On *Le*- Ternary Semi groups-I

C. Sreemannarayana, D. Madhusudhana Rao, P. Sivaprasad, T. Nageswara Rao, K. Anuradha

***Abstract*: We investigated and studied about *Poe*-ternary semi groups, *le*-ternary semi groups and $\bigvee e$-ternary semi groups. Mathematical Subject Classification: 06F99, 06F05, 20M10.**

Index Terms*: *le-ternary semi group, poe-ternary semi groups, ve-ternary semi groups, (u, v, p)-tie, (u, v, p)-quasi tie

I. INTRODUCTION

Sen and Saha in 1986, introduced the concept Γ-semi group as a generalization of semi-group and ternary-semi-group. In 2012, D. M. Rao, subramaneswa rao seetamraju and A. Anjaneyuluu introduced some notions of partially ordered Γ-semi group. In the 2015, D. M. Rao and sivaprasad introduced some concepts of ordered ternary-semirings.

II. PRELIMINARIES

Def 1.1: A ternary-semi-group T is known as a ***partially ordered ternary-semi-group*** if T is ordered set $\ni k \leq l \Rightarrow kmn \leq lmn$, $mkn \leq mln$ and $mnk \leq mnl$ for all $k, l, m, n \in$ T. A partially ordered ternary semi group simply called po-ternary semi group or ordered ternary semi group.

In the following some examples of po-ternary semi groups are given

Ex 1.2 : Let T = { ∅, {*c*}, {*v*}, {*b*}, {*c, v*}, {*v, b*}, {*c, b*}, {*c, v, b*}}. If for all C, V, B ∈ T, CVB = C ∩ V ∩ B and C ≤ V ⟺ C ⊆ V, then T is partially ordered-ternary semi group.

Def 1.3 : A ***poe- ternary-semi-group*** is a *po*-ternary-semi-group *Q* with a greatest element "*e*" (That is $e \geq q;\ \forall q \in Q$).

In a *partially ordered*-ternary-semi-group *Q*, *k* is known as a *right* (respectively. *lateral, left*) *t-Ideal-Element* if $klm \leq k$ (respectively, $lkm \leq k$, $lmk \leq k$) for all $l, m \in M$ and *k* is known as an *t-Ideal-Element* if it is a right, lateral and left t-Ideal-Element. In a *partially-ordered-e*-Ternary-Semi-Group *Q, a* is said to be *right* (respectively. *lateral, left*) *t-Ideal-Element* if $kee \leq k$ (respectively. $eke \leq k$, $eek \leq k$).

Def 1.4 : Suppose that *L* is a semi-lattice under ∨ with a greatest element *e* & at the same time a *partially-ordered*-ternary-semi-group $\ni \forall\, k, l, m, n \in L$, $kl(m \vee n) = klm \vee knl$, $k(l \vee m)n = kln \vee kmn$ and $(k \vee l)mn = kmn \vee lmn$. Then *L* is known as a ***⋁e-ternary semi group***.

Def 1.5 : A lattice ⋁*e*-ternary semi group is known as an ***le-ternary-semi group***. The partially order relation ≤ on *L* is defined as $d \leq g$, $d \bigvee g = g$: Then we can express that for any $d, g, h, j \in L$, $d \leq g$ implies $dhj \leq ghj$ & $hjd \leq hjg$.

Ex 1.6: Let *L* be a *po*-ternary-semi group. Let L_1 be the set of all t-ideals of *L*. Then (L_1, ⊆, ⋃, ⋂) is an *le*-ternary-semi group.

III. ON (U, V, P)-TIE IN LE-TERNARY-SEMI GROUPS

If *L* is a ⋁*e*-ternary-semi group and *u, v, p* odd non negative integers,

Def 2.1: An element $r \in Q$ is known as an ***(u, v, p)-t-Ideal-Element*** of *Q* if $r^u r^v r^p \leq r$. r^0 defined as $r^0 r^0 s = r^0 s r^0 = s r^0 r^0 = s$ for $s \in$ Q. For $u = 0$, $v = 0$, $p = 1$(resp. $u = 0$, $v = 1$, $p = 0$ and $u = 1$, $v = 0$, $p = 0$). Then the definition gives the trivial case of right (respectively. lateral and left)-t-Ideal-Elements. It is clear that right (respectively. lateral, left)-t-Ideal-Elements are (*u*, 0, 0) (respectively. (0, *v*, 0), (0, 0, *p*))-t-Ideal-Elements for $u \geq 1$ (respectively. $v \geq 1$, $p \geq 1$) throughout this paper we use *(u, v, p)-t-Ideal-Element* as (*u, v, p*)-tie.

Def 2.2: An *le*-ternary semi group is said to be ***sub idempotent*** provided $aaa \leq a$ for all $a \in$ Q.

Now we proved the statements

(1) Let *Q* is a partially ordered *e*-ternary-semi group & *f, g* two (*u, v, p*)-t-Ideal-Elements of *Q*. Then $f \bigwedge g$ if exists is an (*u, v, p*)-tie of *Q*.

(2) suppose *Q* is a partially ordered *e*-ternary-semi group. Then *l*-power ($l \geq 1$, *l* is odd) of an (*u, v, p*)-t-Ideal-Element is also an (*u, v, p*)-tie.

(3) Suppose *Q* be a partially ordered *e*-ternary-semi group. If *r* is an (*u, v, p*)-tie of *Q* ($u, v, p \geq 1$), then for every $s, t \in Q$ such that $rst \leq r$ (respectively. $srt \leq r$, $str \leq r$), and *rst, srt* and *str* are (*u, v, p*)-ties of *Q*. For $u = v = p = 1$.

(4) Let *Q* be a ⋁*e*-ternary-semi group and *r, s* two left (respectively. lateral, right)-t-Ideal-Elements of *Q*. Then the union $r \bigvee s$ is a sub idempotent (*u, v, p*)-tie, for odd $u \geq 0$, for $v \geq 1$ and for $p \geq 1$ (respectively. $v \geq 0$, $u \geq 1$, $p \geq 1$ and $p \geq 0$, $u \geq 1$, $v \geq 1$).

Note 2.3: $\langle r \rangle_{(u, v, p)}$ will be denoted the principal (*u, v, p*)-tie of *Q* generated by *r*, i.e., the least ⟨*u, v, p*⟩-t-Ideal-Element of *Q* containing *r* & by $A_{(u, v, p)}$ the set of all (*u, v, p*)-ties of *Q*.

Def 2.4: Let Q be a ⋁*e-ternary semi group* and *u, v, p* are odd non negative integers, then Q said to be ***(u, v, p)-regular*** if $r \leq r^u eer^v eer^p$ for all $r \in$ Q.

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C. Sreemannarayana, Research Scholar, Department of Mathematics, KL University, Vaddeswaram, Guntur(Dt), A.P. India.

D. Madhusudhana Rao, Department of Mathematics, VSR & NVR College, Tenali, Guntur(Dt), A.P. India.

P. Sivaprasad, Department of BSH, VFSTR'S University, Vadlamudi, Guntur, A.P. India.

T. Nageswara Rao, Department of Mathematics, KL University, Vaddeswaram, Guntur(Dt), A.P. India

K. Anuradha, Department of Mathematics, Andhra Loyola College, Vijayawada, A.P. India

Th. 2.5: Let Q is a ⋁*e-ternary semi group* and u, v, p are odd non negative integers. Then the following holds:

(1) $(a \vee a^u e v^n e a^p)^u ee = a^u ee$, **for all** $a \in Q$.
(2) $(a \vee a^v e a^u e a^p)^n ee = a^v ee$, **for all** $a \in Q$.
(3) $(a \vee a^p e a^v e a^u)^p ee = a^p ee$, **for all** $a \in Q$.
(4) $\langle a \rangle_{(m, n, p)} = a \vee a^m e a^n e a^p$ **for all** $a \in Q$.

Proof: We prove first three cases for $u, v, p \geq 1$, in case $u = 0$, $v = 0$, $p = 0$ is obvious.

For $a \in Q$, we have

(1) $(a \vee a^u e a^v e a^p)^u ee = (a \vee a^u e a^v e a^p)^{u-1} (a \vee a^u e a^v e a^p) ee$
$= (a \vee a^u e a^v e a^p)^{u-1} (aee \vee a^u e a^v e a^p ee) = (a \vee a^u e a^v e a^p)^{u-1} aee$
$= (a \vee a^u e a^v e a^p)^{u-2} (a \vee a^u e a^v e a^p) aee$
$= (a \vee a^u e a^v e a^p)^{u-2} (aaee \vee a^u e a^v e a^{p+1} ee) = (a \vee a^u e a^v e a^p)^{u-2} a^2 ee$
$= \ldots\ldots\ldots\ldots = a^u ee$.

The proof of (2) and (3) is analogues of (1).

(4) Suppose, $u, v, p \geq 0$ then by condition (1), (2) and (3) we have
$(a \vee a^u e a^v e a^p)^u ee (a \vee a^v e a^u e a^p)^v ee (a \vee a^p e a^v e a^u)^p ee = a^u e a^v e a^p$.
So that $a \vee a^u e a^v e a^p \in A_{(u, v, p)}$. Now if c is a (u, v, p)-t-Ideal-Element of Q containing a, then $a \vee a^u e a^v e a^p \leq c$. The lemma is finished.

Th 2.6: If Q be a ⋁*e-Ternary-Semi-Group.* Q is regular iff $a^u e a^v e a^p = a$ for all $l \in A_{(u, v, p)}$.

Proof: the necessary part is obvious.

For the sufficient part $l \in Q$. $\because \langle l \rangle_{(u, v, p)} \in A_{(u, v, p)}$, we have
$(\langle l \rangle_{(u, v, p)})^u ee (\langle l \rangle_{(u, v, p)})^v ee (\langle l \rangle_{(u, v, p)})^p = \langle l \rangle_{(u, v, p)}$
But by 2.5, we have $(\langle l \rangle_{(u, v, p)})^u ee = l^u ee$, $ee(\langle l \rangle_{(u, v, p)})^v = eel^v$ and $ee(\langle l \rangle_{(u, v, p)})^p = eel^p$. Thus $l \leq l^u eel^v eel^p$ and Q is a (u, v, p)-regular.

Th 2.7: Suppose that Q be a sub idempotent *le*-ternary-semi group. Then Q is (u, v, p)-regular iff $r \wedge m \wedge t = r^u mt \wedge rm^v t \wedge rmt^p$ for all $r \in A_{(u, 0, 0)}$, $m \in A_{(0, v, 0)}$ and $t \in A_{(0, 0, p)}$ and u, v, p are odd non negative integers.

Proof: part 1: Suppose Q be a (u, v, p)-regular *le*-ternary-semi group.

Let $r \in A_{(u, 0, 0)}$, $s \in A_{(0, v, 0)}$ and $t \in A_{(0, 0, p)}$. Then $r^u ee \leq r$, $es^v e \leq s$ and $eet^p \leq t$,

hence $r^u mt \wedge rm^v t \wedge rmt^p \leq r^u ee \wedge em^v e \wedge eet^p \leq r \wedge m \wedge t$.

On the other hand $r \wedge m \wedge t \leq (r \wedge m \wedge t)^u ee (r \wedge m \wedge t)^v ee (r \wedge m \wedge t)^p \leq r^u eem^v eet^p \leq r^u mt$.

Similarly, $r \wedge m \wedge t \leq rm^v t$ and $r \wedge m \wedge t \leq rmt^p$ and hence $r \wedge m \wedge t \leq r^u mt \wedge rm^v t \wedge rmt^p$.

Part-2: If Q is (0, 0, 0)-regular *le*-ternary semi group, then ascertain is true for $u = v = p = 0$.

Let $u \neq 0$, $v = 0$, $p = 0$. If $r \in A_{(u, 0, 0)}$. $\because e$ is a (0, 0, 0)-tie of Q. from given condition we have $r = r^u ee$. So that by th 2.6, Q is $(u, 0, 0)$-regular. In the case $u = 0$, $v \neq 0$ and $p = 0$, the proof is simillar.

Now, let $u \neq 0$, $v \neq 0$, $p \neq 0$. Then Q has the law $r \wedge s \wedge t \leq rst$ $\forall\, r \in A_{(u, 0, 0)}$, $\forall\, s \in A_{(0, v, 0)}$ & $\forall\, t \in A_{(0, 0, p)}$.

Indeed: $r \in A_{(u, 0, 0)}$ then $r \wedge s \wedge t = r^u st \wedge rs^v t \wedge rst^p \leq rst$ ($u \geq 1$, $v \geq 1$, $p \geq 1$ & u, v, p are odd). Let, $r \in Q$. Since $\langle r \rangle_{(u,0, 0)} \in A_{(u, 0, 0)}$ & e is a $(0, 0, p)$-tie of Q, we get from given condition, $\langle r \rangle_{(u,0, 0)} = (\langle r \rangle_{(u,0, 0)})^u ee \wedge \langle r \rangle_{(u,0, 0)} e^v e \wedge \langle r \rangle_{(u,0, 0)} ee^p \leq (\langle r \rangle_{(u,0, 0)})^u ee = r^u ee$ (by 2.5(4), (1)); thus $(\langle r \rangle_{(u,0, 0)})^u = r^u ee$. lly, $\langle r \rangle_{(0, v, 0)} = er^v e$ and $\langle r \rangle_{(0, 0, p)} = eer^p$.

On the other side $r \leq \langle r \rangle_{(u,0, 0)} \wedge \langle r \rangle_{(0, v, 0)} \wedge \langle r \rangle_{(0, 0, p)} \leq \langle r \rangle_{(u,0, 0)} \langle r \rangle_{(0, v, 0)} \langle r \rangle_{(0, 0, p)} = r^u eer^v eer^p$. Therefore, Q is a (u, v, p)-regular.

3. On (u, v, p)-quasi-t-Ideal-Elements in *le*-ternary-semi groups

Def 3.1: Suppose Q is a *partially ordered e*-ternary-semi group. An element $q \in Q$ is known as an (u, v, p)-*quasi-tie* of Q provided $q^u ee \wedge eeq^p \wedge eq^v e$ exists and $q^u ee \wedge eeq^p \wedge eq^v e \leq q$.

Th 3.2: Let Q be a distributive *le*-ternary semi group. Then, an element q is an (u, v, p)-quasi-tie of Q iff there exists an (u, 0, 0)-tie r, an $(0, v, 0)$-tie s and an $(0, 0, p)$-tie t of Q such that $q = r \wedge s \wedge t$.

Proof. (⇒) Let $r \in Q_{(u, 0, 0)}$, $s \in Q_{(0, v, 0)}$ and $t \in Q_{(0, 0, p)}$. Then, since $r, s, t \in Q_{(m, n, p)}$,

we have $r \wedge s \wedge t \in Q_{(u, v, p)}$.

(⇐) Let $q \in Q_{(u, v, p)}$. Then, by 2.5, we have $q = q \vee (q^u ee \wedge eeq^v \wedge eq^p e) = (q \vee q^u ee) \wedge (q \vee eeq^v) \wedge (q \vee eq^p e) = \langle q \rangle_{(u, 0, 0)} \wedge \langle q \rangle_{(0, v, 0)} \wedge \langle q \rangle_{(0, 0, p)}$ where $\langle q \rangle_{(u, 0, 0)} \in Q_{(u, 0, 0)}$, $\langle q \rangle_{(0, v, 0)} \in Q_{(0, u, 0)}$ and $\langle q \rangle_{(0, 0, p)} \in Q_{(0, 0, p)}$.

It is clear that $(u, 0, 0)$ (respectively. $(0, v, 0)$, $(0, 0, p)$)-ties and $(u, 0, 0)$ (respectively. $(0, v, 0)$, $(0, 0, p)$)-quasi-ties are the same.

Th. 3.3: Suppose Q is an *le*-ternary semigoup, $a \in Q$ and u, v, p are odd natural numbers. Then the conditions:

(1) $(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^u ee \leq a^u ee$,
(2) $e(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^v e \leq ea^v e$,
(3) $ee(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^p \leq eea^p$.
(4) $(a)_{(u, v, p)}$ **exists &** $(a)_{(u, v, p)} = a \vee (a^u ee \wedge ea^v e \wedge eea^p)$ **are holds true.**

Proof: since $u = 0$ it is clear. Let $u \geq 1$. Then we have
$(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^u ee = (a \vee (a^u ee \wedge ea^v e \wedge eea^p))^{u-1} (a \vee (a^u ee \wedge ea^v e \wedge eea^p)) ee$
$\leq (a \vee (a^u ee \wedge ea^v e \wedge eea^p))^{u-1} (aee \vee (a^u e^4 \wedge ea^v e^3 \wedge eea^p ee))$
$= (a \vee (a^u ee \wedge ea^v e \wedge eea^p))^{u-1} aee$
$= (a \vee (a^u ee \wedge ea^v e \wedge eea^p))^{u-2} (a \vee (a^u ee \wedge ea^v e \wedge eea^p)) aee$
$\leq (a \vee (a^u ee \wedge ea^v e \wedge eea^p))^{u-2} (a^2 e^2 \vee (a^{u+1} e^4 \wedge ea^{v+1} e^3 \wedge eea^{p+1} ee))$
$= (a \vee (a^u ee \wedge ea^v e \wedge eea^p))^{u-2} a^2 e^2$
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$\leq a^u ee$.

(2), (3) will prove similarly.

(4): Let $u; v, p \geq 0$. From (1), (2) and (3) we have
$(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^u ee \wedge e(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^v e \wedge ee(a \vee (a^u ee \wedge ea^v e \wedge eea^p))^p \leq a^u ee \wedge ea^v e \wedge eea^p$ and therefore $a \vee (a^u ee \wedge ea^v e \wedge eea^p)$ is an (u, v, p)-quasi tie of Q which has a. Here, if b is an (u, v, p)-quasi tie of Q ∋ $b \geq a$, then $a \vee (a^u ee \wedge ea^v e \wedge eea^p) \leq b$.

Note 3.4: In general in *le*-ternary semi groups $\langle l \rangle_{(u, v, p)} \leq (l)_{(u, v, p)}$.
In particular $\langle l \rangle_{(u, 0, 0)} \leq (l)_{(u, 0, 0)}$, $\langle l \rangle_{(0, v, 0p)} \leq (l)_{(0, v, 0)}$ and $\langle l \rangle_{(0, 0, p)} \leq (l)_{(0, 0, p)}$.

Th 3.5: Suppose Q be an *le*-ternary semi group and u, v, p are odd natural numbers. Then

(1) Q is (u, v, p)-regular
(2) $d^u ed^v ed^p = d^u$ **for all** $d \in A_{(u, v, p)}$

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(3) $q^u e q^v e q^p = q^v$ **for all** $q \in Q_{(m, n, p)}$
(4) $(< d >_{(u, v, p)})^u e(< d >_{(u, v, p)})^v e(< d >_{(u, v, p)})^p = < d >_{(u, v, p)}$ **for all** $d \in Q$.
(5) $((d)_{(u, v, p)})^u e((d)_{(u, v, p)})^v e((d)_{(u, v, p)})^p = (d)_{(u, v, p)}$ **for all** $a \in Q$ **are equivalent.**

***Proof*:** (1) ⇒ (2), It is clear by, th. 2.6.
(2) ⇒ (3), it is clear by, note 3.2.
(3)⇒ (1): Suppose $a \in Q$. By th, 3.5, it follows that $< d >_{(u, 0, 0)} \wedge < d >_{(0, v, 0)} \wedge < d >_{(0, 0, p)}$ is a (u, v, p)-quasi tie of Q, by (4) and Th. 2.5,

$$d \le < d >_{(u, 0, 0)} \wedge < d >_{(0, v, 0)} \wedge < d >_{(0, 0, p)}$$
$$= (< d >_{(u, 0, 0)} \wedge < d >_{(0, v, 0)} \wedge < d >_{(0, 0, p)})^u e(< d >_{(u, 0, 0)} \wedge < d >_{(0, v, 0)} \wedge < d >_{(0, 0, p)})^v e(< d >_{(u, 0, 0)} \wedge < d >_{(0, v, 0)} \wedge < d >_{(0, 0, p)})^p$$
$$\le (< d >_{(u, 0, 0)})^u e(< d >_{(0, v, 0)})^v e(< d >_{(0, 0, p)})^p = d^u e d^v e d^p.$$

(2) ⇒ (4) : It is clear.
(4)⇒ (2): If $d \in A_{(u, v, p)}$, then $< d >_{(u, v, p)} = d$ and by (5) $d^u e d^v e d^p = d$.
(3) ⇒ (5): same as the previous case of (u, v, p)-ties.

Note 3.6: In regular *partially ordered e*-ternary semi groups, we have $Z_{(u, v, p)} = Q_{(u, v, p)}$ for every element r of a *partially ordered e*-ternary semi group Q. We have
$r^u ee \wedge er^v e \wedge eer^p \le (r^u ee \wedge er^v e \wedge eer^p)e(r^u ee \wedge er^v e \wedge eea^p)e(r^u ee \wedge er^v e \wedge eea^p)$
$\le r^u eeeer^v eeeer^p \le r^u er^v er^p \le r^u ee \wedge er^v e \wedge eer^p$.
Therefore, in regular *partially ordered e*-ternary semi group, we have $r^u er^v er^p = r^u ee \wedge er^v e \wedge eer^p$.

IV. CONCLUSION

We mainly characterizations of (*u; v, p*)-regular *le*-Ternary-Semi-Group by means of (u, v, p)-ties & (u, v, p)-quasi-ties. Also, we study the behavior of sub idempotent (*u; v, p*)-ties.

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