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GPS Signal in Space Error Analysis and Positioning Performance using IWO for Southern Region of Indian Subcontinent

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***Abstract:** To determine the position of user, Global Positioning System (GPS) uses the signals transmitted from the satellites. The positioning accuracy of global Positioning System depends on a variety of influence factors. The GPS system performance is assessed with reference to carrier-to-noise density ratio (C/N_0), pseudorange, satellite orbit and satellite clock errors. User navigation error depends on DOP factor, user range and equipment errors. User range errors are also called as Signal in Space Ranging Errors (SISRE). Ephemeris error decided by the broadcast ephemeris data uploaded by the control station. User range error (URE) is widely used to measure the effects of satellite orbit error and clock error on user positioning. Position accuracy of each satellite, geometry of the satellites and measurement of pseudorange are also important factors for determining the accurate position of user.*

I. INTRODUCTION

The Global Positioning System [1] (GPS) is a widely utilized space-based global navigation system. Trilateration is the working principle of GPS, in which to determine exact position it requires minimum of four satellites. It makes utilization of an arrangement of pseudorange estimations that have simultaneously been obtained at a given epoch for all tracked GPS satellites. Control Segment of GPS is used to generate navigation information. Positioning and timing as provided by basic GPS services is a single point positioning (SPP) process. Position accuracy affected by the errors, are divided into signal in space and user equipment errors. User equipment errors are the errors except SISRE. SISRE can be noted as measure of the position accuracy. User range errors are the combination of both satellite orbit[4] and clock errors. SIS errors are due to errors contributed by both Space and control segments, such as errors occurred during estimation and prediction,uncertainty of satellite acceleration, instability of clock, variation in satellite antenna, and imperfections in a signal. UEE is purely based on design of the receiver and the environmental conditions in which a receiver is placed. Ephemeris errors originated whenever navigation data does not consists of correct location of the satellite. User navigation errors obtained from the components of radial, tangential, cross track and clock errors. In general radial errors are smaller than tangential and cross track errors. User equipment error including the ionospheric, tropospheric, multipath, group delay and noise and interference. By using various error correction models tropospheric and ionospheric errors can be corrected. With differential receivers pseudorange corrections can be made.

The user navigation error can be represented by using equation(1)

$$UNE(1\sigma) = GDOP\sqrt{SISRE^2 + UEE^2} \quad (1)$$

Where GDOP = Geometric Dilution of Precision

UERE= UE range error

SISRE is the root mean square value of many signal in space range error values.

2.Signal in Space Range Errors

SIS errors[3] are difficult to describe because they are neither purely stochastic nor purely deterministic. The differences between broadcast and precise clocks are the clock errors, denoted by T in meters. The cross-track error depends on Ascending node's inclination and longitude. GPS SIS URE is the pseudorange inaccuracy due to ephemeris and clock errors. For an arbitrary set of ephemeris and clock errors (R, A,C, T), GPS receivers at different locations on the Earth may experience different SIS UREs[5].

$$SISRE = \sqrt{(R - C_{err})^2 + \left(\frac{1}{49}\right)(A^2 + C^2)} \quad (2)$$

Where

R = radial ephemeris error

A = along track ephemeris error

C = crosstrack ephemeris error

C_{err} = Satellite clock error

Due to the lack of precise clock data, in this paper the orbit-error- global-average rms URE is considered. SISRE of each satellite can be obtained from along track, cross track and radial perturbations at any epoch**.** Below figures (a & b) shows the skyplot with elevation mask 10^o and list of visible satellites.

3D RMS error[2] based on radial ,along intrack and cross track errors is given as equation (3)

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$$RMS_{3D} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(dA_i^2 + dC_i^2 + dR_i^2)} \quad (3)$$

n= Number of epochs for satellite K

dAi = broadcast orbit position error in the along-track direction .

dCi = broadcast orbit position error in the cross-track direction .

dRi = broadcast orbit position error in the radial direction .

The RMS SISRE error is given below equation (4)

$$SISRE_{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(SISRE_i)^2} \quad (4)$$

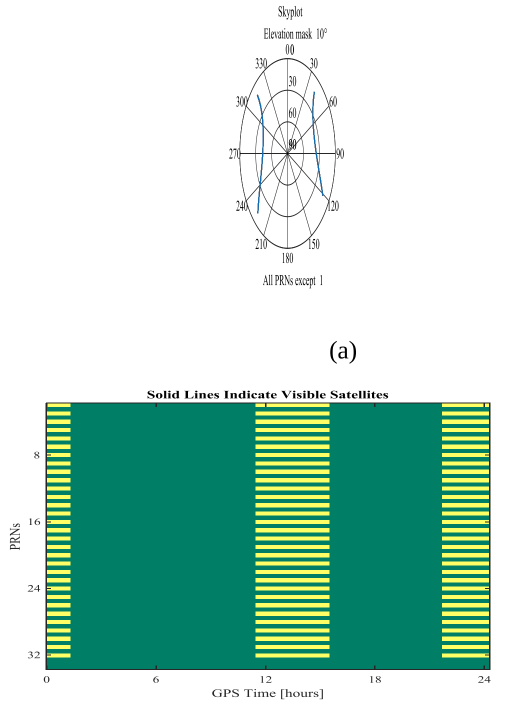
Where n = number of epochs in day

i = epoch

3.GDOP

Geometric DOP is used to state how geometry of satellites will influence the position accuracy. Geometric Dilution of Precision Estimate of satellite conditions for a given area and time, the geometry is said to be feeble and the DOP esteem is high; when far separated, the geometry is solid and the DOP esteem is low. A low DOP esteem speaks to a superior positional accuracy because of the more angle separation between the satellites used to ascertain a unit's position. In order to improve the positioning accuracy of GPS , various GDOP based methods have been developed.

The below formula (Eqn.1) gives the relation between position accuracy,range errors and GDOP . GDOP will be obtained from Equation.5



(a)

(b)

Figure 1: a) Sky Plot b) All visible satellites

$$\sigma_f = \sigma_{UERE} * GDOP \quad (5)$$

Where , σ_f = Position Accuracy

σ_{UERE} =Actual range error (UERE)

$$GDOP = \frac{\left[\sigma_{Ed}^2 + \sigma_{Nd}^2 + \sigma_{Ud}^2 + (c.\delta_T)^2\right]^{1/2}}{\sigma_{UERE}} \quad (6)$$

Where σ_{Ed} , σ_{Nd} and σ_{Ud} are standard deviation values in east ,north and up directions respectively. c is the speed of light (299,792,458 m/s), δ_T is standard deviation in time. In general UERE is in the range of 6 m for P-code usage and 12 m for C/A-code usage.

Table1: GDOP rating

GDOP	Rating
1	Ideal
2-4	Excellent
4-6	Good
6-8	Moderate
8-20	Fair
20-50	Poor

The first step in GPS is measurement of pseudorange which is calculated by finding distance between userand satellite. Pseudorange is given as (Eqn.7), (X_{si},Y_{si},Z_{si}) ~ satellite position and (X_U,Y_U,Z_U) ~ user position.

$$P = \sqrt{(X_{si} - X_U)^2 + (Y_{si} - Y_U)^2 + (Z_{si} - Z_U)^2} \quad (7)$$

From the least squares analysis, the actual value can be considered as sum of a modeled value and an error(Eqn.8).

$$P_{Observed} = P_{Model} + Noise \quad (8) = P(x,y,z,\tau) + v$$

The observed value is given in Eqn.8 , difference between the $P_{Observed}$ and $P_{Computed}$ can be represented in matrix form for m satellites as shown in Eqn.9

$$\Delta P = P_{Observed} - P_{Computed}$$

$$\begin{bmatrix} \Delta P^1 \\ \Delta P^2 \\ \Delta P^3 \\ \vdots \\ \Delta P^m \end{bmatrix} = \begin{bmatrix} \frac{\partial P^1}{\partial x} & \frac{\partial P^1}{\partial y} & \frac{\partial P^1}{\partial z} & \frac{\partial P^1}{\partial \tau} \\ \frac{\partial P^2}{\partial x} & \frac{\partial P^2}{\partial y} & \frac{\partial P^2}{\partial z} & \frac{\partial P^2}{\partial \tau} \\ \frac{\partial P^3}{\partial x} & \frac{\partial P^3}{\partial y} & \frac{\partial P^3}{\partial z} & \frac{\partial P^3}{\partial \tau} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P^m}{\partial x} & \frac{\partial P^m}{\partial y} & \frac{\partial P^m}{\partial z} & \frac{\partial P^m}{\partial \tau} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta \tau \end{bmatrix} + \begin{bmatrix} v^1 \\ v^2 \\ v^3 \\ \vdots \\ v^m \end{bmatrix}$$

(9)

The Eqn 6 is often written in terms of matrix symbols(Eqn 10) (b- Residuals, A_d-Design matrix, v -Noise terms) as

$$b = A_d x + v \quad (10)$$

Observation matrix **A** is purely a function of the direction to each of the satellites as observed from the receiver.

$$A_d = \begin{bmatrix} j_1 & k_1 & l_1 & -1 \\ j_2 & k_2 & l_2 & -1 \\ j_3 & k_3 & l_3 & -1 \\ \vdots & \vdots & \vdots & \vdots \\ j_n & k_n & l_n & -1 \end{bmatrix} \quad (11)$$

J_i, $k_i$and l_i are three components of satellite Si. A least squares solution is as follows

$$x = (A_d{}^T A_d)^{-1} A_d{}^T b \quad (12)$$

GDOP is calculated from the design matrix by using formula Eqn (13)

$$GDOP = \sqrt{trace(A_d{}^T A_d)^{-1}} \quad (13)$$

IWO (Invasive Weed Optimization)

IWO is an explorative and efficient nature inspired algorithm developed by Mehrabian and Lucas developed in 2006 based on colonization of invasive weeds. In IWO, the search space consists of uniformly spreader seeds. Parameter initialization and size of search space are important in Invasive Weed Optimization. Weeds represent the feasible solutions of a problem and set of weeds indicates the population. New set of weeds are produced from existing based on fitness value. The generated new weeds are distributed randomly with zero mean over the search area. The weeds which have better fitness values can produce new seeds, others are being discarded. This will be repeated until it reaches the maximum number of iterations.
First need to define solution space with minimum and maximum values and initialization of parameters must be done in a suitable manner. Generated finite number of weeds distributed over the search space randomly. Each weed takes a random position over D dimensional solution space. Each seed's position is represented as initial solution, containing D values for the D variables, of the optimization problem. The fitness value need to be evaluated for each individual called as plant. Before going to decide new set of seeds ranks will be assigned based on their fitness values. Low rank flowering plants will produce less number of seeds and high rank flowering plants are used to produce more number of seeds.

The newly generated weeds range from W_{min} to W_{max}. New seeds distributed over entire solution space by varying standard deviations. The standard deviation at the present instant can be calculated as [Eqn 14]

$$\sigma_p = \frac{(itr_{\max} - itr)^n}{(itr_{\max})^n}(\sigma_{initial} - \sigma_{final}) + \sigma_{final} \quad (14)$$

Where

$itr_{\max}$ =Maximum number of iterations

$\sigma_{initial}$ =initial standard deviation

σ_{final} =final standard deviation

n= Nonlinear modulation index

After having new position values for all seeds, new seeds will grow to flowering plants and ranked together with their parents based on fitness values. Lower ranking plants will be discarded. This process will be repeated until it reaches its best optimized value or maximum number of iterations. Pseudocode for IWO is given below in algorithm 1.

```
Start
        Initialize population of weeds and parameters
        It_C=1
While  It_C<  It_max
        Sequence_weed= Sort(Population of weeds)
        for i=1:N1
        Fitness(i)=Fitness_fun(Sequence_weed)
        end
Best_Fitness= Max(fitness)
Worst_Fitness=min(fitness)
Std_Update= ((It_Max - it)/(It_Max - 1))^MODI * (std_initial - std_final) + std_final
Ratio = (Fitness – Worst_Fitness)/(Best_Fitness – Worst_Fitness);
W = floor(Wmin + (Wmax - Wmin)*ratio);
   for j = 1:W
   Fitness(j)=Fitness(Sequence_new_seed)
  Pop=join(weed,seed)
        end
end
```

Results and Discussion

Table 2: $SISRE_{RMS}$ Values for all satellites **except(1 & 31)**

Based on C/N_0 values signal quality of GPS satellites can be examined. Figure 2 depicts the Carrier to noise ratio over elevations observed by the GPS receiver data collected on 11th march 2011 located at Andhra University, Visakhapatnam. It is observed that C/N0 is the function of satellite elevation angle. The GPS L1 signal has higher C/N0 Values than the GPS

PRN number			$SISRE_{RMS}$		
2	13	24	0.5319	0.2248	0.1494
3	14	25	0.3640	0.1606	0.0492
4	15	26	1.0636	0.0374	0.5467
5	16	27	0.0874	0.1135	0.6661
6	17	28	0.2184	0.5235	0.4342
7	18	29	0.0852	0.2811	0.0816
8	19	30	0.2670	0.1844	0.4656
9	20	31	0.4782	0.1579	0.2647
10	21		0.4030	0.3623	
11	22		0.3207	0.1758	
12	23		0.0838	0.2086	

higher C/N0 values than the GPS L2.

Table 3: SISRE Values for visible satellites at one epoch

PRN Number	SISRE
2	0.6440
5	0.1485
12	0.0934
15	0.0642
18	0.2324
21	0.4365
25	0.0214
26	0.0251
29	0.2105
Mean	0.2085

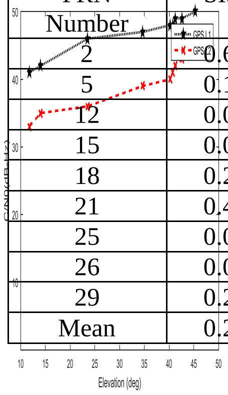


Figure 2: C/N0 values for GPS satellites

The signal in space range error has three important components, which are radial, cross track and along in track errors. Signal in space range errors are calculated for the GPS receiver which is located at (706970.9093, 6035941.0226, and 1930009.5821). The below tables shows the values of these error values. The satellite 15 has low SISRE as 0.0374m and satellite 4 has highest SISRE as 1.0636m during 25th march 2011(Table 2).The mean value at one time instant observed as 0.2085m(Table 3).

These errors can be depicted through below shown figures, how the SISRE values are different for different satellites. And which satellites have less SISRE and High SISRE values(Figure 3). For 24 hours data it is shown that RMS of SISRE of each satellite((Figure 4). Like this,3D error also shown for all satellites except PRN in figure 5.

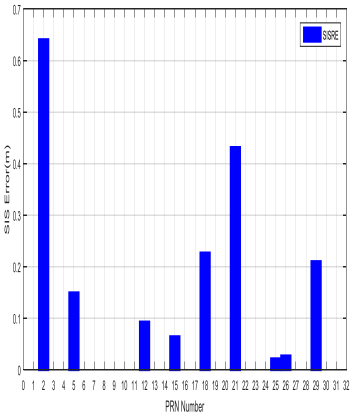


Figure 3: SISRE for visible satellites at one epoch

While observing the SISRE values of various satellites which are visible during time instants on 25th march 2011,even though for one day these values are very less when compared to other errors. But as time increases these errors also effects the accuracy of the position. SISRE depends on Radial, Along In Track, Cross Track and Clock errors.

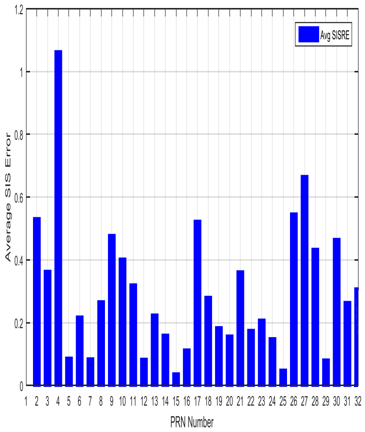


Figure 4: $SISRE_{RMS}$ for all satellites except PRN 1

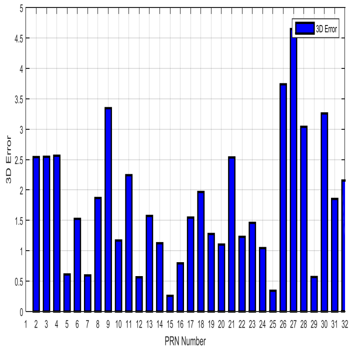


Figure 5: 3D Error for all satellites except PRN 1

Radial,Along In Track and Cross Track errors are as shown in figure 6 for time instants from 451410 to 492390. From the figure it is observed that cross track error range is more when compared to other two errors. SISRE is the one of the error to be considered while determining the accurate position. GDOP also plays vital role in finding accurate position. In this paper to find optimal subset of satellites to give better GDOP invasive weed optimization is used. The optimal subsets along with their GDOP given in below tables. To determine the unknown user position the GPS requires minimum of four satellites. As shown in table 4 for four optimal satellite the GDOP value is 3.0505 and as the user navigation error proportional to the GDOP ,it is very high with four optimal satellites.

If all visible satellites are considered then the GDOP from the table 2.2705 ,as user navigation error is less for this case.

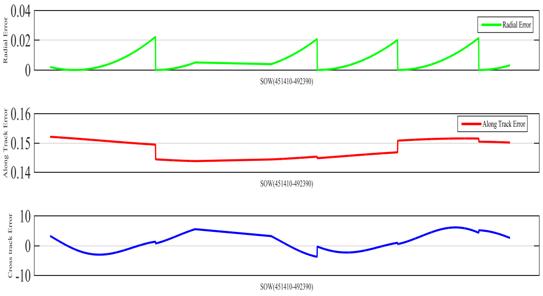


Figure 6: Radial,AlongIntrack and Cross Track Errors

These GDOP values are determined with initial user position (0,0,0).After correcting the user position by using quadratic estimation algorithm, GDOP value are changed as shown in Table 5.With all visible satellites the GDOP is 1.6689.

Table 4: GDOP with Initial user Position (0,0,0)

Number Of Satellites	**GDOP**
4	3.0505
5	2.7688
6	2.5597
7	2.4392
8	2.3235
9	2.2705

Table 5: GDOP with corrected user Position

Number Of Satellites	**GDOP**
4	2.3232
5	2.0906
6	1.9003
7	1.7774
8	1.6987
9	1.6689

The below shown figure 7 depict the fitness value for various subsets of optimal satellites and how they converged. For the subset of seven satellites fitness value converged after fourth iteration.

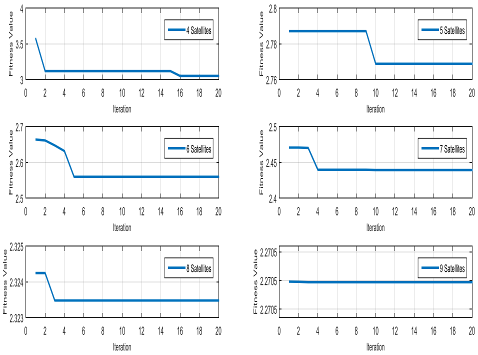


Figure 7: Fitness values of various optimal subsets using Invasive Weed Optimization

Navigation error depends on GDOP value. If four satellites are considered, navigation error is as shown in below table 6. Here for the receiver which is located at (706970.9093, 6035941.0226, 1930009.5821) total error value is considered about 76.8772 m without signal in space error value.

Table 6: User Navigation Error

Number of Satellites	GDOP	User Navigation Error
4 Optimal satellites	2.3232	178.35
7 Optimal satellites	1.7774	136.38
All Visible satellites	1.6689	129.92

CONCLUSIONS

The analysis of signal in space errors is described throughout this paper .Results has shown that SISRE differs significantly for different satellites. SISRE's behavior mainly depends on satellite clock error. This paper discussed about influence of SISRE and as well as GDOP on determination of accurate position. And how minimum GDOP is considered based on optimal satellites using IWO is discussed. Results have shown that all visible satellites combination gave good GDOP value. It is suggested that to reduce position error all visible satellites information need to be considered.

REFERENCES

1. Kaplan E., and Hegarty C. (2006) Understanding GPS: Principles and Applications. Artech House.
2. Warren D., and Raquet J. (2002) Broadcast vs Precise GPS Ephemerides: A Historical Perspective, Proceedings of the 2002 National Technical Meeting of The Institute of Navigation, San Diego, CA, January 2002, p. 733-741.
3. T. Walter, J. Blanch, and P. Enge, "Evaluation of signal in space error bounds to support aviation integrity,"
4. in Proceedings of the 22nd International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS 2009), Savannah, GA, September 2009, pp. 1317–1329.
5. J. C. Cohenour and F. van Graas, "GPS orbit and clock error distributions," NAVIGATION, Journal of the Institute of Navigation, acceptted April 2009.
6. L. Heng, G. X. Gao, T. Walter, and P. Enge, "Statistical characterization of GPS signal-in-space errors," in Proceedings of the 2011 International Technical Meeting of the Institute of Navigation (ION ITM 2011), San
7. Diego, CA, January 2011.