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# Development of the Mathematical Model of a Hydrodynamic Cavitations Device

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*Abstract*: ***The article provides information on the use of cavitation phenomena in the processing technology of fluid systems in various areas - ecology, energy, chemistry, biotechnology, food industry, etc. As a result of a comparative analysis of the effects on the liquid ultrasonic and hydrodynamic cavitation, the choice was made in favor of hydrodynamic. Attention is offered to develop a mathematical model of the described workflow, based on the material, energy and heat balance of the flow using the equations of fluid dynamics, to determine the lateral size of the hydrodynamic cavitation devices of a continuous operating principle with various cavitation exciter- nozzles and a hydrodynamic lattice. The workflow of the proposed cavitation device is fundamentally different from traditional cavitation devices, in which only the cavitation effect on the flow is realized. In the proposed cavitation device, the cavitation effect is enhanced by the shock effect of a pressure jump during the transition of a supersonic to subsonic flow. And it is the formation of cavitation phenomena with the help of a cavitator that allows transfering the flow to two-phase, in which the speed of sound is much less than in each of the phases (gas and liquid), and, thus, to achieve supersonic flow in a homogeneous two-phase medium camera (throat) goes into subsonic through a pressure jump. A detailed description of the working process of a hydrodynamic cavitation device with various cavitation exciteris given in the form of a nozzle and a hydrodynamic lattice. A sufficiently complete algorithm for developing a mathematical model for calculating the transverse dimensions of cavitation devices is given, and practical recommendations are proposed on the selection and calculation of the optimal parameters of the device.***

*Index Terms*: ***cavitation, hydrodynamic device, mathematical model, supersonic flow, sound velocity, Mach number, pressure jump, cavitation number, workflow***

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## I. INTRODUCTION

Improving production efficiency is inevitably associated with the search and development of new high-performance equipment and new technologies. Modern technologies and equipment aimed at obtaining high-quality products must fully meet the requirements of energy and resource conservation, environmental safety, and be competitive in both the domestic and foreign markets. This is possible if they are based on progressive innovative ideas. The scientific search for progressive methods of processing liquid media showed that cavitation technologies, as well as devices implementing these technologies, can be one of the most effective methods [1–4]. The widespread use of cavitation phenomena is due to the need for such technological operations as mixing, dispersing, heating and mass transferring, etc. At present, cavitation technologies provide excellent results for the conversion of gaseous, solid and liquid media. These technologies are used to prepare mixtures resistant to delamination [3, 5], homogeneous solutions, emulsions, suspensions and dispersions from various products, to activate enzymes and accelerate processes by maintaining catalytic reactions [6, 7], for wastewater treatment, and also for water purification in water treatment systems [7–9]. Cavitational impact on the processed environment allows intensifying many technological processes in the metalworking, petroleum, chemical industry, in power engineering, pharmaceuticals, biotechnology and environmental engineering. Cavitation technologies are widely used in the food industry for the preparation of working liquids [1, 10]: preparation of cavitationally activated water for baking and pastry dough, activation of baking yeast, technology for preparing reverse fat-water emulsions [3], cavitation processing of brines for meat products [11] , whole milk and yoghurt [12], water in the water treatment system, etc. [7–9]. Cavitation devices, as a rule, are part of continuous technological systems of production, therefore, if improvement in some of their characteristics is achieved, a significant technical and economic effect can be obtained. Thus, the improvement of existing and development of new advanced technologies and devices based on them is an urgent objective. Information sources provide examples of the most frequent use of acoustic cavitation [13–17]. This phenomenon occurs under the influence of oscillations of ultrasonic frequency and a strictly specified pressure amplitude, the so-called sound pressure, which are forcedly distributed in a liquid medium.

The most common in the ultrasonic range are electroacoustic transducers - piezoelectric and magnetostrictive cavitation generators, which use the direct magnetostrictive and piezoelectric effect in alternating magnetic and electric fields. When an acoustic wave is rarefied in a fluid, cavities are formed that are filled with saturated steam; when compressed by the action of pressure and surface tension, they collapse. At the interface, vapor condensation occurs.

Despite a fairly wide range of excitation frequency converters - from 8 to 44 kHz and above, analysis of the workflow and technical characteristics of ultrasonic reactors shows that their use has several disadvantages, and the potential possibilities of cavitation phenomena in such devices are not fully utilized. Thus, the treatment of the working fluid in an ultrasonic reactor involves careful selection of the parameters of the reactor itself — frequency, external and sound pressure, and the medium to be treated — temperature of the liquid medium, its density, viscosity, surface tension, pressure of saturated vapor of the liquid, and solubility of gas in it. Otherwise, cavitational phenomena may be poorly expressed or not developed at all, since there is some critical value of static pressure, below which, with an increase in static pressure, the efficiency of cavitation increases, and above which decreases, that is, in each case, careful analysis and selection parameters is necessary. Therefore, it seems more convenient to use hydrodynamic cavitation devices.

Cavitation is a complex wave hydrodynamic phenomenon: when the pressure drops to the saturated vapor pressure, vapor or vapor-gas (the pressure drop also leads to the release of previously dissolved gas in the liquid) cavities are formed in the liquid. These cavities quickly appear, and quickly collapse under the action of increased pressure and forces surface tension, and steam condenses at the phase interface. When the bubbles collapse, considerable energy is released. It was established experimentally that when the vapor bubbles collapse, intense cumulative jets form (the speed of cumulative jets is 300 - 500 m/s), which, when encountered, generate a high-frequency oscillatory process and, as a result, sharp point increases in pressure and temperature (up to $4 \cdot 10^3$ MPa and $10^4$ $^0$C respectively) [18]. Externally, the cavitation process resembles the process of bubble boiling. And although the liquid itself does not heat up, it acquires the properties of a boiling liquid. Such a liquid (in particular, water) dissolves salts well, intensively reacts to the hydration reaction of biopolymers of food raw materials (combining them with water molecules), extracts vitamins and minerals from it, without destroying its natural structure. This change in the properties of a liquid occurs as a result of the physicochemical changes in its structure, the mechanism of which “starts” cavitation when water enters a so-called thermodynamically non-equilibrium state. And if the result of “cavitation work” - the formation of a two-phase medium – is used, then under the conditions of a uniform two-phase flow, we can form a supersonic flow passing under the friction of the working chamber into a subsonic one through a pressure jump [1, 2].

The aim of this work is to develop a mathematical model for calculating hydrodynamic cavitation devices of a continuous principle of operation with various cavitation exciterto determine the transverse dimensions of devices, as well as developing practical recommendations for calculating a device with minimal pressure loss and choosing optimal parameters for cavitation and supersonic flow in a homogeneous two-phase medium.

## II. MATERIALS AND METHODS

### Mathematical model

Cavitation causes interesting phenomena in the fluid that can be used in various technologies. But the fact that cavitation creates vapor-gas bubbles and a vapor-gas-liquid mixture is formed, in which the achievement of the speed of sound is quite a real task, since the speed of sound in homogeneous two-phase mixtures is much less than the speed of sound, not only in liquid but also in gas, leads to the idea to supplement the cavitation effect on the flow with the shock effect of a pressure jump during the transition of a supersonic flow into a subsonic flow in the working chamber of the device. It was this idea that lay down in the description of the workflow and the development on its basis of a mathematical model for the calculation of hydrodynamic cavitation devices of a continuous principle of operation. In developing the mathematical model, the fundamental laws of conservation of mass and energy, the basic equations of fluid dynamics, were used. When deriving equations for determining transverse dimensions, the idea was put not only to minimize losses in the device, taking into account the physics of the two-phase flow in the working chamber, but also to recommend effective formation of cavitation. The latter is realized as a result of using the recommended optimal cavitation parameters when developing a mathematical model [1, 2]. The equations characterizing the flow with the drip state of the mixture can be written for the medium as a whole. The region of turbulent vapor-gas-liquid flow can be described using the equation of momentum.

### Workflow and schematic diagrams

## III. RESULTS AND DISCUSSION

The main elements of continuous hydrodynamic cavitation devices are: confuser for preliminary acceleration of the flow; the working chamber (throat), where the working process of the device is carried out, at the beginning of the throat the cavitation exciterare located; adiffuser is located behind the working chamber, in which a partial deceleration of the flow occurs to speeds that are acceptable for the further transportation of the flow in pipelines. cavitation exciterin the flow can be either a nozzle (and then the mixer is called a jet cavitation device), or a hydrodynamic lattice composed of streamlined bodies (a cavitation device with a hydrodynamic lattice). In work [1], a schematic diagram of a jet cavitation device consisting of a supply confuser 1, a nozzle 2, a working chamber (throat) 3 and a diffuser 4 ¬ is shown in Figure 1.

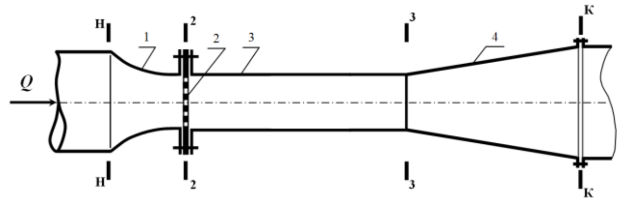


**Figure 1 - Schematic diagram of the jet cavitation device:** 1 - confuser; 2 - nozzle; 3 - working chamber (throat); 4 - diffuser

The stream enters confuser 1, where it is accelerated and pressure is reduced. Next, the flow from confuser 1 enters the nozzle 2. At the exit of the flow from the nozzle device, a sharp drop in pressure occurs up to the pressure of saturated vapor in the jet boundary layer at the nozzle exit. Due to the fact that the flow rate is quite high, there is a predominantly vapor cavitation, since vapor cavitation — an instantaneous expansion of the bubbles — is a rapid process. The pressure drop leads to the transfer of a part of the liquid phase into the vapor phase and the formation, thereby, of a high-speed vapor-liquid flow. The initial heat of vaporization is generated by a nozzle device due to the conversion of part of the mechanical energy into heat during the transformation of the potential energy of the flow into kinetic. In order to intensify this process, it is advisable to distribute the cavitation foci evenly over the normal flow section, and, if possible, to increase their number. It is obvious that for the realization of this aim a multi-jet nozzle with evenly spaced holes, forming several high-speed jets, can be used. The creation of high-speed jets by a nozzle leads to the appearance of cavitation, primarily in the boundary layers. Cavitation occurs in low-pressure centers of turbulent eddies that are formed in the boundary layer. This is due to the fact that between the high-speed jet and a relatively slow moving stream in the boundary layer, there are areas with large shear stresses, which, in turn, leads to a turbulent flow and, as a result, to the pulsation of pressure and velocity. The overlap of the boundary layers of several jets formed by a multi-jet nozzle leads to an additional flow turbulence. The pressure in the eddies cores of the turbulent boundary layer is less than the average pressure. Thus, turbulence can cause instantaneous pressure pulsations at a point and create areas of reduced pressure, which, in turn, creates additional conditions for the development of cavitation.

The pressure drop in the nozzle device also leads to the release of the air that was previously dissolved in stream. Therefore, the cavitation process is close to bubble boiling. Moreover, additional energy for its maintenance is drawn from the flow due to its supercooling. This is evidenced by the experimental data obtained as a result of tests of liquid-gas jet pumps and cavitation mixers [19]. Due to the uniform distribution of cavitation foci over the normal (live) flow cross section at some distance from the nozzle 2, a supersonic vapor-gas-liquid flow forms in the working chamber 3, which then goes into a subsonic in isentropic pressure jump, which ensures intensive fragmentation of liquid additives and their introduction into the carrier medium, thus, at the output section of the chamber 3 fine emulsion is formed. After the working chamber, the flow enters the diffuser 4, where part of the kinetic energy of the flow is converted into potential. The pressure increases to a value less than before the hydrodynamic cavitation device. Obviously, the task of calculation is reduced to determining the dimensions of the cavitation device, in which the achievement of the technological task is carried out with minimal pressure loss ($P_H - P_K$) (see Figure 1) and, consequently, the minimum energy consumption. In this case, both the transverse and longitudinal dimensions of the cavitation device are important, because the choice of transverse dimensions is due to the acceleration of the flow to critical speeds and the formation of a supersonic vapor-liquid flow in the working chamber. The choice of longitudinal dimensions is predetermined by the position of the pressure jump in the flow part of the device. The overestimation of these dimensions leads to an increase in material consumption and additional energy costs, and an understatement leads to inefficient operation of the device. The development of an economical device requires the calculation and comparison of the regime and geometrical parameters of cavitation devices with various cavitationalexciter in the stream.

Traditional designs of hydrodynamic cavitation devices have cavitation exciters in the form of lattice. Lattices are made of cylinders with a smooth or corrugated surface, sometimes in the working chamber a straight ledge is placed - the source of cavitation phenomena. In devices of this type, cavitation foci appear in the vortex wake behind cavitation exciters. A schematic diagram of a cavitation device with a hydrodynamic lattice composed of straight circular cylinders is shown in Figure 2 [1]. The main elements are confuser 1, cavitation lattice 2, working chamber 3 and diffuser 4. The working process in such a device is not fundamentally different from the process in a cavitation device with a multi-jet nozzle, but in this variant cavitation centers do not appear in the jet boundary layer, but in the vortex track behind the cylinders.

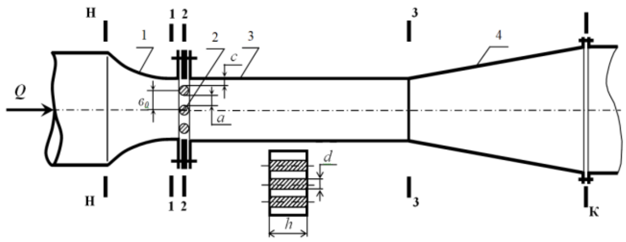


Figure 2 - Diagram of a cavitation device with a hydrodynamic lattice: 1 - confuser; 2 - hydrodynamic lattice; 3 - working chamber (throat); 4 – diffuser

**Mathematical model of the working process of a hydrodynamic cavitation device**

The mathematical model of the working process of a hydrodynamic cavitation device is based on the basic equations of fluid dynamics, the fundamental laws of conservation of mass and energy, and reliable empirical data.

The initial equations describing the workflow in the device are:

- the balance of expenditure equation:

$$\rho_{Ci} V_{Ci} A_i = const; \qquad (1)$$

- D. Bernoulli's equation for the flow with the drip state of the mixture in the area between the normal sections H – H and 2–2 (see Figure 1 and Figure 2):

$$p_H + \frac{\alpha_H \rho_C V_{CH}^2}{2} = p_2 + \frac{\alpha_2 \rho_C V_{C2}^2}{2} + \zeta_{con} \frac{\rho_C V_{C1}^2}{2} + \zeta_{cav} \frac{\rho_C V_{C2}^2}{2}; \quad (2)$$

- the balance equation for the specific energy of the flow with the drip state of the mixture for sections of 3–3 and K – K (see Figure 1 and Figure 2):

$$P_3 + \frac{\alpha_3 \rho_C V_{C3}^2}{2} = P_K + \frac{\alpha_K \rho_C V_{CK}^2}{2} + \zeta_{dif} \frac{\rho_C V_{C3}^2}{2}; \quad (3)$$

-equation of the amount of motion for the control compartment bounded by sections 2–2 and 3–3 and the inner surface of the working chamber:

$$\alpha_3' \rho_{C3} Q_{C3} V_{C3} - \alpha_2' \rho_{C2} Q_{C2} V_{C2} = P_2 \cdot A_2 - P_3 \cdot A_3 - F_{TP}. \quad (4)$$

Here, $F_{TP} = \tau \cdot \Pi \cdot l_{23}$ is the friction force in the section of the channel $l_{23}$ between sections 2–2 and 3–3, we express it through the shear stress on the wall of the channel:

$$F_{TP} = \tau \cdot \Pi \cdot l_{23} \quad (5)$$

In the above equations, the following notation is used: P - perimeter of the channel section, $\tau$ - shear stress; $\rho_C$ - the flux density of the mixture of liquids; $\alpha$ and $\alpha'$ - the coefficients of kinetic energy and momentum; $A_i, V_i, P_i, Q_i$ - the area of the normal section, flow rate, pressure and volume flow in the $i$ section, where $i$ -designation (number) of the normal section on the circuit diagram (see Figure 1 and Figure 2); $\zeta_{con}, \zeta_{dif}$ - coefficients of hydraulic resistance of confuser 1 and diffuser 4; $\zeta_{cav}$ - coefficient of hydraulic resistance of the cavitator 2 (see Figure 1 and Figure 2), in our case - a nozzle or a hydrodynamic lattice. Moreover, in the case of a nozzle device $\zeta_{cav} = \zeta_{noz}$, and in the case of a cavitation lattice $\zeta_{cav} = \zeta_l$, where $\zeta_{noz}$ and $\zeta_l$ - the hydraulic drag coefficients of the nozzle and the lattice, reduced to a velocity head in cross section 2–2.

The subscript "c" means the ratio of the parameter to the entire flow of the mixture; the subscript "n", "k" or a digital index means that it belongs to a specific section in the schematic diagram (see Figure 1 and Figure 2). In the subsequent calculations, we assume $\alpha$ and $\alpha' \cong 1$; we neglect the dynamic pressure $\alpha_H \rho_C V_{CH}^2 / 2$ of the mixture flow in the initial section H – H in equation (2).

To determine the magnitude of the friction voltage in a high-speed vapor-gas-liquid flow, a simple relationship can be used:

$$\tau = \tau_{жc} \cdot Q_l / (Q_g + Q_l), \quad (6)$$

where $Qg$ and $Ql$ - volumetric flow of liquid and gas in the area under consideration; $\tau_{жc}$- shear stress, calculated under the assumption that a single-phase fluid flow flows through the same channel with a volume-average velocity of the mixture $V_m = (Q_g + Q_l) / A$

Expressing shear stresses $\tau_{жc}$ through the coefficient of hydraulic friction $\lambda$ according to the formula:

$$\tau_{жc} = \lambda \cdot \rho_C \cdot V_{C3}^2 / 8, \quad (7)$$

and also the coefficient of hydraulic resistance of the throat $\zeta_{th}$ through $\zeta_{th} = \lambda \cdot l/D$, taking into account the equality of areas of sections 2–2 and 3–3 $A_2 = A_3$, transform equation (4) taking into account expressions (5) - (7) to the following:

$$P_2 - P_3 = \rho_{C3} \cdot V_{C3}^2 - \rho_{C2} \cdot V_{C3} \cdot V_{C2} + (\zeta_Г \cdot \rho_{C3} \cdot V_{C3}^2)/2$$

Adding term by term equation (2) and (3), taking into account equation (8), we arrive at the following equation:

$$P_H + \frac{\rho_C V_{CH}^2}{2} = P_K + \frac{\rho_C V_{CK}^2}{2} + \zeta_{con} \frac{\rho_C V_{C1}^2}{2} + \zeta_{cav} \frac{\rho_C V_{C2}^2}{2} + \zeta_{th} \frac{\rho_C V_{C3}^2}{2} + \zeta_{dif} \frac{\rho_C V_{C3}^2}{2} + \Delta P_{B.T}. \quad (9)$$

In the last equation $\Delta P_{B.T} = \rho_C \frac{(V_{C2} - V_{C3})^2}{2}$ - the loss of specific energy during a sudden deceleration of the flow from the speed $V_{C2}$ to the speed $V_{C3}$, density $\rho_{C2} = \rho_{C3} = \rho_C$.

**To the calculation of the hydrodynamic jet cavitation device**

If a nozzle is used as a cavitation device, then the system of equations (1) - (9) must be supplemented by equations describing the working process in the jet device. These equations are:

- analytical determination of the cavitation number for the jet boundary layer [20]:

$$\sigma = (P_2 - P_{min}) / \frac{\rho_C V_{C2}^2}{2} \quad (10)$$

Here $P_{min}$ - the pressure in the jet boundary layer at the nozzle exit. $P_{min} = P_{s.v}$, where $P_{s.v}$ is the saturated vapor pressure of the liquid additive.

-semi-empirical formula by V.K. Pomnov, which establishes the relationship between the number of cavitation $\sigma$ and the ratio of the areas of normal sections of a jet of fluid behind the nozzle $A_0$ and the working chamber $A_3$ [21]. Let $\Omega = A_0/A_3 = A_2/A_3$– the relative area of the nozzle, then

at $0 < \Omega \le 0,5$ $\sigma = 0,07 + 1,36 \cdot \Omega \cdot (1 - \Omega)$;
at $0,5 < \Omega < 1,0$ $\sigma = 0,41$. (11)

The combination of equations (1) - (11) and a number of transformations, taking into account the equality of the areas of normal flow sections $A_H = A_K u A_1 = A_3$, lead to the expression [1, 2]:

$$\frac{P_H - P_K}{P_H - P_{H.П.}} = \frac{\zeta_{res} + (\zeta_{con} + \zeta_{dif} + \zeta_{th}) \Omega^2 + (1 - \Omega)^2}{1 + \sigma + \zeta_{res} + \zeta_{con} \cdot \Omega^2}, \quad (12)$$

which is basic when calculating a mixer with a multi-jet nozzle. Expression (12) allows, at a known absolute pressure in front of the $P_H$ mixer and selected drag coefficients of the elements of the flow part, to determine the relative area of the nozzle at which the pressure loss in the mixer ($P_H$–$P_K$) will be no higher than a specified value.

Figure 3 shows graphs of the change in relative pressure drop $(P_H-P_K)/(P_H-P_{H.\Pi})$as a function of the relative area of the nozzle Ω, calculated by equation (12) over the entire practical range of variation of the drag coefficient of the working chamber (throat) $\zeta_{th}$ = 0.08 ... 1 and hydraulically perfect profiling of the remaining elements of the flow part of the mixer $\zeta_{con}$= 0,15; $\zeta_{noz}$ = 0,10; $\zeta_{dif}$ = 0,25. Analysis of expression (12) and graphs (see Figure 3) showed that to the minimum relative pressure drop $(P_H-P_K)/(P_H-P_{H.\Pi})$, *and therefore* the minimum range of losses in the mixercorresponds to the following range of changes in the relative area of the nozzle $0{,}45 \le \Omega \le 0{,}7$. Therefore, when calculating hydrodynamic cavitation mixers, it is advisable to choose the geometric characteristics of this particular series [1, 2].

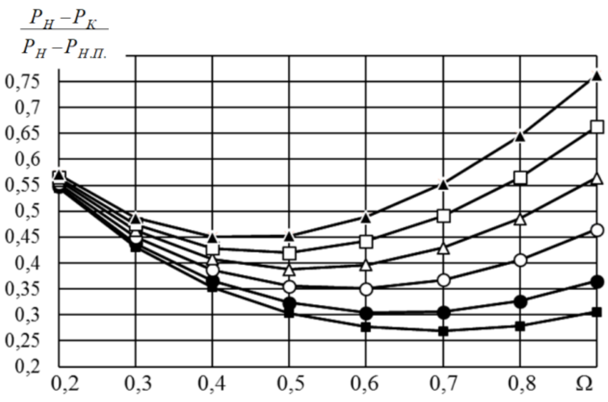


Figure 3 - Dependence of the relative pressure drop on Ω

**To the calculation of a cavitation device with a hydrodynamic lattice**

When calculating a device with a hydrodynamic lattice, we use the following characteristics (see Figure 2):

λ=*l*/*d* – the relative length of the cavitation cavity, where *l* is the visually observed length of the cavitation plume, *d* – the characteristic size of the streamlined body (diameter of the cylinder);

*a*/*d* – the relative distance between the cylinders;

*c*/*d* – the relative distance from the hydrodynamic lattice to the working chamber (throat); *n* – the number of cylinders;

cavitation numbers:

$$\sigma^{'} = (P_2 - P_{\min})/(\rho V_{C2}^2/2) \quad (13)$$

$$\sigma^{''} = (P_\infty - P_{\min})/\rho V_{C1}^2/2 = (P_1 - P_{\min})/(\rho V_{C1}^2/2) \quad (14)$$

Here $P_{\min}$ = $P_{s.v}$, where $P_{s.v}$ is the saturated vapor pressure.

Based on the available publications, we will identify the optimal characteristics of the cavitation flow around the hydrodynamic lattice. The optimal parameters correspond to the maximum development of cavitation and are necessary for orientation towards a reliable calculation of the cavitation device.

Analysis of work [22] made it possible to find the dependence of the optimal value of the cavitation number $\sigma^{'}$ and the relative length of the cavitation cavity λ on the free stream velocity $V_1$ - shown in Figure 4. The analysis data are presented in Table 1.

**Table 1 - Dependence of the number of cavitation · ' on the free stream velocity**

| | | | |
|---|---|---|---|
| $V_1$, m/s | 31,5 | 20 | 10 |
| $\sigma^{'}_{opt}$ | 0,67…0,5 | 0,5…0,4 | 0,33…0,28 |
| $\lambda_{opt}$ | 1,5…2,0 | 2,0…2,5 | 3,0…3,5 |

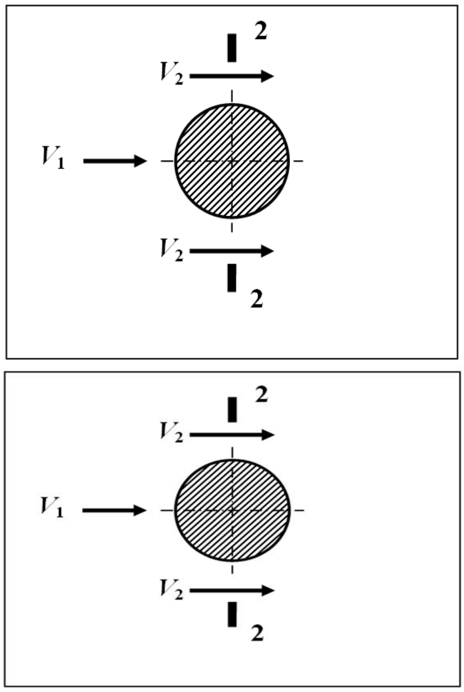


Figure 4 - Flow diagram

The optimal values of the relative distance between the centers of the smooth cylinders of the hydrodynamic lattice with the free stream velocity $V_1$ = 16 m / s and the lattice parameters: n = 3; d = 6, 8 and 10 mm; *h*/*d* = 1.0; *c*/*d*= 0.5:

$$(a/d)_{opt} = 0.5$$

Moreover, as noted in this work, when the free stream velocity of flow is $V_1$ = 15–17 m / s the optimal value of the relative length of the cavitation cavity is $\lambda_{opt}$= 1.5–2.0.

In another paper [23], when the free stream velocity of flow is $V_1$ = 50 m / s and the lattice parameters: n = 3; *d* = 6 and 10 mm the following optimal values of the relative distance between the centers of the cylinders are obtained:

for cylinders with a smooth surface $(a/d)_{opt}$ = 0.4;

for corrugated cylinders

$(a/d)_{opt}$ = 0.8.

In this case, as noted in [23], $\lambda_{opt}$ = 2.5 at $V_2$= 50 m / s.

Data characterizing the dependence of the optimal number of cavitation $\sigma''_{opt}$ on the free stream velocityof flow $V_1$ are presented in table 2.

**Table 2 - Dependence of the number of cavitation $\sigma''$ on the free stream velocity of the flow**

| $V_1$, m/s | 6…15 | 16 | 14 |
|---|---|---|---|
| $\sigma''_{opt}$ | 1,15…1,35 | 1,3 | 1,1 |

When calculating a mixer with a hydrodynamic cavitation lattice, the system of basic equations (1) - (9) must be supplemented by equations that take into account the specific workflow of such a mixer. These equations are the following equations:

-an equation that establishes a relationship between pressure in section 2–2 and the number of cavitation $\sigma'$ of equation (13):

$$P_2=P_{Н.П.}+\sigma'\cdot\frac{\rho_C V_{C2}^2}{2} \qquad (15)$$

-an equation that establishes a relationship between pressure in cross section 1–1 and the number of cavitation $\sigma''$ of equation (14):

$$P_1=P_{Н.П}+\sigma''\frac{\rho_C V_{C1}^2}{2} \qquad (16)$$

-D. Bernoulli's equation for the H – H and 1–1 sections (see Figure 2):

$$P_H=P_1+\frac{\rho_C V_{C1}^2}{2}+\zeta con\frac{\rho_C V_{C1}^2}{2} \qquad (17)$$

Substitute (15) into (2). We take into account that the majority of reference data for the drag coefficient of the hydrodynamic lattice - in our case $\zeta_{cav}$ - are given for the dynamic pressure calculated by the average speed before the obstacle, i.e. before the hydrodynamic lattice or $V_{C1}$, therefore the last term of equation (2) in the case of a device with a hydrodynamic lattice should be written as $\zeta cav\frac{\rho_C V_{C1}^2}{2}$ . With these amendments, and also taking into account equation (1), we obtain the following formula:

$$P_H-P_{Н.П.}=\frac{\rho_C V_{C2}^2}{2}(1+\sigma')+\frac{\rho_C V_{C1}^2}{2}(\zeta_{con}+\zeta_{cav}),$$

or

$$P_H-P_{Н.П.}=\frac{\rho_C V_{C1}^2}{2}\left[\frac{(1+\sigma')}{\Omega^2}+(\zeta_{con}+\zeta_{cav})\right] \qquad (18)$$

Substituting (16) into (17), taking into account equation (1), we arrive at the dependence:

$$P_H=P_{Н.П.}+\frac{\rho_C V_{C1}^2}{2}\sigma''+\frac{\rho_C V_{C1}^2}{2}+\zeta_{con}\frac{\rho_C V_{C1}^2}{2}, \text{ or}$$

$$P_H-P_{Н.П.}=\frac{\rho_C V_{C1}^2}{2}\cdot(1+\sigma''+\zeta_{con}) \qquad (19)$$

Comparison of equations (18) and (19) gives the relationship between the cavitation numbers $\sigma'$ and $\sigma''$:

$$\frac{1+\sigma'}{\Omega^2}+\xi_{cav}=1+\sigma'' \qquad (20)$$

The combination of equations (1) - (9), and also (15) - (20) leads to the following equation:

$$\frac{P_H-P_K}{P_H-P_{Н.П.}}=\frac{(\zeta_{con}+\zeta_{cav}+\zeta_Г+\zeta_{dif}+(1-\Omega)^2/\Omega^2)}{(1+\sigma')/\Omega^2+(\zeta_{con}+\zeta_{cav})}, \qquad (21)$$

which is the main calculation equation for a hydrodynamic lattice cavitation device.

According to equation (21), it is possible at a known absolute pressure in front of the cavitation device PH and the chosen drag coefficients of the elements of the flow path to determine the relative area of the cavitation lattice at which the pressure loss on the cavitation device ($P_H$–$P_K$) will be minimal. Figure 5 shows the graphs of the change in relative pressure drop $(P_H–P_K)/(P_H–P_{Н.П})$ as a function of the relative area of the cavitation lattice Ω, calculated by equation (21) in the practical range of variation of the drag coefficient of the working chamber $\zeta_{th}$ = 0, 08 ... 1, hydraulically perfect profiling of the elements of the flow part of the cavitation device - confuser, diffuser $\zeta_{con}$ = 0,15; $\zeta_{dif}$ = 0,25and for one value of the optimal value of the cavitation number $\sigma'$ recommended by [22]. It is clear that for other recommendations of the optimal values of the cavitation parameters, the nature of the curves takes on other values, but repeats the form shown in Figure 5. The coefficient of resistance of the cavitator$\zeta_{cav}$(hydrodynamic lattice) can be determined for $h/d$ =5 и$a/в_0$≥ 0,5 ( see figure 2) according to the Kirchmer formula:

$$\xi_{cav}=\frac{\Delta P}{\rho_C V_{C1}^2/2}=\beta_1 k_1 \mathrm{Sin}\theta \qquad (22)$$

Here, θ is the angle of inclination of the rod of the hydrodynamic lattice to the flow (to the velocity $V_{C1}$), $\beta_1$=1,73; $k_1=(в_0/a-1)^{4/3}$, the data for calculating the coefficient $k_1$are presented in table 3:

Table 3 - Dependence of $a/в_0$ on the coefficient $k_1$

| $a/в_0$ | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
|---|---|---|---|---|---|---|
| $k_1$ | 1,00 | 0,586351 | 0,324649 | 0,157563 | 0,052744 | 0 |

Table 4 shows the values of the drag coefficients of the hydrodynamic lattice:

**Table 4 - Values of the drag coefficient of the $\zeta_{cav}$ hydrodynamic lattice**

| $a/в_0$ | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
|---|---|---|---|---|---|---|
| $\zeta_{cav}$ | 1,73 | 1,014387 | 0,561642 | 0,272584 | 0,091248 | 0 |

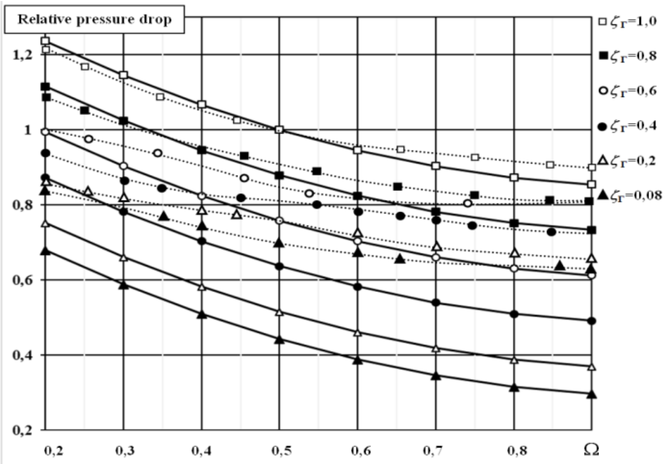


Figure 5 - Dependence of the relative pressure drop on the relative area of the hydrodynamic lattice

Solid lines correspond to the minimum value of the drag coefficient of the hydrodynamic lattice ($\zeta_{cav}$= 0 ... 0.091248), dashed lines - to the maximum value $\zeta_{cav}$= 1.67. For other values of the drag coefficients of the hydrodynamic lattice in the range of 0.091248<$\zeta_{cav}$<1.67, the curves lie in the region between the solid and dashed lines for the corresponding resistance values of the working chamber (throat) $\zeta_{\Gamma}$ - not shown in the figure. It is logical that with an increase in the relative area of the cavitation lattice Ω the relative pressure drop across the device decreases monotonically. This is due to the fact that in the general case the drag coefficient of a hydrodynamic lattice depends on the ratio of the living section or relative area, the shape of the edges of the holes, and the Reynolds number. And with increasing Ω, the total area of the lattice holes increases in relation to the area of the lattice, the losses on the device decrease. We are primarily interested in the development of the working process along the way: preliminary flow acceleration (confuser) → pressure drop to saturated saturated vapor pressure, formation of a supersonic vapor-gas-liquid flow (hydrodynamic lattice) → achievement of a critical state by the flow, realization of pressure jump (device working chamber) → partial transformation of the kinetic energy of the flow in the potential (diffuser). Despite the increased values of the relative pressure drop in a cavitation device with small values of the relative lattice area Ω, cavitation devices with a hydrodynamic lattice calculated for small values of Ω can be recommended for practical use, since in this case the velocity in section is 2–2 (see figure 4) reaches the maximum value, the pressure at the same time drops to the pressure of saturated vapor, cavitation foci appear in the vortex wake behind the cavitation exciter (lattice), supersonic vapor-gas flow is formed and a workflow occurs following the above described algorithm. Moreover, even at a relatively low speed in front of the lattice $V_1$, if the coefficients of the living section of the lattice are small, the flow velocity in its holes (especially in the most compressed section of the streams in the holes) reaches the speed of sound, and the Mach numbers are close to one, respectively, and the drag coefficient of the lattice begins to depend on the Mach number, which is taken into account by the amendment to the influence of the Mach number in formula 22.

## IV. CONCLUSION

The paper describes the workflow and presents the data for the development of a mathematical model for a hydrodynamic cavitation device of a continuous principle of operation with various cavitation excitersin the form of a nozzle and a hydrodynamic lattice. The workflow of this cavitation device takes place in a mode different from traditional hydrodynamic devices [9, 24–26], namely, cavitators not only cause a pressure drop to saturated vapor pressure, at which vapor or vapor-gas cavitiesand the formation of a two-phase flow occur, but forma supersonic flow of a uniform two-phase flow. The developed mathematical model for two cases of cavitators — the nozzle and the hydrodynamic lattice — describes the flow of a supersonic flow, which then passes into a subsonic flow through a pressure jump. It is the transition of the flow through the pressure jump that allows organizing an impact on the flow at a qualitatively different, higher energy level - the cavitation effect on the flow is enhanced by the shock effect of the pressure jump during the transition of a supersonic flow into a subsonic one [1]. The mathematical model of the workflow presented in this paper, based on the material, energy, and heat balance of the flow, gives a fairly complete description of all the equations necessary for calculating the lateral dimensions of hydrodynamic cavitation devices with various cavitation exciters in the form of a nozzle and hydrodynamic lattice. The calculations shown in this paper (in the form of graphs of the relative pressure drop across the device as a function of the relative area of the cavitators) in accordance with the developed mathematical model for two cases of cavitators (nozzle and hydrodynamic lattice) make it possible to work out practical tips for calculating and using such devices — given in relevant subsections. Earlier, in [1], the development of a mathematical model was briefly considered, a mathematical model for calculating the longitudinal dimensions of hydrodynamic cavitation devices of continuous action with minimal power consumption was given. Thus, the mathematical model is closed and complete - it makes it possible to calculate both the transverse and longitudinal dimensions of hydrodynamic cavitation devices with various cavitationexciter. The developed mathematical model for calculating the cavitation effect on the flow, enhanced by the powerful effect of a pressure jump during the transition of a supersonic to subsonic flow, allows calculating devices that can be widely used in technological processes of mixing, homogenization, dispersion and emulsification [27]. In comparison with other methods of cavitation treatment [9, 24–26], this method provides a more powerful effect on the flow and carries out the technological process of mixing, emulsifying, etc., at a qualitatively different level.

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