On *Le*- Ternary Semi groups-II

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Abstract*: The ternary semi group of all bi-ties (K) of le-ternary semi group K is a band iff K is both regular as well as intraregular. Here, we show (K) is a band iff it is normal band. Our main aim to prove (1) Suppose K be an le-ternary semi group. Then (K)(K)ℜ(K) ⊆ 𝔅(k). Moreover, K is regular, then (k) = (K)(K)ℜ(K). (2) Let K is a regular le-ternary semi group. Then (K), (K), &ℜ(K) are bands. (3) An le-ternary semi group K is regular as well as intraregular iff (k) is a band (4) Suppose that K be an le-ternary semi group. Then K is both regular as well as intraregular iff (k) is a normal band. (5) An le-ternary semi group K is left duo iff kle ≤ lek ≤ ekl for all k, l ∈ K (6) An le-ternary semi group K is duo iff kle = lek = ekl for all k, l ∈ K. (7) An le-ternary semi group K is regular left duo iff (k) is a left normal band. (8) An le-ternary semi group K is regular right duo iff (k) is a right normal band.

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***Index Terms*: duo, regular, bi-ideal element, intra-regular, locally testable, normal band, and regular**

I. INTRODUCTION

In commutative rings the ideal theory was observed by W. Kurll. In 1952 the structure of the *le*-semi group introduced by R.A. Good and D.R. Hughes. Here, we characterize the *le*-ternary semi group K such that (K) is in each of the sub-varieties of normal bands.

II. PRELIMINARIES

Definition 2.1: An *le*-ternary semi group M is a structure (M, [ ], ∧, ∨, *e*) ∋ (M, [ ]) is a ternary semi group, (M, ∧, ∨, *e*) is a lattice with *e* which is known as greatest element and for all *p, q, r, s* ∈ M, [*pq*(*r* ∨*s*)] = [*pqr*] ∨[*pqs*], [*p*(*r* ∨*s*)*q*] = [*prq*] ∨[*psq*] and [(*r* ∨*s*)*pq*] = [*rpq*] ∨ [*spq*]. In this paper M will stand for *le*-ternary semi group and tie means t-ideal element, unless otherwise stated.

Example 2.3: Let *M* be a *po*-ternary-semi group. Let M_1 be the set of all ideals of *M*. Then (M_1, ⊆, ⋃, ⋂) is a *le*-ternary-semi group.

The relation ≤ on the set M is defined by for *p, q* ∈ M, *p* ≤ *q* if *p* ∨ *q* = *q*. Here the ternary operation is distributive over the lattice join, then ≤ is compatible with the ternary operation in M, i.e,, ∀ *p, q, r, s* ∈ M, *p* ≤ *q* ⇒ *rsp* ≤ rs*q*, *rps* ≤ *rqs* and *prs* ≤ *qrs*.

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Definition 2.4: An element *m* of M is known as *regular* if *m* ≤ *mem*, and *intraregular* if $m \le em^3e$. If ∀ *k* ∈ M is regular (intraregular) then *le*-ternary semi group M is defined to be *regular*(*intraregular*). Then *k* is

(1) a ternary sub semi group if $k^3 \le k$;
(2) a left *tie* if *eek* ≤ *k*
(3) a lateral *tie* if *eke* ≤ *k*
(4) a right *tie* if *kee* ≤ *k*
(5) a bi-*tie* if it is a ternary sub semi group element and *kekek* ≤ *k*.

Definition 2.5: Let *m* ∈ M. Then $n = m \vee m^3 \vee memem$ is the least bi-tie in M ∋ *m* ≤ *n*. We call $m \vee m^3 \vee memem$ the *bi-tie generated by m*, and denoted this by 𝔅(*m*). Thus *m* ∈ M is a bi-tie iff 𝔅(*m*) = *m*.

Definition 2.6: A ternary semi group K is known as *regular* if *d* ∈ K there exist *t* ∈ K such that *d* = *dtd* and K is called a *band* if $d^3 = d$ for all *d* ∈ K. A band K is right normal band if for all *defgh* = *efdgh* = *fdegh*. A band K is left normal band if for all *defgh* = *deghf* = *dehfg*. A band K is normal if for all *d, e, f, g, h* ∈ K, *defgh* = *dfgeh* = *dgefh*. A ternary sub semi group J of K is called *bi-ideal* of K if JKJKJ ⊆ J. A ternary semi group is known as *locally finite* if each finite generated ternary sub semi group if finite. A ternary semi group in which for all idempotent *g* ∈ K, *gKg* is semi lattice and which is locally finite is known as a *locally testable ternary semi group*.

Lemma 2.7: A regular ternary semi group K is locally testable iff *gKg* is a simulative for all idempotent *g* ∈ K.

III. BI-T-IDEAL ELEMENTS (BI-*TIE*) TERNARY SUB-SEMI GROUP

The set of all left, lateral, right & bi-ties of K are denoted as 𝔏(K), 𝔐(K), ℜ(K) & 𝔅(K) respectively, then all the sets are non-empty because *e* is a left, lateral, right and bi-tie of K.

Th 3.1: Suppose K be a *le*-ternary semi group, then K is regular iff the set of all bi-tie ternary semi group (K) is regular.

***Proof*:** Let K is regular, *k* ∈ (K). Therefore *k* is a bi-tie, *kekek* ≤ *k*. Other side, *k* ≤ *kekek* form the regularity of K. Hence we get *k* = *kekek* and hence *k* is a regular element in (K), here, *e* is also bi-tie in K.

Again, let (K) is a regular ternary semi group. Consider, *j* ∈ *K*, then $𝔅(j) = j \vee j^3 \vee jejej \in 𝔅(K)$ and hence *j* ∈ 𝔅(K) such that $j \vee j^3 \vee jejej = (j \vee j^3 \vee jejej)k(j \vee j^3 \vee jejej)k(j \vee j^3 \vee jejej) \le (j \vee j^3 \vee jejej)e(j \vee j^3 \vee jejej)e(j \vee j^3 \vee jejej) \le jejej$. Therefore, *j* ≤ *jejej*. Hence K is a regular *le*-ternary semi group.

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Suppose K is a regular *le*-ternary semi group, then for very $k \in$ K, $k \leq kekek \Rightarrow k^3 \leq kkkekek \leq kekek$. Therefore, the bi-tie (k) generated by k reduces to the form (k) = $kekek$. Therefore, in a regular *le*-ternary semi group K, an element $j \in$ K is a bi-tie iff $j = lmr$ for some left tie, lateral tie m and right tie r.

Th 3.2: Suppose K be an *le*-ternary semi group. Then (K)(K)$\mathfrak{R}$(K) ⊆ $\mathfrak{B}$(*k*). Moreover, K is regular, then (*k*) = (K)(K)$\mathfrak{R}$(K).

Th 3.3: Suppose that K is a regular *le*-ternary semi group. Then (K), (K), &$\mathfrak{R}$(K) are bands.

Th 3.4: An *le*-ternary semi group K is both regular as well as intraregular iff (*k*) is a band.

Th 3.5: Suppose that K be an *le*-ternary semi group. Then K is regular as well as intraregular iff (*k*) is a normal band.

Proof: assume $k, l, m, n \in (k)$. Then $(klk)(kmk)(knk) = kl(kkmkk)nk \leq kl(onk) \leq klk$. Similarly, $(klk)(kmk)(knk) \leq kmk$ and $(klk)(kmk)(knk) \leq knk$. Hence $(klk)(kmk)(knk) \leq (klk) \wedge (kmk) \wedge (knk)$. Now let $v = (klk) \wedge (kmk) \wedge (knk)$. Then, $v \leq klk$, $v \leq kmk$ & $v \leq knk$. Since K is both regular as well as intra-regular, therefore, (k) is a band.
Now, $vevev = (klk \wedge kmk) \wedge knk)e(klk \wedge kmk) \wedge knk)e(klk \wedge kmk) \wedge knk)$

$=(klkeklk \wedge klkekmk \wedge klkeknk \wedge kmkeklk \wedge kmkekmk \wedge kmkeknk \wedge knkeklk \wedge knkekmk \wedge knkeknk)e(klk \wedge kmk) \wedge knk)$

$=klkeklkeklk \wedge klkeklkekmk \wedge klkeklkeknk \wedge klkekmkeklk \wedge klkekmkekmk \wedge klkekmkeknk \wedge klkeknkeklk \wedge klkeknkekmk \wedge klkeknkeknk \wedge kmkeklkeklk \wedge kmkeklkekmk \wedge kmkeklkeknk \wedge kmkekmkeklk \wedge kmkekmkekmk \wedge kmkekmkeknk \wedge kmkeknkeklk \wedge kmkeknkekmk \wedge kmkeknkeknk \wedge knkeklkeklk \wedge knkeklkekmk \wedge knkeklkeknk \wedge knkekmkeklk \wedge knkekmkekmk \wedge knkekmkeknk \wedge knkeknkeklk \wedge knkeknkekmk \wedge knkeknkeknk$

$\leq klk \wedge klkeklkekmk \wedge klkeklkeknk \wedge klkekmkeklk \wedge klkekmkekmk \wedge klkekmkeknk \wedge klkeknkeklk \wedge klkeknkekmk \wedge klkeknkeknk \wedge kmkeklkeklk \wedge kmkeklkekmk \wedge kmkeklkeknk \wedge kmkekmkeklk \wedge kmk \wedge kmkekmkeknk \wedge kmkeknkeklk \wedge kmkeknkekmk \wedge kmkeknkeknk \wedge knkeklkeklk \wedge knkeklkekmk \wedge knkeklkeknk \wedge knkekmkeklk \wedge knkekmkekmk \wedge knkekmkeknk \wedge knkeknkeklk \wedge knkeknkekmk \wedge knk$

$\leq klk \wedge kmk \wedge knk = v$ implies that $v \in \mathfrak{B}(k)$ implies that $v = v^3 \leq (klk)(kmk)(knk)$. Therefore, $(klk) \wedge (kmk) \wedge (knk) \leq (klk)(kmk)(knk)$ and hence $(klk) \wedge (kmk) \wedge (knk) = (klk)(kmk)(knk)$.

Then $k(k)$k = { klk / $l \in \mathfrak{B}(k)$} is a semi lattice for all $l \in \mathfrak{B}(k)$. Hence, (k) is a locally testable ternary semi group. Since a locally testable ternary semi group is a band iff it is a normal band. Therefore, (k) is a normal band.
Converse of the th will prove from th 3.4.

Definition 3.6: A partially ordered ternary semi group K is known as *left* (*right*) *duo* if every left (right) t-ideal of K is a right (left) t-ideal of K and K is known as *duo* if K is both left and right duo.

Th 3.7: An *le*-ternary semi group K is left duo iff $kle \leq lek \leq ekl$ for all *k, l* ∈ K.

Proof: Suppose K is left duo & $k, l \in$ K. Then the left t-ideal $(k]_l = \{p \in$ K / $p \leq lmk$ for $l, m \in$ K$\}$ generated by k is a right t-ideal. Hence $kle \in (k]_l \Rightarrow$ there is some $m, n \in$ K $\ni kle \leq mnk \Rightarrow$ that $kle \leq lek$. Similarly we can show that the remaining parts. Therefore, $kle \leq lek \leq ekl$.

For the other part, Suppose $\mathcal{L}$ be a left t-ideal of K & $k \in \mathcal{L}$. Then $\forall$ $q, r \in$ K, as $kqr \leq kqe \leq ekq \in \mathcal{L} \Rightarrow kqr \in \mathcal{L}$. Thus $\mathcal{L}$ is a right t-ideal of K and hence K is left duo.

Th 3.8: An *le*-ternary semi group K is duo iff $kle = lek = ekl$ for all *k, l* ∈ K.

Let K is a regular left duo *le*-ternary semi group. Then $\forall$ $k \in$ K, $k \leq kekek \leq (kee)ekek \leq ek^3e$ shows that K is intraregular. Hence,(k) is a band.

Th 3.9: An *le*-ternary semi group K is regular left duo iff (*k*) is a left-normal-band.

Proof: Assume that K is regular left duo. Then (k) is a band. Let $k, l, m, n, p \in (k)$. Then $klmnp = (klmnp)(klmnp)(klmnp)$
$= kkklmnpklmnpklmnp \leq kk(kee)npklmnpklmnp$
$\leq kk(eek)npklmnpklmnp \leq kk(eke)npklmnpklmnp$
$\leq knpklmnpklmnp \leq lmknp$. Similarly, $lmknp$
$\leq mklnp \Rightarrow klmnp = lmknp$.
Similarly $lmknp = mklnp$ implies that $klmnp = lmknp = mklnp$ and hence (k) is normal band.

Now, suppose that (k) is a left normal band. Then K is regular. Also for every $k \in$ K, both eek and $kekek$ are bi-ties of K, therefore, $(kee)(kee)(kee) = k(eek)(eek)ee \leq k \leq kee$ which shows that K is left duo.

Th 3.10: An *le*-ternary semi group K is regular right duo iff (k) **is a right normal band**.

Note 3.11: A band is a semi-lattice iff it is a left as well as a right normal band.

Th 3.12: An *le*-ternary semi group K is regular duo iff (k) is a semi lattice.

Th 3.13: If K is an *le*-ternary semi group. Then (K) is a rectangular band iff K is regular & $eke = ele$ ∀ *k, l* ∈ K.

Proof: Suppose (K) is a rectangular band, $k, l \in$ K. ∵ (K) is a band, hence K is regular. Therefore, $(k) = kek$ & $(l) = lel$. Then $(k) = (k)(l)\mathfrak{B}(k) \Rightarrow k \leq kek = (kek)(lel)(kek) \leq ele$. Hence $eke \leq e^2le^2 \leq ele$. Similarly, $(l) = (l)(k)\mathfrak{B}(l)$
$\Rightarrow ele \leq eke$. Therefore, $eke = ele$ ∀ $k, l \in$ K.
Conversely, If $k \in$ K. ∵ K is regular, hence $k \leq kek \leq kekek \leq kek^3ek$. Therefore, $k \leq ek^3e$, hence K is a intraregular and hence (K) is a band. By th. 3.4, now let k, l are two bi-tie of K. ∵ k is a bi-tie & K is regular, then $kek = k$ and hence $k = kek = kekek = keklkek = klk$ and hence $\mathfrak{B}$(K) is rectangular band.

IV. CONCLUSION

Since, we proved (K) is a locally testable ternary semi group and hence a normal-band if K is both regular & intraregular.

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