cdr^5, cdr^6\}$
$|wz| = 5$
$f(vz) = \{bdr^2, bdr^3, bdr^4, bdr^5, bdr^6\}$
$|vz| = 5$

Thus , the cardinality of edge set labels of graph G are either 7 or 5 , that is 2n+1 and 2n-1

Definition 2.4 -:

A product set labeling f :V(G)→ P(N) of a graph G is said to be a geometric product set labeling ,if the set labels of all the vertices and edges with respect to the function f are in GP.

Theorem 3 -:

If the label size of the vertices in a geometric product set abeled graph is n , then the label size of the edges will be 2n-1.

Proof -: Let G be a geometric product set labeled graph and f be the geometric product set labeling of the graph G under which the set labels of all the vertices and edges are in GP. Let u and v be any two adjacent vertices in G and uv denote the edge connecting u and v and f(u) and f(v) be the set labels of the vertices u and v under f .Suppose that the label size of the two vertices u and and V be three ,since the minimum possible label size is three. let ,

$f(u) = \{ar, ar^2, ar^3 \ldots\ldots ar^n\}$
$f(v) = \{br, br^2, br^3 \ldots\ldots br^n\}$

On Computing for the edge uv, we get

$f(uv) = \{abr^2, abr^3, abr^4, abr^5, abr^6 \ldots\ldots abr^n, abr^{n+1}, abr^{n+2}, \ldots\ldots\ldots\ldots abr^{2n}\}$

it has (2n)-1 elements

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