On Le- Ternary Semi groups-II

C. Sreemannarayana, D. Madhusudhana Rao, P. Sivaprasad, M. Sajani Lavanya, K. Anuradha

Abstract: The ternary semi group of all bi-ties (K) of le-ternary semi group K is a band if K is both regular as well as intraregular. Here, we show (K) is a band iff it is normal band. Our main aim to prove (1) Suppose K be an le-ternary semi group. Then (K) (K(K)) is a band. Moreover, K is regular, then (K) = (K)(K) (K). (2) Let K is a regular le-ternary semi group. Then (K), (K), & (K) are bands. (3) An le-ternary semi group K is regular as well as intraregular iff (K) is a band (4) Suppose that K be an le-ternary semi group. Then K is both regular as well as intraregular iff (K) is a normal band. (5) An le-ternary semi group K is left duo iff (K) ≤ (K) ≤ (K) for all k, l ∈ K (6) An le-ternary semi group K is duo iff (K) = (K) = (K) for all k, l ∈ K. (7) An le-ternary semi group K is regular left duo iff (K) is a left normal band. (8) An le-ternary semi group K is regular right duo if (K) is a right normal band.

Mathematical Subject Classification: 06F99, 06F05, 20M10.

Index Terms: duo, regular, bi-ideal element, intra-regular, locally testable, normal band, and regular

I. INTRODUCTION

In commutative rings the ideal theory was observed by W. Kurtil. In 1952 the structure of the le-semi group introduced by R.A. Good and D.R. Hughes. Here, we characterize the le-ternary semi group K such that (K) is in each of the sub-varieties of normal bands.

II. PRELIMINARIES

Definition 2.1: An le-ternary semi group M is a structure (M, [ ], Λ, V, e) ⊆ (M, [ ]) is ternary semi group, (M, Λ, V, e) is a lattice with e which is known as greatest element and for all p, q, r ∈ M, [pq(r vs)] = [pq][r vsq], [ps][r vsq] = [psq][r vsq] and [r vsq] = [r v s]q. In this paper M will stand for le-ternary semi group and tie means t-ideal element, unless otherwise stated.

Example 2.3: Let M be a po-ternary-semi group. Let M be the set of all ideals of M. Then (M, ⊆, U, ∩) is le-ternary-semi group.

The relation ≤ on the set M is defined by for p, q, r ∈ M, p ≤ q if p V q = q. Here the ternary operation is distributive over the lattice join, then ≤ is compatible with the ternary operation in M, i.e., ∀ p, q, r, s ∈ M, p ≤ q ⇒ rsp ≤ rsq, rps ≤ rsq and prs ≤ qrs.

Revised Manuscript Received on December 22, 2018.

C. Sreemannarayana, Research Scholar, Department of Mathematics, KL University, Vaddeswaram, Guntur(Dt.), A.P. India.

D. Madhusudhana Rao, Department of Mathematics, VSR & NVR College, Tenali, Guntur(Dt.), A.P. India.

P. Sivaprasad, Department of BSH,VFSTR’S University, Vadlamudi, Guntur, A.P. India.

M. Sajani Lavanya Department of Mathematics, AC College, Guntur, A.P. India.

K. Anuradha, Department of Mathematics, Andhra Loyola College, Vijayawada, A.P. India.

Definition 2.4: An element m of M is known as regular if m ≤ mm, and intraregular if m ≤ emm. If ∀ k ∈ M is regular (intraregular) then le-ternary semi group M is defined to be regular(intraregular). Then K is

1. a le-ternary sub semi group if k ≤ k;
2. a left tie if ek ≤ k
3. a regular tie if eke ≤ k
4. a right tie if kek ≤ k
5. a bi-tie if it is a ternary sub semi group element and kekek ≤ k.

Definition 2.5: Let m ∈ M. Then n = m V m V mem is the least bi-tie in M ⊊ m ≤ n. We call m V m V mem the bi-tie generated by m, and denoted this by B(m). Thus m ∈ M is a bi-tie iff B(m) = m.

Definition 2.6: A ternary semi group K is known as regular if d ∈ K there exist t ∈ K such that d = dtd and K is called a band if d = d for all d ∈ K. A band K is right normal band if for all defgh = defgh = ddefgh. A band K is left normal band if for all defgh = defgh = deghf. A band K is normal if for all d, e, f, g, h ∈ K, defgh = ddefgh = deghf. A ternary sub semi group J of K is called bi-ideal of K if JKJK ⊊ J. A ternary semi group is known as locally finite if each finite generated ternary sub semi group if finite. A ternary semi group in which for all idempotent g ∈ K, Kg is semi lattice and which is locally finite is known as a locally testable ternary semi group.

Lemma 2.7: A regular ternary semi group K is locally testable iff Kg is a semi automorphic for all idempotent g ∈ K.

III. BI-T-IDEAL ELEMENTS (BI-TIE) TERNARY SUB-SEMI GROUP

The set of all left, lateral, right & bi-ties of K are denoted as L(K), B(K), R(K) & S(K) respectively, then all the sets are non-empty because e is a left, lateral, right and bi-tie of K.

Th 3.1: Suppose K be a le-ternary semi group, then K is regular iff the set of all bi-tie ternary semi group (K) is regular.

Proof: Let K is regular, k ∈ K. Therefore k is a bi-tie, kekek ≤ k. Other size, k ≤ kekek form the regularity of K. Hence we get k = kekek and hence k is a regular element in (K), here, e is also bi-tie in K.

Again, let (K) is a regular ternary semi group. Consider, j ∈ K, then B(K) = j V j V jejej ∈ S(K) and hence j ∈ S(K) such that j V j V jejej ≤ j V j V jejej ≤ j V j V jejej ≤ j V j V jejej ≤ j V j V jejej ∈ (j V j V jejej) V (j V j V jejej) ≤ jejej. Therefore, j ≤ jejej. Hence K is a regular le-ternary semi group.
Suppose K is a regular le-ternary semi group, then for very k ∈ K, k ≤ kekek ⇒ k' ≤ kkkek ≤ kekek. Therefore, the bi-tie (k) generated by k reduces to the form (k) = kekek. Therefore, in a regular le-ternary semi group K, an element j ∈ K is a bi-tie iff j = lmr for some left tie m and right tie r.

Th 3.2: Suppose K be an le-ternary semi group. Then (K)(K) ⊆ (K). Moreover, K is regular, then (k) is a regular le-ternary semi group.

Th 3.3: Suppose that K is a regular le-ternary semi group. Then (K), (K), & (K) are bands.

Th 3.4: An le-ternary semi group K is both regular as well as interregular iff (k) is a band.

Th 3.5: Suppose that K be an le-ternary semi group. Then K is regular as well as interregular iff (k) is a normal band.

**Proof:** assume k, l, m, n ∈ (k). Then (klk)(kmk)(knk) = klk(kmknk) ≤ klk(omk) ≤ klk. Similarly, (klk)(kmk)(knk) ≤ knk and (klk)(kmk)(knk) ≤ knk. Hence (klk)(kmk)(knk) ≤ (klk) ∧ (kmk) ∧ (knk). Now let v = (klk) ∨ (kmk) ∨ (knk). Then, v ≤ klk, v ≤ kmk & v ≤ knk. Since K is both regular as well as inter-regular, therefore, (k) is a band.

Now, v = (klk) ∨ (kmk) ∨ (knk) ≤ (klk) ∧ (kmk) ∧ (knk) ≤ (klk) ∧ (kmk) ∧ (knk)

Th 3.7: An le-ternary semi group K is left duo iff (klk) ≤ ek for all k, l ∈ K.

**Proof:** Suppose K is left duo & k, l ∈ K. Then the left t-ideal (klk) = {p ∈ K / p ≤ lmk for l, m ∈ K} generated by k is a right t-ideal. Hence kkl ∈ (klk) ⇒ there is some m, n ∈ K & k ≤ lmk ⇒ that kkl ≤ ek. Similarly we can show that the remaining parts. Therefore, kkl ≤ ek ≤ ekl .

For the other part, Suppose L be a left t-ideal of K & k ∈ L. Then ∀ q, r ∈ K, as kqr ≤ kqe ≤ ekq ∈ L ⇒ kqr ∈ L. Thus L is a right t-ideal of K and hence K is left duo.

Th 3.8: An le-ternary semi group K is duo iff kle = ekl for all k, l ∈ K.

Let K be a regular left duo le-ternary semi group. Then ∀ k ∈ K, k ≤ kekek ≤ (keekek) ≤ ek for K shows that K is intraregular. Hence (k) is a band.

Th 3.9: An le-ternary semi group K is regular left duo iff (k) is a left-normal-band.

**Proof:** Assume that K is regular left duo. Then (k) is a band. Let k, l, m, n, p ∈ (k). Then klmp = (klmp)klmp = kkkmpklmpklmp ≤ kklmmmpmmmpklmp ≤ kklmpklmpklmp ≤ kapklmnpklmpklmp ≤ lmnp. Similarly, lmnp ≤ mklnp ⇒ klmp = lmnp. Similarly lmnp = mklnp implies that klmp = lmnp and hence (k) is normal band.

Note 3.11: A band is a semi-lattice iff it is a left as well as a right normal band.

Th 3.10: An le-ternary semi group K is regular right duo iff (k) is a right normal band.

Th 3.12: An le-ternary semi group K is regular duo iff (k) is a semi lattice.

Th 3.13: If K is an le-ternary semi group. Then (K) is a rectangular band if K is regular & eke = ele ∀ k, l ∈ K.

**Proof:** Suppose (K) is a rectangular band, k, l ∈ K. (k) is a band, hence K is regular. Therefore, (k) = kek & (l) = lel. Then (k) = (k)(l)B(k) ⇒ k ≤ kek = (kek)(l)kek ≤ ele. Hence ekek ≤ ekek ≤ ele. Similarly, (l) = (l)B(l) ⇒ ekel ≤ ekek. Therefore, ekel = ele ∀ k, l ∈ K.

Conversely, If K ∈ (k) is regular, hence k ≤ kek ≤ kekek ≤ ekek. Hence k, l ≤ ekek & hence k = kekek = kekek = klk & hence B(K) is rectangular band.

**IV. CONCLUSION**

Since, we proved (K) is a locally testable ternary semi group and hence a normal-band if K is both regular & intraregular.

[Image]
ACKNOWLEDGMENT

The Authors are very greatful who supported to write this paper.

REFERENCES