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Comparision of Transition Curves in Terms of Lateral Acceleration and Lateral Jerk

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Abstract: ***This study describes about the modification of transition curves and their suitability in Highway and Railway alignment. The development of new transition curve is useful in the design and construction of new roads and also in the repair of existing roads. Transition may refer to the change of road segment used i.e., from straight road to curved, or curved to straight or from curved to curved of different radii. After straight alignment if a circular curve is introduced suddenly, it leads to jerks and directly causes discomfort to passengers. Transition curves give smooth and gradual introduction of super elevation which avoids these problems. More over when transitions are not given due to the sudden change of curvature, velocity of the vehicle also changes which results in accidents. For the comfort and safety of passengers these curves are designed in the road geometry. The different types of transition curves are clothoid, cubic parabola, spiral, sinusoidal and bloss curve. Clothoid curves are vastly used for setting out the transition curve, they are easy to set and are further called as ideal curves. In the present study cubic spirals and quadratic spirals are studied and then they are compared to the clothoid curves in terms of lateral acceleration and lateral jerk. The cubic spiral forms the first order of approximation of the spiral in which the curvature is linearly related to the path. In this study the three curves are compared to find the differences and conclusions are given based on the results.***

Keywords: transition curve, clothoids, cubic spiral, quadratic spirals, lateral jerk, superelevation.

I. INTRODUCTION

Horizontal and vertical transition curves are used in the design of highways. Rapid developments in technology resulted in use of high speed vehicles which in turn increased the design speed limit of the basic highways. Road geometry plays an important role in the highway alignment. The road structure also has an equal importance. As known there are two types of pavements flexible and rigid. For laying both the pavements soil stabilization is very important.

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The preliminary studies on the construction area are very important. The soil type of the subgrade is to be considered. For example black cotton soil is less used for the construction of roads and foundations due to their expansive nature and also has high potential for shrinkage. (Shaik 2017). Based on the type of soil many methods are introduced to stabilize the soil based on the need. Some fiber reinforced polymers are also used to stabilize the soil (Thahira 2018). For the rigid pavements to increase the strength of the structure several admixtures are added (Ravindran 2016, VenkatReddy 2018). Design of horizontal transition curve plays a crucial role for the high speed road projects (Walton 2015). Moreover these curves are the route elements for constructing new road projects or for reforming the existing roads. Regular curves observed in highways are circular curves and transition curves. Circular curves have a constant radius and constant curvature which forces the user to reduce the speed. Whereas transition curves gradual change in curvature, this allows the high speed vehicles to travel through the curve without changing the velocity of the vehicle. Moreover this factor also reduces the sudden shock experiences of the user due to centrifugal force (Levent 2011). The simple solution to reduce the centrifugal force is to place a curve which has a gradual curvature change between a curve and an alignment (Pirti 2009). Transition curves reduces the centrifugal force gradually along its length, curvature change always depends directly on the curvature length. (Levent 2011). When transition is designed for a straight line to a circular curve, this transition is used by the drivers to decrease the speed while entering the curve and to increase the speed while leaving to change of vehicle speed abruptly (Gintalas et al. 2008). the curve (Dell'Acqua, Russo 2010). It is noted that many accidents are occurred on the horizontal curves and especially at the beginning and end of the horizontal curves. Improper understanding of the road user in a transition zone causes such kind of accidents due One of such transition curve which is widely used is clothoid curve. This curve has an even change of curvature, even change of lateral acceleration and has low third order parameters. But practically the user may experience some unfavorable stresses. In some transition curves though the third order parameters are continuous they significantly result in higher values. (Levent 2011). According to Crews (2009) for the tangential deflections ranging from 0 to 90 degrees it is suggested to use the clothoid curves because it provides a rational course of curvature. Though the clothoid curves are harder to tabulate and stake the data, they are the most common curves used for horizontal transitions. Noting some complications with the curves some new transition curves are derived putting clothoids as base like cubic spirals, quadratic spirals, bloss curves, sinusoidal and co-sinusoidal curves for the highway. And then all the curves are studied to compare each other with regard to basic design components of a horizontal transitional curve. After studying the parameters for the desired area suitable and comfortable transition curve is designed.

Wenderlein (1968, 1970) proposed a first alternative to the traditional solutions of transition curves i.e., using the general characteristics of transition curves and ensure that there is a continuous change of curvature along the curvilinear transition. This states that general transition curves can create or are used create a viable transition curve according to the requirement. By using general transition curves the curvilinear transitions among rectilinear directions can be drawn using a single equation (Wenderlein 1968, 1970). In these curves, the curvature of the starting point remains same and the end point is zero. Between the start and end points there is only one extreme of curvature (Grabowski 1984) Horizontal Transition curves are used even in designing the railways. As the speed of the rails increases the geometry of the transition curves must be designed more carefully. Many new methods are developed to connect the curves of different curvature with optimal geometry [Long 2010– Kobryn (2014)]. For high speed railway tracks the geometry design is particularly very important for the efficient transition. These are done based on hyperbolic functions, these are continuous and have very close values to the clothoid cuves. The hyperbolic transition curves depends on the strating point of the curve [Megyeri 1998]. For the design of smart cities all the modern design techniques are used to ease the mobility of the users (VenkatReddy 2017). In this paper for a road transition curve, the three curves clothoid, cubic spiral and quadratic spiral are studied using the available formulae for curvature, lateral acceleration and transversal jerk. And then the most suitable curve for the study area is suggested.

II. METHODS

2.1 NORTHING AND EASTING VALUES:

2.1.1 CLOTHOID:

Clothoid curve is also called as Euler spiral, some other names are Spiro's and Cornu spirals. In this curves the curvature changes linearly with its curve length that can be easily stated as the curvature of the circular curve is equal to the reciprocal of its radius. Diffraction computation is the application of Euler's spiral. These curves are widely used in both railway engineering and highway engineering to connect and transit the geometry between the tangent and the circular curve. A simple principle which defines the geometry of the Euler spiral is that the linear variation of the curvature of the transition curve between a tangent and the circular curve (Euler 2000).

- The curvature begins with zero at the straight section and then increases linearly along its length
- The point at which the Euler spiral meets the circular curve, the curvature of the Euler spiral is equal to that of latter.

The formula for the deflection is given as

$$\theta = \frac{l^2}{2RL} + C$$

θ- the deflection angle from the tangent to a point on the spiral

The total X and Y Cartesian coordinates are found using cos and sin functions respectively, with the deflection angle. These values can also be found using the formulae (Calogero 1969)

$$TotalX = L*[1 - \frac{L^2}{40R^2} + \frac{L^4}{3456R^4} - ...]$$

$$TotalY = \frac{L^2}{6R}[1 - \frac{L^2}{56R^2} + \frac{L^4}{7040R^4} - ...]$$

2.1.2 CUBIC SPIRALS:

A cubic spiral is also a simple polynomial cubic. This cubic spiral forms the first order of approximation of the spiral in which the curvature is linearly related to the path which is a clothoid. Hence with this feature this curve is preferred to be used in designing of railways and new modern highways (Levent 2011) . This is a set of trajectories in which their direction functions θ are cubic. The entire cubic spiral has an infinite length or can be called as a continuous spiral. From this spiral the useful part is cut at two inflection points and this curve has a finite length. The curvature function of the cubic spiral at the desired length is given by

$$K(s) = A * s * (l - s)$$

Constant to be determined.

At the inflection point (s = 0 and s = l), the curvature of the spiral becomes zero. The constant A of a cubic spiral joining the separated configurations has a relative angle of α, which is given as

$$\alpha = \theta(l) - \theta(0)$$

Then the curvature can be calculated as

$$K(s) = \frac{6\alpha}{l^3} s(l - s)$$

If the length of the cubic spiral is l, its size is calculated as,

$$D(\alpha) = 2\int_0^{\frac{l}{2}} \cos\left(\alpha\left(\frac{3}{2} - 2t^2\right)\right) t\,dt$$

There is no closed equation to find or represent the size of a cubic spiral. All the cubic spirals are similar, there is a pre-calculated D(α) table, with this we can evaluate the relation of l and d, by α using the following equation.

$$l = \frac{\boldsymbol{d}}{\boldsymbol{D(\alpha)}}$$

As we know that the cubic spiral is a first order of approximation to the clothoids, the following northing and easting values are found.

Let us assume that

$$Sin\theta = \theta$$

Then,

$$\frac{dy}{dl} = sin\theta = \theta = \frac{l^2}{2RL}$$

Integrate both sides

$$\int \frac{dy}{dl} = \int \frac{l^2}{2RL}$$

$$y = \frac{l^3}{6RL}$$

$$\alpha = \frac{\theta}{3} = \frac{l^2}{6R}$$

Relation between parameters:

Most of the parameters of the cubic spiral are similar to clothoid. And the ones which are different are given below. Firstly there is no difference between X and total X values, because we did not assume anything about cosθ.

$$x = \int_0^L \cos(\frac{l^2}{2L^2R^2})dl$$

$$x = l*[1 - \frac{l^4}{40R^2L^2} + \frac{l^8}{3456R^4L^4} - ...]$$

At l = L (full length of transition)

$$TotalX = L*[1 - \frac{L^2}{40R^2} + \frac{L^4}{3456R^4} - ...]$$

$$y = \frac{l^3}{6RL}$$

At l = L (full length of transition)

$$TotalY = \frac{L^2}{6R}$$

$$\boxed{\begin{aligned} \tan\alpha &= \frac{y}{x} \\ \alpha &\cong \frac{\theta}{3} - \delta \end{aligned}}$$ polar deflection angle

2.1.3 QUADRATIC SPIRALS:

The formulation for the quadratic spirals is almost easy as compared to other curves. The only change and important factor to be assumed is the length of the section taken . here

If l > L/2, then

The equation for the quadratic spirals becomes

$$\theta = \frac{(L-2l)^3 + 4l^3}{6RL^2}$$

Differentiating with respect to l we get equation for 1/r, where r is the radius of curvature at any given point.

$$\therefore r = \frac{RL^2}{L^2 - 2(L-l)^2}$$

Else θ is taken as the equation for the quadratic curve

$$\theta = \frac{2l^3}{3RL^2}$$

Sl.no.	Transition Curve	θ	X	Y
1	Clothoid	$\theta = \frac{l^2}{2RL}$	$X = \int_0^L cos\theta dl$	$Y = \int_0^L Sin\theta dl$
2	Cubic spirals	$\theta = \frac{l^3}{6RL}$	$X = \int_0^L cos\theta dl$	$Y = \frac{l^3}{6RL}$
3	Quadratic Spirals	$\theta = \frac{2l^3}{3RL^2}$	$X = \int_0^L cos\theta dl$	$Y = \int_0^L Sin\theta dl$

2.2 LATERAL ACCELERATION

The velocity of a moving object is completely changed by simply changing its speed or by changing its direction. Let a object be moved with non uniform velocity like a train whose velocity is constantly changing. Sometimes velocity increases constantly and after sometime it starts to move with a constant velocity. The change of the velocity at different times is explained on the basis of acceleration **(Levent 2011)**.

When considering a uniform circular motion, the object obviously moves with zero acceleration and constant velocity but the direction is changed. It is to be noted that the direction of the velocity is in tangential direction(Long 2010). The relation between the exerted force and the direction of acceleration is in the direction of force, this is stated in the Newton's second law. Generally in a uniform circular motion, the direction of acceleration is towards the center of the circle which is called as centripetal force or in other words as Lateral acceleration. . The law states that the lateral acceleration completely depends on the radius of the circular path and the speed of the vehicle. And this is essential for a uniform circular motion. When the tangential acceleration is in the direction of tangent of the circle then the lateral acceleration is directed towards the radius of the circle. When centripetal acceleration is discussed as a mathematical expression we get some better idea. A lateral acceleration as discussed earlier is obtained only when an object moves on a curve. To firstly understand the concept let us assume the curve here as a circle. In the case of a circular motion the velocity occurred is called as angular velocity (Gaetano 2011).. Angular velocity is measured as the angle covered by the object per unit time. It is denoted by the Greek letter ω. And the formula can be given as

$$\omega = \theta t$$

Where θ is the angle rotated in the time t

When the overall motion is circular, at a certain instant the object may have a linear velocity *v* in the direction i.e, tangent to the circle of radius *r*. And let *i* be the distance covered by the object. Then θ is given as

θ = *ir*

hence ω is given as, ω = (*ir*)**t*

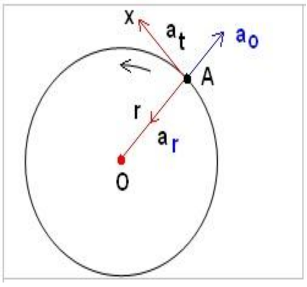
= (*i***t*)**r*

ω = *V r*

an angular acceleration is defined as the rate of change of angular velocity with respect to time. Angular acceleration is denoted as α.

$$\omega = \theta * dt$$
$$\alpha = d\omega * dt$$
$$= (\theta * dt) * (dt)$$
$$\omega = \theta * dt^2$$

The above formula is used to find the gradual change lateral acceleration of the transition curve Generally, the object in circular motion is subjected to two types of acceleration.



Here let an object is tied at A, one end of the string is rotated by keeping the other end as center. When the string is rotated at a certain speed mostly fast, the string completely gets stressed out and the length of the string becomes the radius of rotation. Firstly a force is exerted on the object from the center to the end and here acceleration is faced by the object and to counteract this force a tension force is developed in the opposite direction (Ergin 2005). This tension force is called as the centripetal force and further the acceleration developed here is called as the lateral acceleration. Hence the formula for lateral acceleration can be given as

$$a_r = \frac{v^2}{r}$$

v – the velocity of the object in the curve

r – radius of the curve.

In the transition curves as the radius changes the lateral acceleration also changes accordingly. And it is based on the type of the transition adopted.

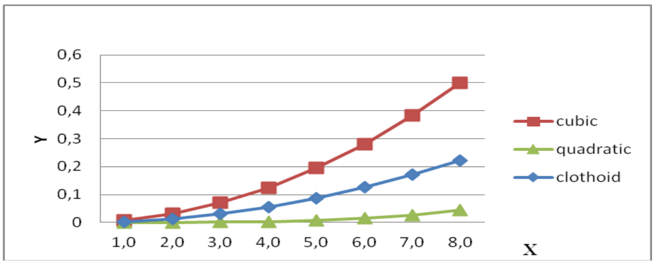
2.3 LATERAL JERKS

In the case of transition curves, the criteria to evaluate the performance and the alignment geometry in terms of comfort are JERK. On the horizontal curve the radial acceleration formed by the centrifugal force directly affects the road safety and directly reduces the travel comfort of the vehicle. Jerk is defined as the radial acceleration changing per unit time occurring in horizontal curves (Lajos 2013). At the time of designing the curve we take jerk criteria for road safety and comfort which are very important (Gaetano 2011). Jerk is the third derivative of position. The second derivative of position is the velocity, finally we can say that position divided by time is velocity and velocity divided by time is acceleration and finally acceleration divided by time is Jerk (Kenjiro 1999).

III. RESULTS AND DISCUSSION:

3.1 NORTHING AND EASTING:

Finding the length of the transition curve is the first step to design the geometry. Here the length of the curve is first calculated using the available formulae, based on super elevation, speed and road widening, and then length of the curve is fixed. After calculating the length it must be divided into 'n' equal parts and then the values of X and Y are calculated. This is the general method which we follow for setting the curve. Here in this study we divided the length into 8 equal parts and then the values of curvature are calculated. A graph is drawn in the Cartesian plane to check the differences among the curvatures of three different curves.

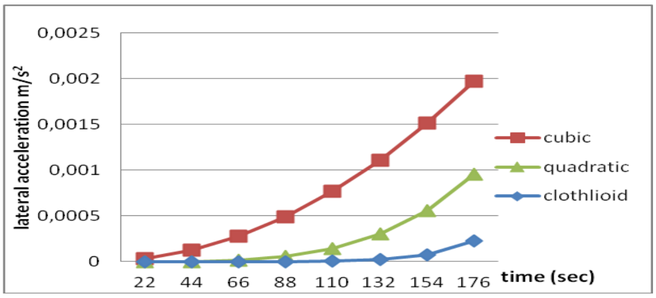


Graph 1: Northing and Easting values of transition curves

The northing values have a linear change in curvature. Cubic spiral is observed to have high northing value and is observed to be suitable when there is a transition required between two curves of different radii. Since the northing values tend to form a circular curve after the transition. Clothoid curve is observed to be neutral. The quadratic spiral is having very less northing values. And it is observed that these curves can be used when connecting a circular curve with a straight line in a very less distance.

3.2 LATERAL ACCELERATION:

Lateral acceleration is calculated along the total length of the transition curve. Acceleration is time and speed dependent. Hence the velocity of the vehicles using the curves is always an important factor to be considered. The design speed of the road is taken and then using the angular variation, lateral acceleration is calculated. Then a graph is plotted between time on X-axis and obtained lateral acceleration on Y-axis.



Graph 2: Lateral acceleration of transition curves

The obtained graphs are very linear throughout the curve length. Clothoid curve have very low changes in lateral acceleration. This denotes that the vehicle can easily move along the transition without much change in speed. Quadratic spirals are also observed to have less change in acceleration but compared to the clothoids the accelerations are equal up to half the length of the transition curve. Later the acceleration values gradually increased which states that the user may not change the speed of the vehicle suddenly as the transition starts. In the case of cubic spirals there is a huge change in acceleration throughout the length of the curve. It constantly increased along with the time.

3.3 LATERAL JERK

Jerk factor always affects the travelers experience on the transition curve. Here the user must be very alert for the safe journey. Jerk directly depends on the acceleration of the vehicle and the time taken to cross the transition curve.

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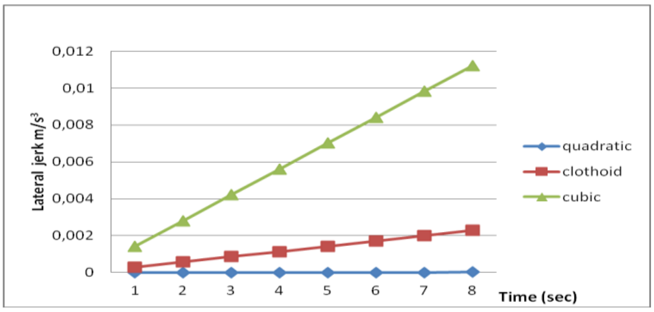

Graph 3: Lateral jerk of transition curves

Though the lateral acceleration of the clothoids curve is less, the value of jerk is high. This may be because of the time taken by the vehicle to travel along the transition curve. The cubic spirals have high values of lateral jerks. They linearly increase with time. The quadratic spirals have very less values of jerks compared to the others and we can say zero value of jerk is obtained. Hence the user may not feel any discomfort throughout the length of the transition curve. And the rate of accidents can be reduced

IV. CONCLUSIONS:

The computation of the clothoid curves is easy when compared to other curves.
Clothoid curves which are widely used in road geometry have very less changes in the lateral acceleration through the length of the curve. Though the lateral acceleration of clothoids curves are less, the lateral jerks are more in the clothoids curves. Cubic spirals always have high northing values, lateral acceleration and lateral jerks which is not recommend.
Quadratic spirals are easy to compute than the cubic spirals. Since the lateral accelerations and lateral jerks are very less, hence this curve is considered to be better than the clothoids and cubic spirals.

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