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Parametric Instability and Property Variation Analysis of a Rotating Cantilever FGO Beam

Surya Narayan Padhi, Trilochan Rout, K. S. Raghu Ram

Abstract*: This report is presented on the parametric excitation and dynamic stability of functionally graded ordinary (FGO) rotating cantilever Timoshenko beam. The equation of motion is derived using Finite element method in conjunction with Hamilton's principle. Floquet's theory is used to establish the stability boundaries. It is assumed that the properties along the depth of the FGO material beam follows the power law with different indices as well as exponential distribution law. The elastic property variation using power law at different indices and a comparison of elastic property variation between using power law at n=0.5 and exponential law along the thickness of FGO beam have been investigated. The properties drawn by Exponential distribution confirms better stability compared to properties drawn by power law.

***Index Terms*: Exponential distribution, FGO beam, load factor, Power law, Stability.**

I. INTRODUCTION

Vibrating structures under rotation such as compressors, motors, pumps and micro-electro-mechanical systems is a naturally occurring phenomenon and results severe vibration in a structural resonant mode with an excitation by harmonic loading because of imbalanced rotor or variable fluid dynamic force, which causes heavy mechanical damage. Thus, the understanding of stability and dynamic response of rotating structures in service is highly important to avoid the risk of such resonance problems. In real life, the above mentioned rotating structures are normally pre-twisted and the cross-section is asymmetric in nature. However, Prismatic beams under rotation may be used as a sample model and compared at par with the actual rotating structures for investigation of stability and dynamic response. The research on functionally graded materials (FGMs) is rapidly growing because of its ability to meet desired material properties in contrast to the conventional homogeneous and layered composite materials which suffer from debonding, huge residual stress, locally large plastic deformations etc. An FGM can be a good replacement for the material of rotating beams. The present research work has been carried out with a good amount of literature survey on the rotating beam structures and reported below.

Brown et al. [1] have used the finite element method to study the parametric instability of uniform bars. Eisenberger et al. [2] have presented and compared the two methods for solving the eigenvalue problems of vibrations and stability of a beam on a variable Winkler elastic foundation. Heyliger et al. [3] have studied the influence of in-plane inertia and slenderness ratio on the non-linear frequency for beams with different support conditions. Datta et al. [4] have studied the parametric instability behaviour of a non-prismatic bar with localized zones of damage resting on an elastic foundation by using finite element analysis. Masashi [5] has examined the effects of coordinate system on the accuracy of corotational formulation for planar Bernoulli-Euler's beam. Murin [6] has studied the Cartesian stiffness matrix using methods of differential geometry. Kosmatka [7] developed the linear flexural stiffness, incremental stiffness, mass, and consistent force matrices for a simple two-node Timoshenko beam element based upon Hamilton's principle. Lee [8] has reported on the stability of a rotating cantilever beam using Hamilton's principle and assumed mode method. It is found that a rotating beam is not likely to experience parametric instability when the beam is short and the rotational speed of the beam is large. Dufour and Berlioz [9] have verified their simulated results of the investigation on the stability of an axial loaded beam with periodic force and torque.

Sabuncu et al. [10] have studied the dynamic stability of a blade having asymmetric aerofoil cross-section subjected to lateral parametric excitation using the finite element method considering the effects of the shear coefficient, the beam length, coupling due to the centre of flexure distance from the centroid and rotation on the stability and found that as the length of the beam decreases, the effect of the shear coefficient on stability becomes significant and with an increase in the rotational speed, the blade becomes more stable. Aminbaghai et al. [11] contributed on modelling and simulation of a free vibration of the 2D functionally graded material (FGM) beams with continuous spatial variation of material properties and reported that the continuous variation of the effective elasticity modulus and mass density can be caused by continuous variation of both the volume fraction and material properties of the FGM constituents in the transversal and longitudinal direction.

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***** Correspondence Author
Dr.Surya Narayan Padhi*, Dept.of Mechanical Engg., Koneru Lakshmaiah Education Foundation, Guntur, India.
Dr.Trilochan Rout, Dept.of Mechanical Engg., Parala Maharaja Engineering College, Berhampur,India
Dr. K. S. Raghu Ram, Dept. of Mechanical Engg.,Vignan's Institute of Information Technology, Visakhapatnam, India.

Shafiei et al. [12] have made an exhaustive study on the small scale effect on vibrational behavior of a rotary tapered axially functionally graded (AFG) microbeam on the basis of Timoshenko and Euler-Bernoulli beam and modified couple stress theories using Hamilton's principle to derive the equations for cantilever and propped cantilever boundary conditions and the generalized differential quadrature method (GDQM) to solve the equations.

Shafiei et al. [13] have studied the vibration behavior of the two-dimensional functionally graded nano and microbeams which are made of two kinds of porous materials for the first time, based on Timoshenko beam theory modelling the 2D-FGMs according to the power law. Azimi et al.[14] have done the vibration analysis of rotating, functionally graded Timoshenko nano-beams under an in-plane nonlinear thermal loading using Eringen's nonlocal elasticity theory. Though many researchers have reported on static and dynamic stability of ordinary beams plentily, the literature on dynamic stability of functionally graded rotating beams reported are not enough to the best of the authors' knowledge. In the present article, a functionally graded rotating ordinary beam with fixed-free support condition is considered for dynamic stability analysis.

II. FORMULATION

An FGO beam with alumina as top skin, steel as bottom skin is shown in Fig. 1(a). One end of the beam is clamped and other end kept free. A pulsating axial force P(t) = Ps + P(t)SinΩt , is applied on the beam and acting along its neutral axis. Where Ps is the static component of the axial force, P(t), Ω , and t are respectively the amplitude, frequency and time of the dynamic component of the force. Fig. 1(b) shows the two noded finite element coordinate system used to derive the governing equations of motion. Fig.1(b) shows the expression for the displacements on (x-y) plane (reference plane) at the centre of the longitudinal axis. The thickness coordinate is measured as 'z' from the reference plane. The axial displacement and the transverse displacement of a point on the reference plane are, u and w respectively and ϕ is rotation of cross-sectional plane with respect to the un-deformed configuration. Figure 1(c) shows a two nodded beam finite element having three degrees of freedom per node.

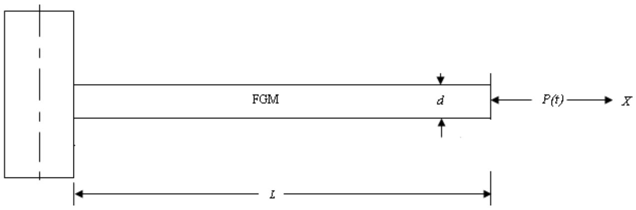

Figure 1(a) Functionally graded ordinary beam subjected to dynamic axial load.

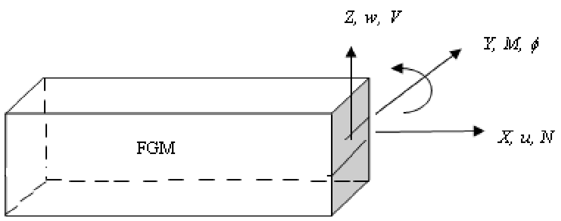

Figure 1(b) The coordinate system with generalized forces and displacements for the FGO beam element.

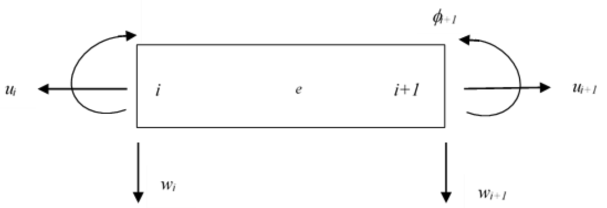

Figure 1(c) Beam element showing generalized degrees of freedom for ith element.

A. Shape Functions

According to the first order Timoshenko beam theory the displacement fields are expressed as

$$U(x,y,z,t) = u(x,t) - z\phi(x,t),$$
$$W(x,y,z,t) = w(x,t), \tag{1}$$

Where U = axial displacement and W = transverse displacement of a material point. The respective linear strains are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial \phi}{\partial x}, \quad \gamma_{xz} = -\phi + \frac{\partial w}{\partial x} \tag{2}$$

The matrix form of stress-strain relation is

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} E(z) & 0 \\ 0 & kG(z) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} \tag{3}$$

Where $\sigma_{xx}, \tau_{xz}, E(z), G(z)$ and k are the normal stress on x-x plane, shear stress in x-z plane, Young's modulus, shear modulus and shear correction factor respectively. The variation of material properties along the thickness of the FGM beam governed by

(i) Exponential law is given by

$$R(z) = R_t \exp(-e(1-2z/h)) \tag{4}$$

$$e = \frac{1}{2}\log\left(\frac{R_t}{R_b}\right), \text{ and}$$

(ii) Power law is given by

$$R(z) = (R_t - R_b)\left(\frac{z}{h} + \frac{1}{2}\right)^n + R_b \tag{5}$$

Where, $R(z)$ can be any one of the material properties such as, E, G and ρ etc., denote the values of The corresponding properties at top and bottommost layer of the beam are represented by R_t and R_b respectively, and the power index is n. The change in the values of E of FGM governed by power law along the thickness with different indices and a comparison between power law and exponential law is shown in Fig. 2(a) and Fig. 2(b) respectively.

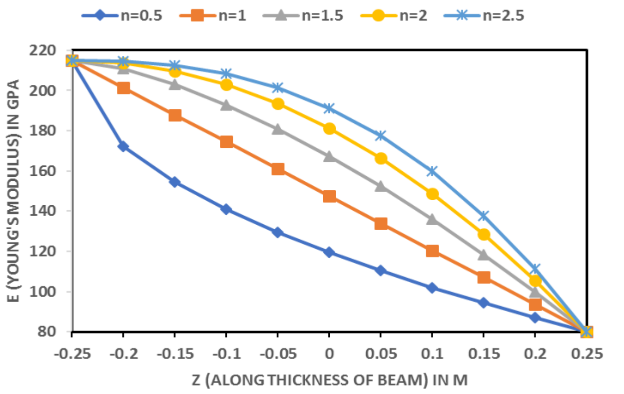

Fig. 2(a) Change in Young's modulus (E) along depth(Z) of FGM beam with steel-rich bottom and alumina-rich top according to power law with various indices.

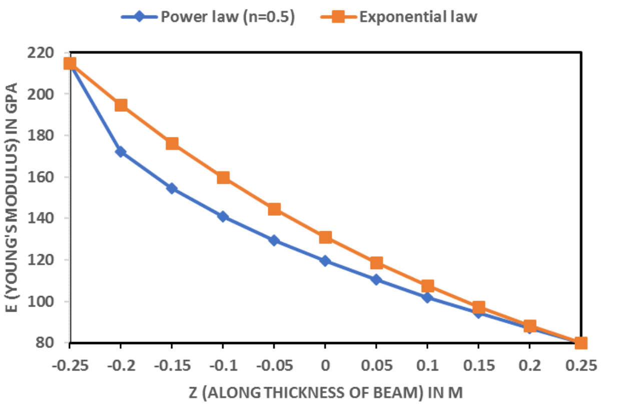

Fig. 2(b) Comparison of change in Young's modulus (E) along depth(Z) of FGM beam with steel-rich bottom and alumina-rich top according to power law at n = 0.5 and exponential law.

Now the shape function can be expressed as

$$\aleph(x) = [\aleph_u(x) \quad \aleph_w(x) \quad \aleph_\phi(x)]^T \qquad (6)$$

where, $\aleph_u(x)$, $\aleph_w(x)$, $\aleph_\phi(x)$ are the shape functions for the axial, transverse and rotational degree of freedom respectively.

B. Element Elastic Stiffness Matrix

The element elastic stiffness matrix is given by the relation

$$[k_e]\{\hat{u}\} = \{F\} \qquad (7)$$

where, $\{F\}$ = nodal load vector and $[k_e]$ = element elastic stiffness matrix.

C. Element Mass Matrix

The element mass matrix is given by

$$T = \frac{1}{2}\{\dot{\hat{u}}\}^T[m]\{\dot{\hat{u}}\} \qquad (8)$$

D. Element Centrifugal Stiffness Matrix

The ith element of the beam is subjected to centrifugal force which can be expressed as

$$F_c = \int_{x_i}^{x_i+l}\int_{-h/2}^{h/2} b\rho(z)\tilde{N}^2(R+x)dzdx \qquad (9)$$

Where x_i = the distance between i^th node and axis of rotation, $\tilde{N}$ and R are the angular velocity and radius of hub. Work due to centrifugal force is

$$W_c = \frac{1}{2}\int_0^l F_c\left(\frac{dw}{dx}\right)^2 dx = \frac{1}{2}\{\hat{u}\}[k_c]\{\hat{u}\} \qquad (10)$$

Where,

$$[k_c] = \int_0^l F_c[\aleph'_w]^T[\aleph'_w]dx \qquad (11)$$

E. Element Geometric Stiffness Matrix

The work done due to axial load P may be written as

$$W_p = \frac{1}{2}\int_0^l P\left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (12)$$

Substituting the value of w from eq. (6) into eq. (12) the work done can be expressed as

$$W_p = \frac{P}{2}\int_0^l \{\hat{u}\}^T[\aleph'_w]^T[\aleph'_w]\{\hat{u}\}dx$$

$$= \frac{P}{2}\{\hat{u}\}[k_g]\{\hat{u}\} \qquad (13)$$

here,

$$[k_g] = \int_0^l [\aleph'_w]^T[\aleph'_w]dx \qquad (14)$$

Where, $[k_g]$ = geometric stiffness matrix of the element.

III. EQUATION OF MOTION

Using Hamilton's principle.

$$\delta\int_{t_1}^{t_2}(T - S + W_p - W_c)dt = 0 \qquad (15)$$

Substituting Eqns (7, 8, 10 and 13) into Eqn (15) and rewritten in Eqn (16)

$$[m]\{\ddot{\hat{u}}\} + [[k_{ef}] - P(t)[k_g]]\{\hat{u}\} = 0 \qquad (16)$$

$$[m]\{\ddot{\hat{u}}\} + [[k_{ef}] - P^{\oplus}(\alpha + \beta_d\cos\Omega t)[k_g]]\{\hat{u}\} = 0 \qquad (17)$$

$$\left[k_{ef}\right] = \left[k_e\right]+\left[k_c\right] \qquad (18)$$

where, $\left[k_e\right]$, $\left[k_c\right]$, $\left[m\right]$ and $\left[k_g\right]$ are elastic stiffness matrix, centrifugal stiffness matrix, mass matrix and geometric stiffness matrix respectively. $\left[k_{ef}\right]$ is the effective stiffness matrix. Assembling the element matrices as used in eq. (17), the equation of motion in global matrix form for the beam, can be expressed as

$$\left[M\right]\left\{\ddot{\hat{U}}\right\}+\left[\left[K_{ef}\right]-P^{\oplus}\left(\alpha+\beta_d\cos\Omega t\right)\left[K_g\right]\right]\left\{\hat{U}\right\}=0 \qquad (19)$$

Where $\left[M\right]$, $\left[K_{ef}\right]$, $\left[K_g\right]$ are global mass, effective stiffness and geometric stiffness matrices respectively and $\left[\hat{U}\right]$ is global displacement vector. Equation (19) represents a system of second order differential equations with periodic coefficients of the Mathieu Hill type. Floquet Theory has been used to distinguish between the dynamic stability and instability zones as follows. A solution with period 2T which is of practical importance is represented by

$$\hat{U}(t)=c_1\sin\frac{\Omega t}{2}+d_1\cos\frac{\Omega t}{2} \qquad (20)$$

Substituting eq. (20) into eq. (19) and solving the boundary solutions with period 2T. The resulting equation is given by

$$\left(\left[K_{ef}\right]-(\alpha\pm\beta_d/2)P^{\oplus}\left[K_g\right]-\frac{\Omega^2}{4}\left[M\right]\right)\left\{\hat{U}\right\}=0 \qquad (21)$$

Equation (21) ends up with an eigenvalue problem with known quantities $P^{\oplus}$, α , β_d . Where $P^{\oplus}$ is the critical buckling load .

The plus and minus sign in the eq. (21) results with two sets of eigenvalues (Ω) binding the regions of instability and can be determined from the solution of the above equation

$$\left|\left[K_{ef}\right]-(\alpha\pm\beta_d/2)P^{\oplus}\left[K_g\right]-\frac{\Omega^2}{4}\left[M\right]\right|=0 \qquad (22)$$

Free Vibration

The eq. (22) can be written for a problem of free vibration by substituting α =0, β_d =0, and $\omega=\frac{\Omega}{2}$

$$\left|\left[K_{ef}\right]-\omega^2\left[M\right]\right|=0 \qquad (23)$$

The values of the natural frequencies $\{\omega\}$ can be obtained by solving eq. (23). The frequency analysis is beyond the scope of this article.

B. Static Stability

The eq. (22) can be written for a problem of static stability by substituting α =1, β_d =0, and $\omega=0$

$$\left|\left[K_{ef}\right]-P^{\oplus}\left[K_g\right]\right|=0 \qquad (24)$$

The values of buckling loads can be obtained by solving eq. (24). Analysis on buckling load is not included in this article.

C. Regions of Instability

ω_1 and $P^{\oplus}$ are calculated from eq. (23) and eq. (24) for an isotropic steel beam with identical geometry and end conditions ignoring the centrifugal force.

Choosing $\Omega=\left(\frac{\Omega}{\omega_1}\right)\omega_1$, eq. (22) can be rewritten as

$$\left|\left[K_{ef}\right]-(\alpha\pm\beta_d/2)P^{\oplus}\left[K_g\right]-\left(\frac{\Omega}{\omega_1}\right)^2\frac{\omega_1^2}{4}\left[M\right]\right|=0 \qquad (25)$$

For fixed values of α , β_d , $P^{\oplus}$, and ω_1, the eq. (25) can be solved for two sets of values of $\left(\frac{\Omega}{\omega_1}\right)$ and a plot between β_d and $\left(\frac{\Omega}{\omega_1}\right)$ can be drawn which will give the zone of dynamic instability.

IV. RESULTS AND DISCUSSION

An FGO rotating cantilever beam of steel-alumina with length 1m and width 100mm is taken for the parametric instability analysis. The beam is steel-rich bottom and alumina-rich top. The mechanical properties of the two phases of the beam are considered as given in the following table.

Table 1. Material properties of Steel-Alumina FGO beam.

Properties of steel	Properties of alumina
Young's modulus E =2.1x10^{11} Pa	Young's modulus E =3.9x10^{11} Pa
Shear modulus G =0.8x10^{11} Pa	Shear modulus G =1.37x10^{11} Pa
Mass density ρ =7.85x10^{3}kg/m^3	Mass density ρ =3.9x10^{3}kg/m^3
Poisson's ratio v is assumed as 0.3,shear correction factor k=(5+v)/(6+v)=0.8667	
Static load factor α =0.1	
Critical buckling load, $P^{\oplus}$ =6.49x10^7 N	
Fundamental natural frequency ω_1 =1253.1 rad/s	

FGO beams with properties along the depth by power law with index n=1.5, n=2.5 and by exponential law have been investigated for dynamic stability. It has been found that stability is increased because of either the instability regions being more and more away from the dynamic load factor axis or decrease in the area of the instability regions. Fig. 3(a) and fig. 3(b) respectively are plotted to depict the influence of property distribution on the instability region for first mode and second mode. It is obvious from the plot that the area of the instability region of FGO beam by exponential law is the smallest among the three and is placed farthest from the dynamic load factor axis. Thus FGO beam by exponential law is found to be the most stable beam for both the modes.

The area of instability region of FGO beam with n=2.5 is the largest for both the modes. Hence it is the least stable beam.

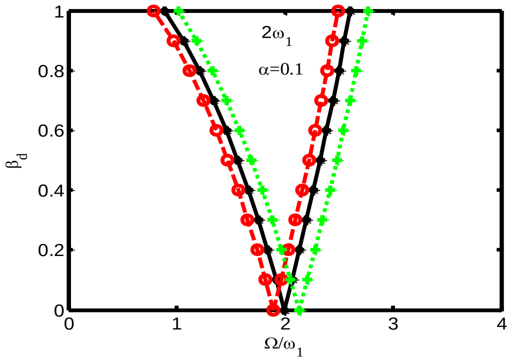

Figure 3(a). Dependence of first mode instability region on property distribution of steel-alumina FGO beam for δ=0.1, s =0.2, ν =1.15 (*n=1.5, °n=2.5, +exp. law)

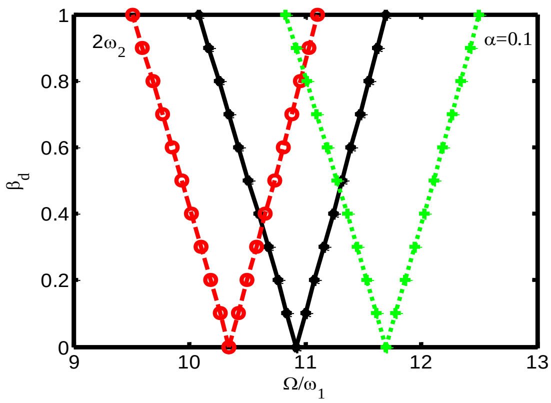

Figure 3(b). Dependence of second mode instability region on property distribution of steel-alumina FGO beam for δ=0.1, s =0.2, ν =1.15 (*n=1.5, °n=2.5, +exp. law)

V.CONCLUSION

Finite element method is used to analyze the dynamic stability of FGO rotating cantilever beams. The variation of material properties along the depth of FGO beam is assumed to be following either exponential law or power law. The dependence of property distribution and parametric instability on beam geometry of the beams is investigated.

Between the Exponential distribution and Power law, Exponential distribution of material properties along the depth of the beam seems to have better dynamic stability than the properties distributed by power law for FGO beams.

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AUTHORS PROFILE

Dr. Surya Narayan Padhi is working as a Professor in the Department of Mechanical Engineering, K L Deemed to be University, Guntur, India. He is a Doctor of Philosophy in Mechanical Engineering from Centurion University of Technology and Management,India. He has completed his M.Tech. in Materials Engineering from National Institute of Technology, Karnataka,India having twenty years of teaching experience in undergraduate and post graduate engineering education. He is a life member of ISTE, Corporate member of Institution of Engineers (India) and member of various professional Institutions. He has already published 20 papers in international journal and conferences. He is also an editorial member of various Journals.He has begged the “Research Ratna” international award for outstanding research on Feb 26th,2019.

Dr. Trilochan Rout is working as an Asst. Professor in the Department of Mechanical Engineering, Parala Maharaja Engineering College, India. He is a Doctor of Philosophy in Mechanical Engineering from National Institute of Technology, Rourkela, India. He has completed his M.Tech. in Machine Design from Jadavpur University,India having eighteen years of teaching experience in undergraduate and post graduate engineering education. He is a life member of ISTE, Corporate member of Institution of Engineers (India) and member of various professional Institutions. He has already published 13 papers in various national and international journals.

Dr. K.S.Raghuram is working as an Asso. Professor in the Department of Mechanical Engineering, Vignan’s Institute of Information Technology, India. He is a Doctor of Philosophy in Mechanical Engineering and M.Tech. in Machine Design from JNTUK, Kakinada, India having eighteen years of teaching experience in undergraduate and post graduate engineering education. He is a member of various professional Institutions. He has already published 15 papers in various national and international journals. He has also authored 3 books on Auto power plant, Basic Mechanical and Electrical Engineering and Auto Transmission System. He has bagged the “Research Ratna” international award for outstanding research on Feb 26th,2019.