

Performance Evaluation of Nature Inspired Meta-Heuristic Algorithms using Rosenbrock, Rastrigin and Sphere Test Function for Optimization

Deepika Dhawan, Rajeshwar Singh

Abstract: Due to a large pool of optimization algorithms available, it is a daunting task for a researcher to select the best algorithm for their optimization problem. Nature inspired meta-heuristic algorithms have emerged and are very popular among researchers for their research. The meta-heuristic algorithms are used, for finding and generating partial search algorithm that is supposed to provide a sufficiently good solution to any optimization problem, especially with imperfect information. In this research paper, we have chosen two widely researched and rapidly developing algorithms namely Firefly algorithms (FA) and Artificial Bee Colony (ABC) Algorithms to test on different test functions. Therefore, in this research paper the efficiency of Artificial Bee Colony and Firefly algorithms are rigorously tested on three most used test function for optimization namely Rosenbrock test function, Rastrigin test function and Sphere test function. The best cost for both the algorithms on, all three test functions are presented and compared to find which algorithm is the best along with the test function for finding an optimal solution to meta-heuristic implementation.

Index Terms: Nature-inspired, Meta-heuristic, ABC Algorithms, Firefly Algorithms, Optimization.

I. INTRODUCTION

Computer Optimization Algorithms which are used to solve optimization problems can be roughly categorized into two parts: *Exact algorithms or deterministic algorithm and heuristics algorithms* [1]. The Exact algorithms yield the solutions in a determined time and do not put a load on the computation complexity; they yield a solution in a finite time. Also, Heuristic Algorithms are applied to give solutions to NP-Hard problems. The solutions to these problems are not found in a deterministic time, hence the name NP-Hard where NP stands for Nondeterministic Polynomial. In this category, the time required for a problem to be solved cannot be guaranteed. Also, these algorithms find the solution which is "good" in a reasonable amount of time. Heuristic algorithms are very specific for a problem, whereas, Meta-heuristic is a high language framework which provides guidelines to

develop a heuristic optimization problem. Meta-heuristic algorithms are used for finding and generating partial search algorithms which provide solutions to mostly imperfect information or limited capacity of computation. Unlike heuristic algorithms meta-heuristic algorithm does not have to be fully modified to work on any other optimization problem, they are hardly needed to modify to execute on other optimization problems. In meta-heuristic algorithms, there are many metaphor based algorithms that are applied to give a solution to optimization in computer science and engineering. To list a few, Simulated Annealing (SA), Ant Colony Optimization(ACO), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), Firefly Algorithm (FA), etc. In this research paper, we are using the two of the above-stated metaphor based metaheuristic to evaluate on various test function for optimization as they are widely used among researcher for their research. The purpose of this paper is to evaluate the algorithms on various test function so that we can compare the results (best cost) of the algorithms and we can infer from the results that which algorithm would behave well on certain test function. For this purpose, we have taken Artificial Bee Colony (ABC) [2] and Firefly Algorithm (FA) [3], as they have gained some serious attention of researchers in the past few years [4-8]. Artificial Bee Colony is a metaphor based metaheuristic algorithm which mimics the behavior on honey bee swarm. It was proposed by Derviş Karaboğain year 2005 [2]. The Firefly Algorithms are also metaphor related meta-heuristic algorithms that mimic the characteristics of firefly swarm and the way they act is that the firefly's flash communicates to other fireflies [3]. Related work of this research paper is discussed in section 2 and details of ABC Meta-Heuristics in Section 3. In Section 4, we present the Firefly Algorithm and Section 5 has various optimization functions used. Evaluation and results are presented in Section 6 and the conclusion in Section 7.

II. RELATED WORK

In paper [9], authors have introduced a new algorithm to tackle the multicast QoS problem. The key point of the FA algorithm is to compute the most feasible path which successfully full-fills the requirements of the system and helps to give solutions. The simulation results show the efficacy of the algorithm are drawing attention to meet routing challenges that occur in video, text and image communications and it is really very difficult to find a solution for Quality of Service in multi-cast routing problems.

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In paper [10], authors are presenting a brilliant approach which is based on ABC to overcome challenges of routing.

The artificial bees are used to alter the position of food sources in order to discover new food places. Authors are using Fitness function to calculate the location of the source with optimal value. The obtained optimal value is considered an optimized solution. Authors in paper [11] are discussing improvements which can be implemented with Artificial Bee Colony Algorithms (ABC) to solve Vehicle Routing using Time Window. They are proposing improvements over weak searchability and slow search speed of Artificial Bee Colony Algorithms. First of all the single search mode is changed into a three-way search method, which enhances the optimization depth of the algorithm. Secondly, by using multiple neighbor-hood searches of new food sources has enhanced the survival of new food sources and increased the diversity of populations. Next to it is the global optimal solution which is recorded by setting and updating the bulletin board. In paper [12], authors are presenting modified Firefly algorithms. It is basically a Firefly Algorithm and Particle Swarm Optimization (PSO). Firefly algorithms (FA) falls under the category of nature-inspired Algorithms of Swarm Intelligence (SI), it means the outcome of algorithms depend on the attraction of a firefly to the luminance signal of other firefly. Searching behavior in standard algorithm is improvised by the modified velocity concept of Particle Swarm Optimization and is used in the modified algorithm. The research work in [13] focuses on Function Optimization (FO). This is the suitably applied continuous optimization work to give the best suitable results to obtain the optimal solution of a function. Nature Inspired Algorithms (NIA) becomes more famous to find a solution to FO. In this paper, authors are simply comparing the results of different NIAs by using simulation on standard benchmark functions. The paper [14], attracts attention towards two major issues in multicast routing, one is load balancing and another one is transmission delay. Authors are taking bi-objective optimization challenges for network routing using multicast, where the average bandwidth utilization ratio and the average transmission delay are two objectives for minimization. Authors compare outcomes based on Pareto dominance, which helps to enhance the local exploitation.

III. ABC META-HEURISTIC [2]

It is a process that matches the activities of honey bees, as these bees collectively accomplish their task through social cooperation. In the ABC algorithm, there are three types of bees: employed bees, onlooker bees, and scout bees. The job of the employed bees is to search for food in the vicinity of the food source. Also, it is assumed that there is only one artificial employed bee for each food source [4, 5]. Then the information for the food source is then forwarded onto the onlooker bees. The onlooker bees then select the higher quality (fitness) food source rather than the lower quality ones. The scout bees are the employed bees who have abandoned their food source and search for a new one, once they find a new food source they memorize the location of the new food source and forget the old food source's location. The number of onlooker bees and the employed bees is equal to the number of the solution in the swarm. The artificial bee colony generates a randomly distributed initial population (which means the bees in the simulation are placed randomly,

it may or may not be near the food source) of SN food sources (solution). Process for bees to exchange all the information about the food sources to the other bees is that when the employed bees go to their food source they roam around in the area near the food source, the onlooker bees then watches the employed bee flying and dancing near the food source and get the information by co-operation that food is present at that source, and then chooses the food source from the probability value p_i associated with it, which is calculated by the following expression $P_i = \text{fit}_i / \sum \text{fit}_n$, where value of n is 1 to SN (Number of food source), fit_i - fitness value and i - employed bee evaluation.

ABC Algorithms steps are:

- Initialized Food source for each employed bee
- Repeat
 - a. Every employed bee visit to the food source they memorized, and dance in the hive, if they find its nectar value
 - b. Every onlooker bee sees the dance of the employed bee and selects the source depending on the dance, and then goes to the source, and evaluates its nectar amount.
 - c. Unknown food sources are left permanently and the scout bees are able to search new food source.
 - d. The best food source found so far is registered, if it has high value than that of previous.
- Until requirements are met.

Mathematically ABC Defined: The employed bees searching in the vicinity the food resources at x_i will search for better food resource at new location v_i . the identification of the new food source will be evaluated by the equation (1) [7]

$$v_{ij} = x_{ij} + \Phi_{ij}(x_{ij} - x_{kj}) \quad j = 1, 2, \dots, n; \quad k = 1, 2, \dots, SN \quad (1)$$

The new location vector of the bees is $v_i = [v_{i1}, v_{i2}, \dots, v_{in}]$, $x_i = [x_{i1}, x_{i2}, \dots, x_{in}]$ is the location vector of the i^{th} bee, k where k not equal to j is a correct random number in $[1, SN]$ and Φ_{ij} a random number uniformly distributed between $[-1, 1]$ The selection of the x_{ij} is done using equation (2) [7]

$$x_{ij} = L_j + \text{rand}(0,1) \times (U_j - L_j) \quad (2)$$

The U_j and L_j are the upper bound and lower bound in equation (2). When a new location of the food source is found, its optimization/befitting rate is calculated using equation (3). [7]

$$\begin{aligned} \text{Fit}_i &= 1/1+f_i, \text{ if } f_i \geq 0 \text{ else,} \\ \text{Fit}_i &= 1+\text{abs}(f_i) \end{aligned} \quad (3)$$

For onlookers, the food source selection is calculated using equation (4) [7]

$$P_i = \text{fit}_i / \sum \text{fit}_n \quad (4)$$



Parameters during implementation: We have implemented the meta-heuristic in MATLAB. During the implementation, the number of maximum iteration is 50 and population size 50. Another value needed is the coefficient for Acceleration, we have taken it as 1, and the abandonment limit parameter is evaluated by $(0.6 * \text{Decision Variable} * \text{population size})$.

IV. FIREFLY ALGORITHMS [3]

It is an algorithm which mimics the characteristics of the firefly. As the ABC this algorithm is also metaphor based. It is a meta-heuristic algorithm which is inspired by the flashing of the light from fireflies. Aim of the firefly's flash is to act as a communication signal to attract other fireflies. This algorithm simulates the flash pattern of a firefly. They use flashing as a signal to attract other mates and also search for prey. While formulating the algorithm the following assumptions were made:

- Each firefly will be attracted to all other fireflies.
- The value of attraction of the fireflies is a direct function of the brightness of the fireflies' light, i.e. the more rhythmic light's intensity is the more other fireflies will be attracted to it.
- Lesser bright fireflies are attracted to and go to a brighter one. However, the intensity of the light decreases as the space between the two fireflies increases, i.e. if the brighter firefly is flying away from another firefly the intensity of the light will decrease.
- It wanders in the vicinity if no more bright firefly is found.

Thus, it can reach an optimum result using the above-stated assumptions.

FA defined mathematically: The two factors in the Firefly algorithm are change in luminous density and creation of attraction for other fire flies. That is to say that low light fireflies get attracted to other high light fireflies and this process continues until all of the fireflies have gathered in one place, which is supposed to be a global optimum. As mentioned above luminous density is in proportion to the creation of attraction to other fireflies. Therefore, we can write

$$\beta = \beta_0 e^{-\gamma r^2} \tag{5}$$

In equation (5) γ is the light absorption coefficient and β_0 is the attractiveness at $r = 0$. Also, r_{ij} is the Euclid distance between two fireflies, which is evaluated as shown in equation (6)

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^n (x_{ik} - x_{jk})^2} \tag{6}$$

The update of the movements of the fireflies following the above-stated procedure takes place using equation (7)

$$x_i \leftarrow x_i + \beta = \beta_0 e^{-\gamma r^2} (x_j - x_i) + a(\text{rand} - 1/2) \tag{7}$$

In equation (7) α , β_0 and γ are considered constant, α , β_0 select from $[0, 1]$ and γ is selected from $[0, \infty]$. [8]

Implementation of Firefly Algorithms: Firefly Algorithms is implemented as described in the following paragraphs. Maximum iterations is set to 100, population size is set to 50,

light absorption coefficient is set to 1, attraction coefficient is set to 2, mutation coefficient is set to 0.2, mutation coefficient damping ratio is set to 0.98 and the uniform mutation range is defined as $0.8 * (\text{variable upper bound} - \text{variable lower bound})$.

V. VARIOUS TEST FUNCTION FOR OPTIMIZATION

Test functions in mathematics and computer science are used useful for evaluation of characteristics of optimization algorithms such as convergence rate, precision, robustness. In this part we have compared the algorithms on three different test function first is Rosenbrock function, it is a non-convex function used to evaluate the performance of the optimization algorithm. Next is the sphere function which is uni-modal, has d dimensions, it is also used to evaluate the performance of the optimization algorithm. The last function we have used is the Rastrigin function; this function is non-convex and is also used to evaluate the performance of optimization algorithms.

Rosenbrock Test function: The Rosenbrock is a non-convex function, which is used for the performance test problem for the optimization algorithm. It was proposed by Howard H. Rosenbrock. The function is defined by

$$f(x) = \sum_{i=1}^{N-1} [100(x_{i+1} - x^2)^2 + (1 - x_i)^2] \tag{8}$$

A plot of the Rosenbrock Test Function is Fig.1.

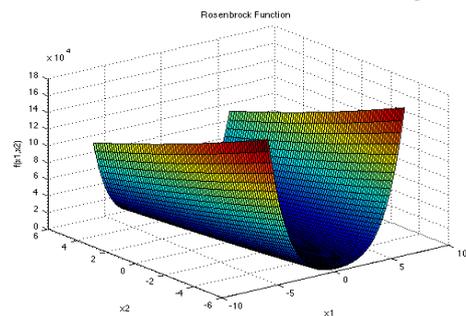


Fig.1: Rosenbrock Test Function.

Sphere Test function: These Test functions have dimensions d and it is local value except for the global one. It is defined by the formula

$$f(x) = \sum_{i=1}^d x_i^2 \tag{9}$$

A plot of the function is in Fig. 2.

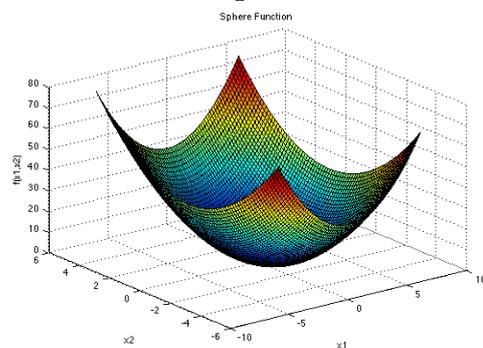


Fig. 2: Sphere Test Function.

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Rastrigin Test function: Rastrigin function is a non-convex and two-dimensional function which is used for performance testing in optimization algorithms. It is defined by

$$f(x) = An + \sum_{i=1}^n [x_i^2 - A \cos(2\pi x_i)] \quad (10)$$

A plot of it is in Fig. 3.

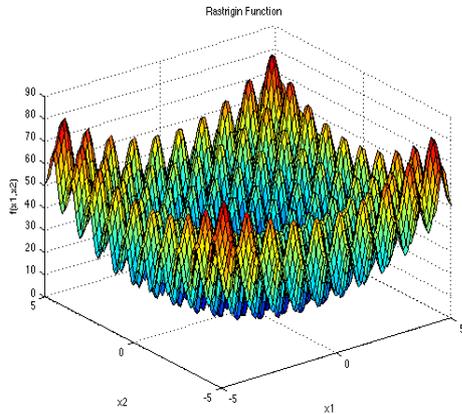


Fig. 3: Rastrigin Test Function.

VI. EVALUATION AND RESULTS

ABC and Firefly are tested using the Rosenbrock, Sphere and the Rastrigin test function. The implementation of the algorithms are very well explained above, the parameters used are also stated along with the implementation. The test functions are briefly described above along with the figures to have an idea of the plot of the function. The MATLAB codes are used for functions implementation.

ABC and FA algorithms are very delicate because of their parameters and any change in the parameter can affect the outcome of the algorithm, so, the optimal parameters are set in order to get the optimal result. In the evaluation, we have tested the algorithms after setting the maximum iteration and population/swarm size to 100 and 50. The population is a constant as we have set it to 50 but the iterations are incremented with 10 each time i.e. first it is simulated on 10, then 20, then 30 and so on up until 100. However, we have presented the output data from 50 to 100 iterations so that our data looks clean

Execution with test function set to Rosenbrock: Upon execution of algorithms with Rosenbrock test function, the data is retrieved is as appended in table.1.

Table.1: Rosenbrock Test Function Convergence data for ABC and FA.

Algorithms	Iterations					
	50	60	70	80	90	100
ABC	1.6268	1.6411	1.2016	1.5757	1.5264	1.4621
FA	0.1874	0.0881	0.1422	0.0662	0.07576	0.0425

The convergence graphs with maximum iterations for both the algorithms are in Fig. 4 (a) & Fig .4 (b).

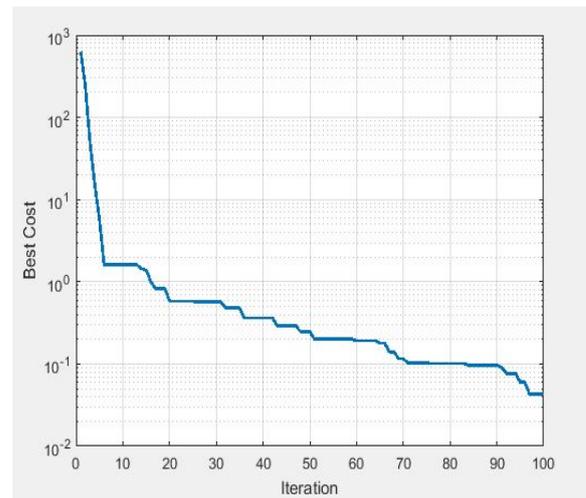


Fig. 4 (a) : ABC Algorithms.

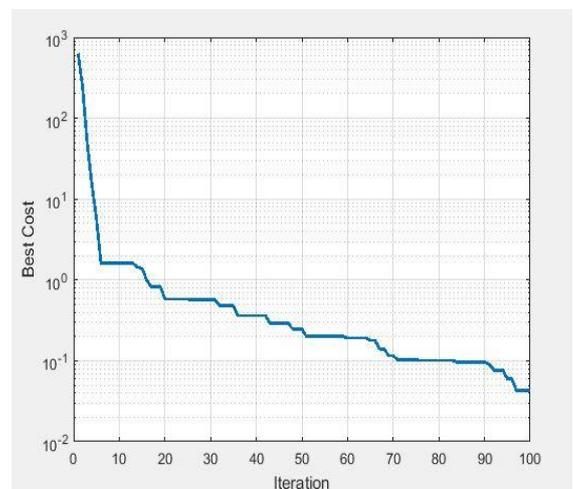


Fig. 4 (b): Firefly Algorithms.

Comparison diagram from the data of table.1 is in Fig. 4 (c).

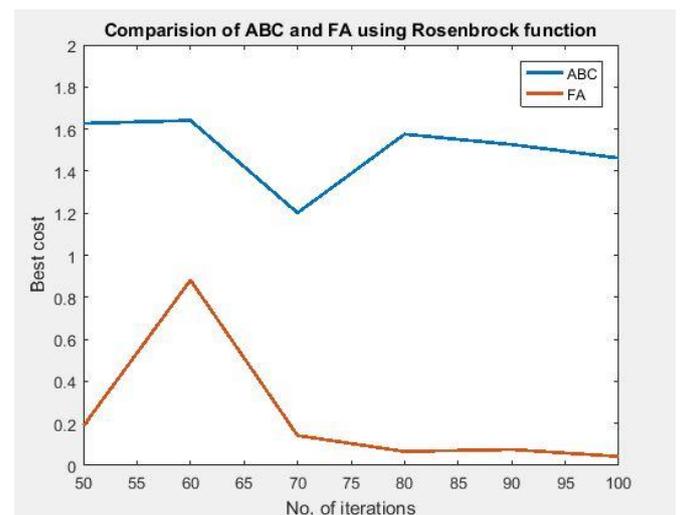


Fig. 4 (c): ABC and FA using Rosenbrock Function

Execution with test function set to Sphere: Upon execution of algorithms with Sphere test function, the data obtained is as appended in table.2.



Table.2: Sphere Test Function Convergence data for ABC and FA.

Algorithms	Iterations					
	50	60	70	80	90	100
ABC	3.0144e ⁻⁵	7.5238e ⁻⁷	1.7323e ⁻⁷	8.5798e ⁻⁹	2.2461e ⁻⁹	4.0359e ⁻¹⁰
FA	2.9248e ⁻⁴	2.3697e ⁻⁴	1.1092e ⁻⁴	8.5121e ⁻⁵	5.0446e ⁻⁵	4.1717e ⁻⁵

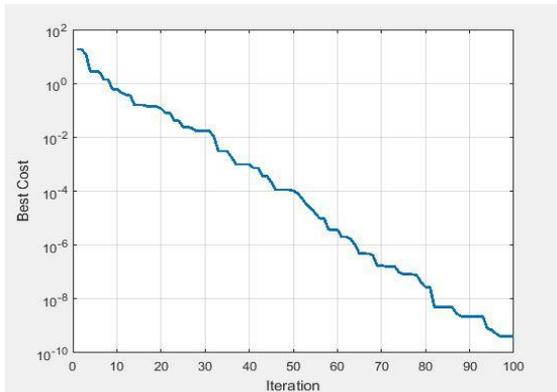


Fig. 5 (a) : ABC Algorithms.

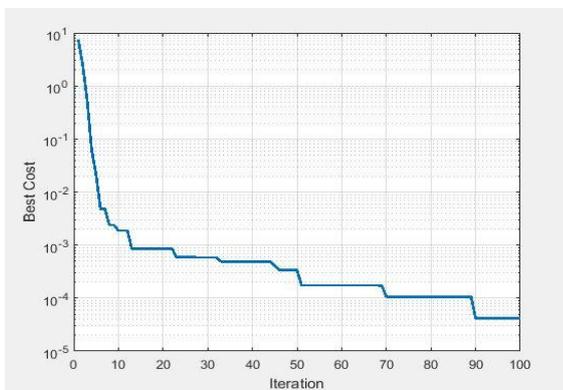


Fig. 5 (b): Firefly Algorithms.

Fig. 5(a) represents the convergence graph of ABC algorithm and Fig.5 (b) represents the convergence graph of Firefly Algorithm. Comparison of ABC and FA using Sphere Test Function is carried out for data of table.2 and is in Fig. 5(c).

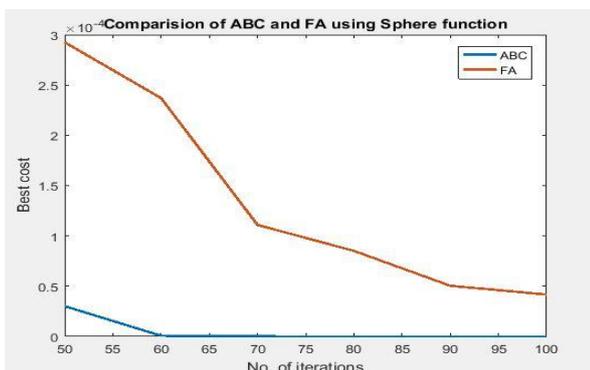


Fig. 5 (c): ABC and FA using Sphere Test Function

Execution with test function set to Rastrigin: Upon execution of algorithms with Rastrigin test function, the data obtained is as appended in table.3.

Table.3: Rastrigin Test Function Convergence data for ABC Algorithm and FA.

Algorithms	Iterations					
	50	60	70	80	90	100
ABC	3.9224	4.1649	3.7726	3.9081	6.3575	4.2936
FA	1.0284	0.1919	0.0246	0.0123	0.0084	0.0037

The convergence graph with maximum iterations for both the algorithms is in Fig. 6(a) and Fig. 6(b)

Fig.6(a): ABC Algorithms

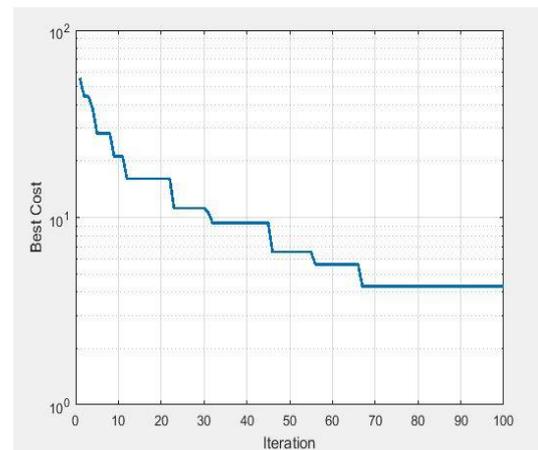
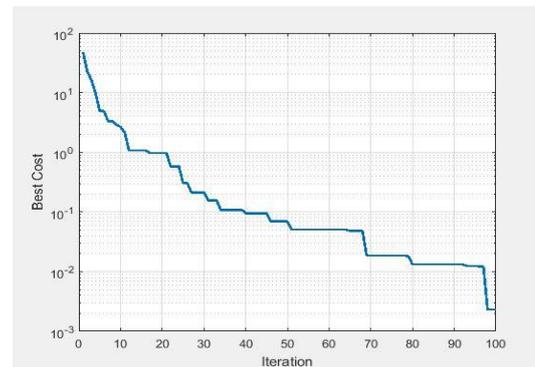


Fig.6(b): Firefly Algorithms.

The data obtained for the Rastrigin Test Function is as shown in table 3. The comparison is carried out and the graph obtained is in Fig. 6 (c).

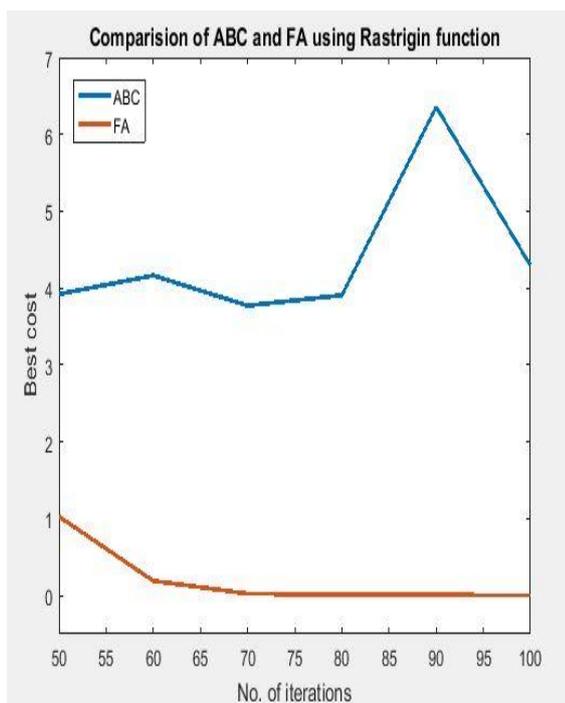


Fig. 6 (c): ABC and FA using Rastrigin Test Function

VII. CONCLUSION

Performance evaluation of nature-inspired ABC Algorithm and Firefly Algorithm has been carried out on three test functions Sphere, Rastrigin and Rosenbrock. It is found that with the rosenbrock test function, Firefly algorithm has yielded a much better cost than the Artificial Bee Colony algorithm. In the convergence diagram, we can see that the FA’s plot has decreased rapidly approximately up to 0.05, whereas, the cost yielded by the ABC is near 1.2, which reveals that the FA performs better in comparison when using the rosenbrock test function. The plots and convergence diagram of ABC and FA using the sphere test function reveal that the ABC algorithm has proven to be the best in this particular case, as it has revealed the best cost that is much more impressive than the FA algorithm has revealed. We can see in the convergence diagrams (5 (a) and 5(b)) that initially FA algorithm was starting to converge faster but then it could not converge as much as the ABC. And in graph 5(c), we can see that near to iteration 70 the FA’s cost risen up. Hence we can be sure and say that the ABC algorithm will reveal better results when using with Sphere test function. The plot 6(c) and convergence diagram 6(a) and 6(b) of ABC and FA using the rastrigin test function reveal that the FA algorithm gives better results as compared to ABC using rastrigin test function. The FA algorithm gives the best cost around 0.00038 while the ABC gives 3.78. Hence we can be sure that the FA algorithm performs well using the rastrigin function in comparison with the ABC algorithm.

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