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FM / FEk /1 Queuing Model with Erlang Service Under Various Types of Fuzzy Numbers

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Abstract: ***In this paper we tend to initiate a strategy to determine the membership functions of substantial state execution proportions in Erlang service model utilizing α cut and DSW (Dong, Shah & Wong) algorithmic rule with Hexagonal, Heptagonal and octagonal fuzzy numbers. Here the inter-entry rate that is poisson and administration rate that is Erlang are Fuzzy natured, where FEk indicates erlang probability dispersion with k exponential number of stages. The numerical precedents are illustrated to check the achievability of the model, FM/FEk/ 1.A relative investigation comparing to individual fuzzy numbers is additionally accomplished for various estimations of α.***

Keywords: ***Hexagonal, Heptagonal and octagonal fuzzy numbers, Erlang service, execution measures, α - cut, DSW algorithm.***

I. INTRODUCTION

Queue contains minimum of one line or minimum of one administration offices beneath loads of principles. The parameters entry rate (λ) and administration rate (μ) are required to pursue dispersion in lining. Kanufmann in 1975 developed introduction to the idea of fuzzy subsets. George et. al. in 1995 described the fuzzy sets and fuzzy logic. Gross et. al. [4] in1985 proposed some fundamentals of queuing theory. Yovgav [7] in 1986 conferred a characterization of the extension principle. Buckley. J, [1] in 1990 presented the elementary queuing theory supported possibility theory. Fuzzy queuing model was introduced by Lie et. al. [5] in 1989. Negi et. al. [6] in 1992 bestowed analysis of queuing systems. Recently, Chen [3] in 2005 projected a constant nonlinear programming approach to fuzzy queues with bulk, service. He developed FM/FM/1/∞/FCFS, wherever FM denotes fuzzified exponential time supported queuing theory. Here fuzzy service rate is delineate by linguistic terms terribly high, high, low, terribly low and moderate. Chen. S.P. [3] in 2006 bestowed an arithmetic programming approach to the machine interference problem with fuzzyparameters. Srinivasan [9] in 2014 introduced a fuzzy queuing model victimization DSW algorithm program. Vasantha Kumar [14] in 2016 proposed a two phase unreliable M/Ek/1 queuing system with server start up ,N-Policy,

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delayed repair and state dependentarrivalrates.K.Usha Prameela &Pavan Kumar[15] in 2019 approached the DSW algorithm to measure the execution proportions of multi-server queuing model with pentagonal fuzzy number.With regards to conventional lining model, both the parameters, that is, the entomb landing times and administration times are required to pursue bound conveyances.

In this paper we tend to project FM/FEk/1 lining model, wherever FM& FEk suggests fuzzified exponential and Erlang conveyance. In phase 2, some fundamental ideas and definitions are exhibited. In area 3, the documentations and presumptions are portrayed. In area 4, the projected lining model is given. In segment 5, the arrangement way to deal with the present model is depicted. In area 6, three numerical models are tackled. In area 7, correlation of 3 fuzzy numbers is postponed. In area 8 the outcomes and discourses are introduced. In phase 9, the model is finished up.

II. ESSENTIAL IDEAS & DEFINITIONS

Fuzzy Number [8]: A fuzzy set Ã outlined on the set of real numbers R is alleged to be fuzzy number if it has the subsequent characteristics Ã Is normal, convex set & the support of Ã is closed and bounded

α-cut [8]: An α- cut of a fuzzy set is a crisp set Aα that contains all the elements of the universal set X that have a membership grade in A in an exceedingly bigger than or equal to fixed value of α.i.e. $A_\alpha = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1\}$

Hexagonal Fuzzy Number:

$$\tilde{A}(x) = \begin{cases} 0 & \text{for } x<a1 \\ \frac{1}{2}\left(\frac{x-a1}{a2-a1}\right) & \text{for } a1\leq x\leq a2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a2}{a3-a2}\right) & \text{for } a2\leq x\leq a3 \\ 1 & \text{for } a3\leq x\leq a4 \\ 1-\frac{1}{2}\left(\frac{x-a4}{a5-a4}\right) & \text{for } a4\leq x\leq a5 \\ \frac{1}{2}\left(\frac{a6-x}{a6-a5}\right) & \text{for } a5\leq x\leq a6 \\ 0 & \text{for } x>a6 \end{cases}$$

Heptagonal membership function is constructed by

$$\gamma_{hep}(x) = \begin{cases} 0, & \text{for } x < c_1 \\ \frac{1}{2}\left(\frac{x-c1}{c2-c1}\right), & \text{for } c_1 \le x \le c_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-c2}{c3-c2}\right), & \text{for } c_2 \le x \le c_3 \\ 1, & \text{for } c_3 \le x \le c_4 \\ 1 - \frac{1}{4}\left(\frac{x-c4}{c5-c4}\right), & \text{for } c_4 \le x \le c_5 \\ \frac{3}{4} - \frac{1}{2}\left(\frac{x-c5}{c6-c5}\right), & \text{for } c_5 \le x \le c_6 \\ \frac{1}{4}\left(\frac{x-c7}{c6-c7}\right), & \text{for } c_6 \le x \le c_7 \\ 0, & \text{for } x > c_7 \end{cases}$$

Octagonal membership function is constructed by

$$\gamma_{oct}(x) = \begin{cases} 0, & \text{for } x < c_1 \\ k\left(\frac{x-c1}{c2-c1}\right), & \text{for } c_1 \le x \le c_2 \\ k, & \text{for } c_2 \le x \le c_3 \\ k + (1-k)\left(\frac{x-c3}{c4-c3}\right), & \text{for } c_3 \le x \le c_4 \\ 1, & \text{for } c_4 \le x \le c_5 \\ k + (1-k)\left(\frac{c6-x}{c6-c5}\right), & \text{for } c_5 \le x \le c_6 \\ k, & \text{for } c_6 \le x \le c_7 \\ k\left(\frac{c8-x7}{c8-c7}\right), & \text{for } c_7 \le x \le c_8 \\ 0, & \text{for } x > c_8 \end{cases}$$

Interval Analysis for Arithmetic

The two interval numbers designated by ordered pairs of real numbers with lower and higher limits be

$G=[a_1, a_2]$, $a_1 \le a_2$

and $H= [b_1, b_2]$, $b_1 \le b_2$

The math property is signified generally with the symbol *, wherever * = [+, -, ×, ÷] and the activity is characterized by

$G * H= [a_1, a_2] * [b_1, b_2]$ where

$[a_1, a_2] + [b_1, b_2] = [a_1+ b_1, a_2+ b_2]$

$[a_1, a_2] - [b_1, b_2] = [a_1- b_1, a_2- b_2]$

$[a_1, a_2] *[b_1, b_2]$ = [min(a1 b1, a1 b2, a2 b1, a2 b2), max (a1 b1, a1 b2, a2 b1, a2 b2)]

$[a_1, a_2] \div [b_1, b_2] = [a_1, a_2] *[1/b_2, 1/b_1]$ provided that 0 does not belong to $[b_1, b_2]$

$A [a_1, a_2] = [\alpha a_1, \alpha a_2]$ for $\alpha>0$ and $[\alpha a_2, \alpha a_1]$ for $\alpha<0$

III. SUSPICIONS AND INSCRIPTIONS

3.1 Suspicions

(i) We tend to think about the infinite capacity *FM/FEk/1* Queuing Model with one server.

(ii) Inter arrival time follows Poisson distribution and.

(iii) Service (Erlang) time follows Exponential distribution.

(iv) Arrival rate, service rate are fuzzy numbers.

3.2 Inscriptions

In the present model, the subsequent notations are used:

μ= Average number of shoppers being serviced per unit of time.

λ =Average number of shoppers arriving per unit of time.

L_S=The average number of shoppers in the system.

L_q =The Average number of customers waiting in the queue.

W_S =Average waiting time of a client in the system.

W_q = Average waiting time of a client in the queue.

X = set of the inter arrival time.

Y = set of the service time.

A = inter arrival time

S = service times.

FM =fuzzified exponential dispersion

F*Ek=* fuzzified Erlang dispersion

IV. PROPOSED QUEUING MODEL - *FM* / F*Ek*/1 [14]

In this model, the client arrives at one server facility with arrival rate (Poisson) λ and service rate (Erlang service) μ. Here the unit is served in K phases. A brand arrival creates k phases of service and departure of 1 unit reduces K-phases of service. The execution proportions of this queuing model are:

Expected range of consumers within the system

$$L_s = \left(\frac{K+1}{2K}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu}$$

Expected range of consumers within the queue

$$L_q = \left(\frac{K+1}{2K}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$$

Expected time a client spends within the queue

$$W_q = \left(\frac{K+1}{2K}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right)$$

Expected time a client spends within the system

$$W_s = W_q + \frac{1}{\mu}$$

V. RESOLUTION PERSPECTIVE

DSW (Dong, Shah and Wong) is an estimated strategy which utilized the intervals at various α-cut estimations in establishing membership functions. The entry time A and overhauled time S are accompanying by the subsequent fuzzy sets

$\bar{A}$= {(a, μÃ(a)) /a∈ X)};

$\bar{S}$ = {(s, μš(s))/s ∈Y)}

Here, X is that the set of the inter entry time and Y is that the set of the overhauled time. μÃ(a) is membership operate of the inter entry time. μš(s) is membership operate of the overhauled time and their αcuts are expressed as:

$\bar{A}$ (α) = {a ∈ X / μÃ (α) ≥ α};

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$\bar{S}$(α) = {s ∈ Y / µš (s) ≥ α}

The DSW calculation contains the accompanying advances:

Stage 1. Settle for α cut associate incentive from 0 ≤ α ≤1.

Stage 2. The intervals within the input membership functions such as on top of α cut esteems are to be processed.

Stage 3. confirm the interval for the output membership operate for the chosen α cut estimations.

Stage 4. reiterate stages 1-3 for numerous estimations of α to finish the α cut portrayal of the arrangement.

VI. NUMERICAL EXAMPLES

6.1 Hexagonal: Contemplate associate integrated queuing system within which the service consists of 2 phases each arrival and service rate are heptagonal fuzzy numbers denoted by λ = {1, 2, 3, 4, 5, 6}, µ = {11, 12, 13, 14, 15, 16}. The confidence interval at α are [1+α, 6-α] & [11+α, 16-α] with k=2.

Expected range of consumers within the system

$$Ls= \left(\frac{K+1}{2K}\right)\left(\frac{x^2}{y(y-x)}\right)+\frac{x}{y}$$

Expected range of consumers within the queue

$$Lq= \left(\frac{K+1}{2K}\right)\left(\frac{x^2}{y(y-x)}\right)$$

Expected time a client spends within the queue

$$Wq= \left(\frac{K+1}{2K}\right)\left(\frac{x}{y(y-x)}\right)$$

Expected time a client spends within the system

$$Ws= W_q+\frac{1}{y}$$

The obtained results are depicted in the following Tables.

Table 1: The α-cuts of L_s & L_q at α-Value

α	Ls	Lq
0	[0.0656 1.0364]	[0.0031,0.4900]
0.1	[0.0730 0.9350]	[0.0038 0.4523]
0.2	[0.0806 0.9350]	[0.0046 0.4171]
0.3	[0.0884 0.8895]	[0.0056 0.3850]
0.4	[0.0963 0.8469]	[0.0066 0.3557]
0.5	[0.1045 0.8070]	[0.0077 0.3288]
0.6	[0.1129 0.7696]	[0.0090 0.3040]
0.7	[0.1215 0.7343]	[0.0104 0.2813]
0.8	[0.1303 0.7010]	[0.0119 0.2604]
0.9	[0.1394 0.6696]	[0.0135 0.2410]
1	[0.1487 0.6398]	[0.0153 0.232]

Table 2: The α-cuts of W_s&$W_q$at α-Value

α	Ws	Wq
0	[0.0656 0.1727]	[0.0031 0.0818]
0.1	[0.0663 0.1667]	[0.0035 0.0766]
0.2	[0.0671 0.1612]	[0.0039 0.0719]
0.3	[0.0680 0.1560]	[0.0043 0.0675]
0.4	[0.0688 0.1512]	[0.0047 0.0635]
0.5	[0.0697 0.1467]	[0.0051 0.0597]
0.6	[0.0705 0.1425]	[0.0056 0.0530]
0.7	[0.0714 0.1385]	[0.0061 0.0530]
0.8	[0.0724 0.1348]	[0.0066 0.0500]
0.9	[0.0733 0.1313]	[0.0071 0.0472]
1	[0.0743 0.1279]	[0.0076 0.0446]

6.2 Heptagonal: Consider an integrated system within which the service consists of 2 phases both arrival and service rate are heptagonal fuzzy numbers denoted by λ = {1,2,3,4,5,6,7}, µ = {11,12,13,14,15,16,17}. The confidence intervals at α are [1+α, 7-α] & [11+α, 17-α] with k=2.

Table 3: The α-cuts of L_q&W_q at α- values

α	Lq	Wq
0	[0.0029,0.0960]	[0.0002,0.0087]
0.1	[0.0038,0.0844]	[0.0002,0.0076]
0.2	[0.0049,0.0740]	[0.0003,0.0066]
0.3	[0.0062,0.0648]	[0.0004,0.0057]
0.4	[0.0077,0.0565]	[0.0006,0.0049]
0.5	[0.0095,0.0492]	[0.0007,0.0042]
0.6	[0.0115,0.0427]	[0.0009,0.0036]
0.7	[0.0139,0.0369]	[0.0011,0.0031]
0.8	[0.0166,0.0318]	[0.0013,0.0026]
0.9	[0.0197,0.0272]	[0.0016,0.0022]
1	[0.0232,0.0232]	[0.0019,0.0019]

Table 4: The α-cuts of L_s&W_s at α- values

α	Ls	Ws
0	[0.0648,0.3841]	[0.0049,0.0349]
0.1	[0.0742,0.3592]	[0.0057,0.0323]
0.2	[0.0842,0.3357]	[0.0065,0.0299]
0.3	[0.0947,0.3134]	[0.0074,0.0277]
0.4	[0.1058,0.2923]	[0.0084,0.0256]
0.5	[0.1175,0.2723]	[0.0094,0.0236]
0.6	[0.1298,0.2533]	[0.0104,0.0218]
0.7	[0.1426,0.2352]	[0.0116,0.0201]
0.8	[0.1562,0.2180]	[0.0128,0.0184]
0.9	[0.1704,0.2016]	[0.0140,0.0169]
1	[0.1853,0.1860]	[0.0154,0.0155

6.3 Octagonal: Consider an integrated system within which the service consists of 2 phases both arrival and service rate are octagonal fuzzy numbers denoted by
λ= {1,2,3,4,5,6,7,8}
µ= {11,12,13,14,15,16,17,18}.
The confidence interval at α are [1+α,8-α] and [11+α,18-α]

Table 5: the α cuts of Lq & Wq at α-Values

α	Lq	Wq
0	[0.0025,1.4545]	[0.0025,0.1818]
0.1	[0.0037,1.1983]	[0.0027,0.1668]
0.2	[0.0037,1.1983]	[0.0030,0.1536]
0.3	[0.0044,1.0931]	[0.0034,0.1420]
0.4	[0.0052,1.000]	[0.0037,0.1316]
0.5	[0.0060,0.9171]	[0.0040,0.1223]
0.6	[0.0070,0.8430]	[0.0044,0.1139]

0.7	[0.0080,0.7764]	[0.0047,0.1064]
0.8	[0.0092,0.7163]	[0.0051,0.0995]
0.9	[0.0104,0.6619]	[0.0055,0.0932]
1	[0.0118,0.6125]	0.0059,0.0875]

Table 6: The α cuts of Ls & Ws at α-Values

α	Ls	Ws
0	[0.0580,0.2102]	[0.0616,0.2102]
0.1	[0.0645,2.0295]	[0.0623,0.2011]
0.2	[0.0711,1.8947]	[0.0630,0.1928]
0.3	[0.0778,1.7745]	[0.0637,0.1852]
0.4	[0.0847,1.6667]	[0.0644,0.1782]
0.5	[0.0917,1.5693]	[0.0652,0.1717]
0.6	[0.0989,1.4809]	[0.0659,0.1658]
0.7	[0.1063,1.4000]	[0.0667,0.1603]
0.8	[0.1138,1.3265]	[0.0675,0.1551]
0.9	[0.1215,1.2585]	[0.0683,0.1503]
1	[0.1294,1.1958]	[0.0692,0.1458]

VII. COMPARISION OF HEXAGONAL, HEPTAGONAL & OCTAGONAL FUZZY NUMBERS AT α-CUT VALUES

In this section, we compare the values of performance measures for hexagonal, heptagonal & octagonal fuzzynumbers, corresponding to various values of α-cut, in Table 7.

Table 7: Comparison of Performance Measures Corresponding to Various Values of α-cuts

α		0	0.5	1.0
Ls	Hexa	[0.0656,1.0364]	[0.1045,0.8070]	[0.1487,0.6398]
	Hepta	[0.**0648,0.3841]**	**[0.1175,0.2723]**	**[0.1853,0.1860]**
	Octa	[0.0580,0.2102]	[0.0917,1.5693]	[0.1294,1.1958]
Lq	Hexa	[0.0031,0.4900]	[0.0077,0.3288]	[0.0153, 0.232]
	Hepta	[0.0029,0.0960]	[0.0095,0.0492]	[0.0232,0.0232]
	Octa	[0.0025,1.4545]	[0.0060,0.9171]	[0.0118,0.6125]
Ws	Hexa	[0.0656,0.1727]	[0.0697,0.1467]	[0.0743,0.1279]
	Hepta	[0.00490.0349]	[0.0094,0.0236]	[0.0154,0.0155]
	Octa	[0.0616,0.2102]	[0.0652,0.1717]	[0.0692,0.1458]
Wq	Hexa	[0.0031,0.0818]	[0.0051,0.0597]	[0.0076,0.0446]
	Hepta	[0.0002,0.007]	[0.00070.0042]	[0.0019,0.0019]
	Octa	[0.0025,0.1818]	[0.0040,0.1223]	[0.0059,0.0875]

VIII. RESULTS & DISCUSSION

Utilizing MATLAB computer code package, we tend to succeed α-cuts of entry rate, overhauled rate and fuzzy anticipated variety of employments in queue at eleven variation levels: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for fuzzy expected variety of employments in queue at various probability α levels are represented in table1-3. The execution estimates for example, anticipated variety of purchasers within the system (Ls), anticipated length of line (Lq), the average holding up time within the system (Ws) and the average holding up time of clients in the line (Wq) likewise inferred in table 1-3.

As an example, while these four execution proportions are fuzzy, the foremost probability price of the expected queue length *Lq* falls between 0.118 and 0.6125 and its value is impossible to fall outside the range of 0.0025 and 1.4545 equally the expected length of the system falls between 0.1294 and 1.1958 and won't fall outside the vary of [0.0580,0.2102] just in case of octogonal.in the same approach in hexagonal& heptagonal also conjointly the vary comes. The above data will be very suitable for designing a queueing system.

IX. CONCLUSION

In this paper, FM/*FEk* /1 queuing model is studied utilizing alpha cut methodology. The inter arrival and service time are fuzzy natured. The performance of this technique additionally fuzzy natured. Numerical example shows the potency of this technique. It is seen that the accomplishment of lining model can be overhauled by growing the quantity of factors. The planned model will assist the industries, wholesalers and retailers in accurately determinative the optimum performance measures of the queuing system. There are numerous aspects that the paper can be extended. One of them is to think about variable random variable, or fuzzy random variable to arrival rate and service rate. Another potential dimensional to extend this paper is to think about intuitionistic fuzzy numbers.

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