Computational Analysis and Simulation of Fractional order PID controller for Ceramic Infrared Heater

Vineet Shekher, Pankaj Kumar, Surya Deo Chaudhary

Abstract*— In current scenario, the study of fractional order PID controller tuning rules of robust control systems for first order plus time delay systems have been developed. In this paper, on the basis of computational scheme, a controller is designed to satisfy the robustness property with respect to gain variation and desired phase margin criteria. In this study, numerical computation of tuning formulae and the relationship between design specification and design parameter are discussed by both taking an example of the ceramic infrared heating system. In the design specification , the controller parameters and the plant conditions, a fair comparison with an optimal design integer order PID (IOPID) controller done via simulation to show the controllers dynamic performance , stability and robustness when the parameters change.

Index terms- PID, IOPID, FOPTD, FOPID, Ceramic Infrared Heater

I. INTRODUCTION

In, the recent year the applications of fractional calculus have been attracting more and more researchers in the field of engineering and science. The orders of fractional calculus are real number. Today, many researchers have focused on fractional order PID controllers and have obtained some useful results. The fractional order PID controller ($PI^{\lambda}D^{\mu}$) was proposed as a generalization of PID controller, where expanding of the derivative and integrals to fractional orders, which are adjusted to the frequency response of the control system directly and continuously.

Revised Manuscript Received on December 22, 2018.
Vineet Shekher,Electrical Engineering Department,Noida Institute of Engineering and Technology,Greater Noida, Uttar pradesh ,Mail: vshekher2407@gmail.com
Pankaj Kumar,Electrical Engineering Department,Birsa Institute of Technology,Sindri, Dhanbad
Surya Deo Chaudhary,Electronics and Communication,Noida Institute of Engineering and Technology,Greater Noida, Uttar pradesh

This paper presents a mathematical computational tuning scheme of FOC for certain temperature systems used in industry.
The main contribution of this paper includes

1. Fractional order PID controller with λ and μ as unity is proposed (IOPID) with mathematical computation for first order plus time delay system has been presented.
2. The fractional order PID controller is proposed with mathematical computation for first order plus time delay system has been presented.
3. According to the systematic design and simulation, a fair comparison of control performance examines with IOPID controller.
4. From the simulation results, it can be seen that FOPID controller outperforms the IOPID controller

II. PROBLEM FORMULATION

A.Design Specification of Control plant and controllers

a) Control Plant:
Because of small delay time in a large number of temperature system plant, so a typical first order plus time delay plant discussed in this paper is

$$G(s)=\frac{k}{Ts+1}e^{-Ls} \qquad (1)$$

Which can be an approximately model of a large number of industrial plants. For the ceramic infrared heating system transfer function with the value of gain (k) variation of 3.96 to 4.2, time constant of 140 sec and lag time of 7 sec. So, the typical FOPLT plant for ceramic infrared heater taken as [1],[2],[3]

$$G_{IRD}(s)=\frac{[3.96-4.2]}{140s+1}e^{-7s} \qquad (2)$$

b) Controllers:
The fractional order proportional integral derivative controller (FOPID) has the following form

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$C(s)=K_p\left(1+\frac{T_i}{s^{\lambda}}+T_d s^{\mu}\right) \qquad (3)$$

Where, $\lambda \in (0,1)$ and $\mu \in (0,1)$

Clearly, this is a specific form of the most common $PI^{\lambda}D^{\mu}$ controller ,which includes an Integrator of order λ and a differentiator of order μ .

By considering the value of $\lambda \in 1$ and $\mu \in 1$, the controller form becomes the IOPID in the following expression as

$$C_{IOPID}(s)=K_p\left(1+\frac{T_i}{s}+T_d s\right) \qquad (4)$$

Where Kp, Ki and Kd represents proportional, integral and derivative gain respectively [4][5][6].

C.Design Consideration:

By considering, the tuning method present by Monje, Vinagre and their colleague used in the paper. Monje and Vinagre et. al, consider the five design criteria algorithm for design specification [7][8][9]. These design criteria obtained by getting the value of required phase margin ϕ_m, critical frequency ω_{cp} point on the Nyquist curve of plant at which

$$\arg[(G(\omega_{cp}))]=-180^{o}$$

and gain margin as

$$g_m=\frac{1}{\left|G(j\omega_{cp})\right|}$$

By getting the phase margin (ϕ_m) and critical frequency (ω_{cp}), five design criteria of Monje – Vinagre et. al methods are given as follows[9][10].

B.Phase Margin and Gain Crossover Frequency

The two frequency domain specifications are used to measure the robustness i.e gain margin and phase margin. The phase margin is related to the damping of the system, thus the following equation should be satisfied

$$\left|C(j\omega_{cg})G(j\omega_{cg})\right|_{dB}=0\ dB$$

and

$$Arg(C(j\omega_{cg})G(j\omega_{cg}))=-\pi+\phi_m \qquad (5)$$

Where ω_{cg} is a gain crossover frequency and ϕ_m is the required phase margin.

C.Robustness due to variation in the gain of Plant

The phase is forced to be flat at ω_{cg} and the phase plot is almost constant within the interval around ω_{cg} to satisfy the following constraint

$$\left(\frac{d(Arg(C(j\omega_{cg})G(j\omega_{cg}))}{d\omega}\right)_{\omega=\omega_{cg}}=0 \qquad (6)$$

As, per the phase plot around the specified frequency ω_{cg} is locally flat, which implies that the system will be more robust to variation of gain and step response is almost constant within the interval with constant overshoots.

D.High Frequency Noise Rejection

The following condition must be satisfied to the robustness due to high frequency noise

$$\left|T(j\omega)=\frac{C(j\omega)G(j\omega)}{1+C(j\omega)G(j\omega)}\right|_{dB}\le A\ dB \qquad (7)$$

Where A is the desired value of the noise attenuation for frequency is $\omega \ge \omega_t$ rad/sec.

E.Good output disturbance rejection

The following constraint must be satisfied to ensure a good output disturbance rejection.

$$\left|S(j\omega)=\frac{1}{1+C(j\omega)G(j\omega)}\right|_{dB}\le B\ dB \qquad (8)$$

Where B is the desirable value of sensitivity function for which the frequency is $\omega \le \omega_s$ rad/sec.

III. DESIGN ANALYSIS

A. Design of IOPID controller

By considering the FOPLD system for ceramic infrared heater, whose open loop transfer function $G_{IRD}(s)$

$$P(s)=C(s)G_{IRD}(s)$$

The frequency response for ceramic IR heater system as

$$G_{IRD}(s)=\frac{k}{Tj\omega+1}e^{-Lj\omega}$$

Where K=3.96 to 4.2, T=140 sec, L=7 sec

The gain and phase of the plant are as follows

$$\left|G_{IRD}(j\omega)\right|=\frac{k}{\sqrt{1+(\omega T)^2}}$$

$$Arg[G_{IRD}(j\omega)]=-\tan^{-1}(\omega T)-L\omega$$

B.Controller Design:

As per the FOPID controller, the value of integrator order ($\lambda=1$) and differentiator order ($\mu=1$) are taken as unity respectively then IOPID controller obtained as[10]

$$C_{IOPID}(s)=K_p\left(1+\frac{T_i}{s}+T_d s\right)$$

In this study , a method has been proposed to obtain the proportional gain constant (K_p) , the constant of integral gain (K_i) and the constant of derivative gain(K_d).Let the ϕ_m be the required phase margin and ω_{cp} be the frequency of the critical point on the Nyquist curve of plant $G_{IRD}(s)$ at which $\arg[(G(\omega_{cp}))] = -180^o$ and define gain margin as

$$g_m = \frac{1}{\left|G_{IRD}(j\omega_{cp})\right|} = k_c$$

Then, in order to make the phase margin of the system equal to ϕ_m and $\left|C(j\omega_{cp})G_{IRD}(j\omega_{cp})\right|_{dB} = 1$, the following equation must be satisfied.

$$C(j\omega_{cp}) = \frac{1}{\left|G_{IRD}(j\omega_{cp})\right|} e^{j\phi_m} = k_c \cos\phi_m + jk_c \sin\phi_m$$

According to IOPID controller transfer function(4), we can get the frequency response as

$$C(j\omega) = K_p + \frac{K_i}{j\omega} + j\omega K_d \qquad (9)$$

The gain and phase of controller are as follow,

$$\left|C(j\omega)\right| = \sqrt{K_p^{\ 2} + \left(\omega K_d - \left(K_i \big/ \omega K_p\right)\right)^2} \qquad (10)$$

$$Arg[C(j\omega)] = \tan^{-1}\left(\frac{K_d\omega^2 - K_i}{\omega K_p}\right) \qquad (11)$$

The open loop frequency response given as

$$P(j\omega) = C(j\omega)G_{IRD}(j\omega)$$

The gain and phase of the open loop frequency response as follows

$$\left|P(j\omega)\right| = \frac{\sqrt{K_p^{\ 2} + \left(\omega K_d - \left(K_i \big/ \omega K_p\right)\right)^2}}{\sqrt{1+(\omega T)^2}} \qquad (12)$$

$$Arg[P(j\omega)] = \tan^{-1}\left(\frac{K_d\omega^2 - K_i}{\omega K_p}\right) - \tan^{-1}(\omega T) - L\omega \qquad (13)$$

According to the design specification (i) and (ii), the robustness to gain variation in the plant, we can establish an equation about K_p as

$$K_p = \frac{1}{k}\sqrt{\frac{B_1}{1+A_1^2}} \qquad (14)$$

$$K_i = \frac{1}{2k}\left[\sqrt{\frac{1+A_1^2}{B_1}}\left(T\omega_{cp} + LB_1\omega_{cp}\right)^2 - A_1\omega_{cp}\sqrt{\frac{B_1}{1+A_1^2}}\right] \qquad (15)$$

$$K_d = \frac{1}{2k}\left[\sqrt{\frac{1+A_1^2}{B_1}}\left(T + LB_1\right)^2 - A_1\omega_{cp}^{-1}\sqrt{\frac{B_1}{1+A_1^2}}\right] \qquad (16)$$

Where $A_1 = \tan\left[\tan^{-1}(\omega_{cp}T) + L\omega_{cp} + \phi_m\right]$

$$B_1 = 1 + \omega_{cp}^{\ 2}T^2$$

From, the Nyquist curve, we set the gain margin and phase margin as follow by taking Nyquist and bode plot of system as $\omega_{cp} = 0.08$ rad/sec, $\phi_m = 60^0$

By solving the equation (14), (15) and (16), we get K_p, K_i and K_d directly

$K_p = 2.825$, $K_i = 0.0855$, $K_d = 9.074$

The IOPID controller obtained as

$$C_{IOPID} = 2.825 + \frac{0.0855}{s} + 9.74s \qquad (17)$$

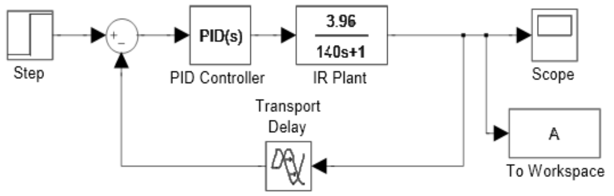


Figure 1. Block diagram of feedback control system with IOPID Controller

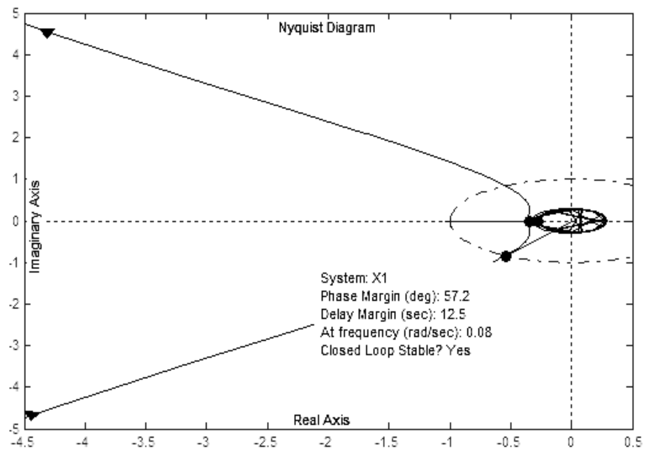


Figure 2. Nyquist plot of system $G_{IRD}(s)$

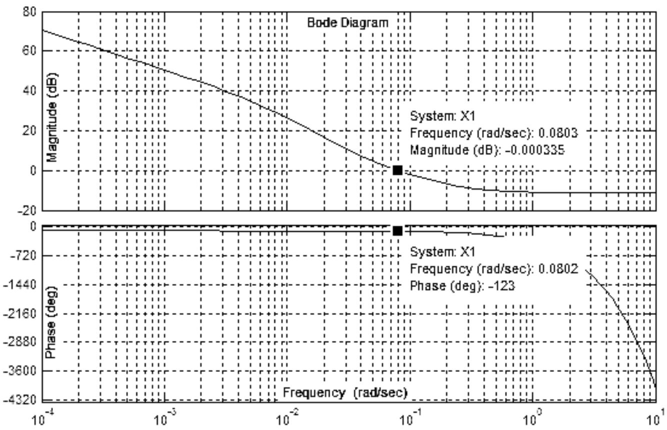


Figure 3. Bode plot of system $G_{IRD}(s)$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

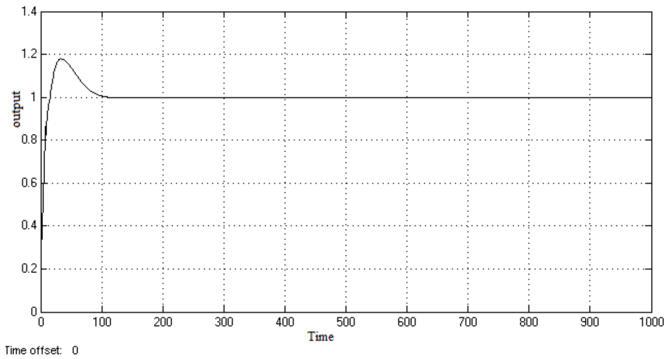


Figure 4. Step response of the system with C_{IOPID} by considering phase margin= (60^0) and ω_{cp}=0.08 rad/sec

C.Design of $PI^{\lambda}D^{\mu}$ Controller

This section represents the development of a tuning method of $PI^{\lambda}D^{\mu}$ controller for first order plus time delay system with gain parameter uncertainty structure. All parameters of the $PI^{\lambda}D^{\mu}$ controller are calculated to satisfy the performance of the plant. Five unknown parameters of the $PI^{\lambda}D^{\mu}$ controller are estimated solving five non-linear equations that satisfy five design criteria [10],[11], [12], [13], [14], [15]. Bode plot of FOPTD systems with gain parameter uncertainty structure are successfully combined with five design criteria to obtain the $PI^{\lambda}D^{\mu}$ controller. The phase and amplitude of the plant in frequency domain taken as,

$$Arg[G_{IRD}(j\omega)] = -\tan^{-1}(\omega T) - L\omega$$

$$|G_{IRD}(j\omega)| = \frac{k}{\sqrt{1+(\omega T)^2}}$$

D.FOPID Controller design

From fractional order PID controller transfer function (3), we can get its frequency response as follows,

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^{\lambda}} + (j\omega)^{\mu} K_d$$

$$C(j\omega) = K_p + (j\omega)^{-\lambda} K_i + (j\omega)^{\mu} K_d$$

$$(j\omega)^{-\lambda} = \omega^{-\lambda}\left(\cos(\frac{\pi}{2}\lambda) - j\sin(\frac{\pi}{2}\lambda)\right)$$

$$= k_p + K_i\omega^{-\lambda}\left(\cos(\frac{\pi}{2}\lambda) + j\sin(-\frac{\pi}{2}\lambda)\right) + K_d\omega^{\mu}\left(\cos(\frac{\pi}{2}\mu) + j\sin(\frac{\pi}{2}\mu)\right)$$

$$= k_p + K_i\omega^{-\lambda}\cos(\frac{\pi}{2}\lambda) + K_d\omega^{\mu}\cos(\frac{\pi}{2}\mu) + j\left(K_i\omega^{-\lambda}\sin(-\frac{\pi}{2}\lambda) + K_d\omega^{\mu}\sin(\frac{\pi}{2}\mu)\right)$$

$$a = k_p + K_i\omega^{-\lambda}\cos(\frac{\pi}{2}\lambda) + K_d\omega^{\mu}\cos(\frac{\pi}{2}\mu) \quad (18)$$

$$b = -K_i\omega^{-\lambda}\sin(\frac{\pi}{2}\lambda) + K_d\omega^{\mu}\sin(\frac{\pi}{2}\mu) \quad (19)$$

$$Arg[C(j\omega)] = \tan^{-1}\left(\frac{b}{a}\right)$$

$$|C(j\omega)| = \sqrt{a^2+b^2}$$

According to specification (a), the phase value $P(j\omega_c)$

$$Arg[P(j\omega_c)] = Arg[C(j\omega_c)] + Arg[G(j\omega_c)] = -\pi + \phi_m$$

$$= \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}(\omega_c T) - L\omega_c = -\pi + \phi_m \quad (20)$$

According to specification (a) we get the magnitude of $P(j\omega_c)$ as

$$|P(j\omega_c)| = |G(j\omega_c)||C(j\omega_c)|$$

$$= \left|\frac{k\sqrt{a^2+b^2}}{\sqrt{1+(\omega t)^2}}\right|_{dB} = 0 \ dB \quad (21)$$

According to specification (b) we get

$$\frac{d}{d\omega}\left(Arg(P(j\omega_c)\right) = 0$$

As

$$\frac{1}{1+(b/a)^2}\left(\frac{a.bu - b.au}{a^2}\right) - \frac{T}{1+\omega_c^2T^2} - L = 0 \quad (22)$$

Where

$$au = -K_i.\lambda.\omega_c^{-\lambda-1}\cos(\frac{\pi}{2}\lambda) + K_d.\mu.\omega_c^{\mu-1}\cos(\frac{\pi}{2}\mu) \quad (23)$$

$$bu = K_i.\lambda.\omega_c^{-\lambda-1}\sin(\frac{\pi}{2}\lambda) + K_d.\mu.\omega_c^{\mu-1}\sin(\frac{\pi}{2}\mu) \quad (24)$$

As per the specification (c) we get the high frequency noise rejection as

$$\left|T(j\omega_c) = \frac{k\sqrt{at^2+bt^2}}{(1+k.at)^2 + j(\omega T + k.bt)^2}\right|_{dB} \le -20dB \quad (25)$$

Where

$$at = K_p + K_i.\lambda.\omega_t^{-\lambda}\cos(\frac{\pi}{2}\lambda) + K_d.\mu.\omega_t^{\mu}\cos(\frac{\pi}{2}\mu) \quad (26)$$

$$bt = -K_i.\lambda.\omega_t^{-\lambda}\sin(\frac{\pi}{2}\lambda) + K_d.\mu.\omega_t^{\mu}\sin(\frac{\pi}{2}\mu) \quad (27)$$

As per the specification (d) we get Good disturbance rejection as

$$\left|S(j\omega_c) = \frac{\sqrt{1+(\omega T)^2}}{\sqrt{(1+K.as)^2+(K.bs+T\omega)^2}}\right|_{dB} \le -20dB \quad (28)$$

Where

$$as = K_p + K_i.\lambda.\omega_s^{-\lambda}\cos(\frac{\pi}{2}\lambda) + K_d.\mu.\omega_s^{\mu}\cos(\frac{\pi}{2}\mu) \quad (29)$$

$$bs = -K_i.\lambda.\omega_s^{-\lambda}\sin(\frac{\pi}{2}\lambda)+K_d.\mu.\omega_s^{\mu}\sin(\frac{\pi}{2}\mu) \qquad (30)$$

Steady state gain k does not have any effect on phase plot of the plant. In order to design robust $PI^{\lambda}D^{\mu}$ controller should be satisfied with transfer function of FOPID, namely ω_{cp} must be taken at the point 'x'. The constraint of phase margin and gain margin should be satisfied at point 'y' shows the minimum phase margin.

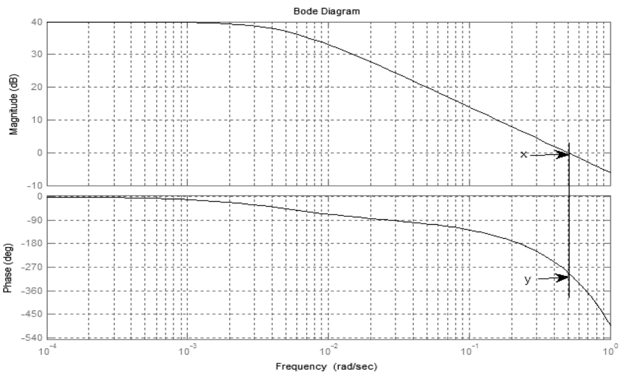


Figure 5. Bode plot of a FOPTD plant

By using Equation (20, 21, 22, 25, 28) five unknown parameter K_p, K_i, K_d, λ *and* μ can be solved by using FMINCON optimization toolbox of Mat Lab. Equation (21) is considered as a main equation and other equations are taken as non-linear constraints for optimization. Value of the all five unknown parameters are calculated to obtain the $PI^{\lambda}D^{\mu}$ controller to control the ceramic IR heater as Kp=0.6073 , Ki=6.1194,Kd=0.2045, λ =0.7815, μ =0.4454 and transfer function of fractional order PID controller given as

$$C(s)_{FOPID} = 0.6073 + \frac{0.2045}{s^{0.7815}} + 6.1194 s^{0.4454}$$

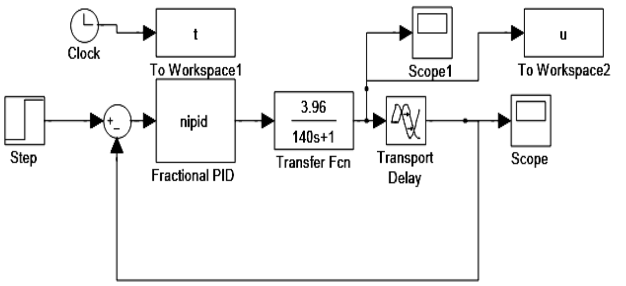


Figure 6 . Block diagram of feedback control system with FOPID controller

Ninteger is a toolbox of Mat Lab intended to help developing fractional (or non-integer) order controllers for single input-single output plant and to access their performance .The step response of the plant with FOPID controller obtained by using 'nintblock' of Mat Lab developed by Valerio, D [16],[17],[18],[19].

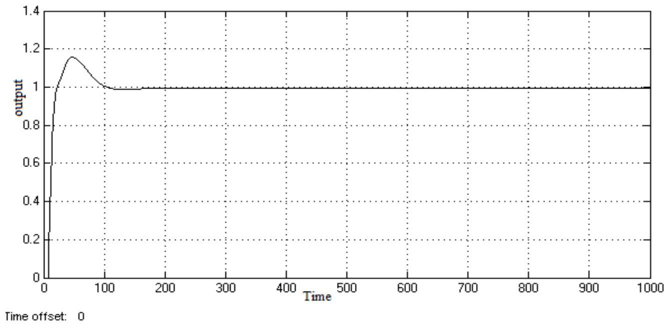


Figure 7. Step response of the system with C_{FOPID} by considering phase margin= (60^0) and ω_{cp}=0.08 rad/sec

The step response of the system shows that the system is more effective and robust to gain change and overshoot of the step responses is almost constant. Bode plot, Magnitude plots of T(s) and S(s) of the system obtained in Mat Lab. It shows that phase of the system are almost flat and almost constant within an interval around ω_{cp} with specified constraints [20][21].

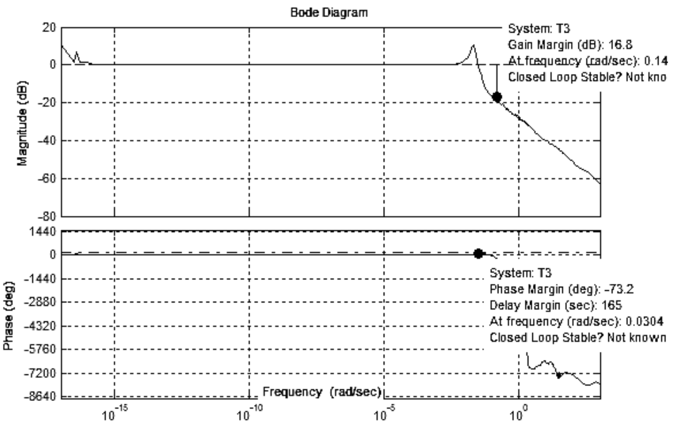
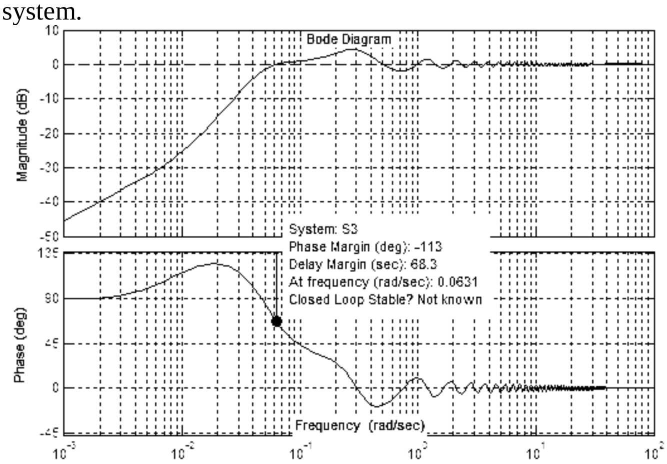


Figure 8. Magnitude of T(s) for C_{FOPID} (s)G(s)

From the figure of bode plot, T(s) and S(s), one can conclude that the controller satisfies the robust performance of the system.



Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Figure (9): Magnitude of S(s) for C_{FOPID} (s)G(s)

IV. Result

Step response specification and performance indices of C_{IOPID} and C_{FOPID} Controller

Step Response Specification	IOPID Controller	FOPID Controller
Rise Time	0.5333	0.6857
Peak Time	6	14
Peak Overshoot (%)	200	600
Settling Time (5%)	6.96	6.88
IAE	9.641	10.24
ISE	18.71	23.26

V. CONCLUSION

In this paper, two methods for tuning of $PI^{\lambda}D^{\mu}$ controller have been proposed. The first method is based on the idea of using unity power for the Integrator and derivative function of $PI^{\lambda}D^{\mu}$. By solving the equation obtained by taking consideration of the constraint, we get the value of the different three parameters optimized to achieve a better step response.

The proposed robust tuning method for a $PI^{\lambda}D^{\mu}$ controller to control first order plus time delay with parameter uncertainty structure designed. The five design constraints for the benefit of the Monje-Vinagre et. al. method was used to derive five non-linear equations. Value of unknown parameters K_p, K_i, K_d, λ *and* μ of the phase extremum of bode envelopes of the plant is used to satisfy robust performance of the system.

The simulation results (rise time, peak time, settling time , peak overshoot and performance indices(IAE,ISE)) show that the proposed method of $PI^{\lambda}D^{\mu}$ controller has better response than IOPID controller for the ceramic IR heater.

REFERENCES

1. Adonis, M and Khan, MTE. 2001. Infrared heating profile controller. Proceedings of the 3rd International Conference on Control Theory and Applications, Dec., 445-449.
2. Adonis, M and Khan, MTE. , "Analysis of the efficacy of a simplified infrared energy management system", Journal of Energy in Southern Africa , Vol 19 No 2 , May 2008
3. Adonis, M and Khan, MTE. , "PID Control of infrared radiative power profile for ceramic emitters"IFAC,2008
4. Podlubny, L. Dorcak, and I. Kostial, "On fractional derivatives, fractional-order dynamic system and PIλDμ -controllers", Proc. of the 36th IEEE CDC, San Diego, 1999.
5. K. J. Astrom and T. Hagglund, PID Controllers: Theory, Design and Tuning, Research Triangle Park, Instrument Society of America, 1995.
6. Podlubny, "Fractional-order Systems and Fractional-order Controllers", The Academy of Sciences Institute of Experimental Physics, UEF-03-94, Kosice, Slovak Republic, 1994.
7. Podlubny, "Fractional-order systems and PIλDμ -controllers," IEEE Trans. Automatic Control, vol. 44, pp. 208-214, 1999.
8. C.A. Monje, B.M. Vinagre, Y.Q. Chen, V. Feliu, P. Lanusse and J.Sabatier, "Proposals for Fractional PIλDμ Tuning", The First IFAC Symposium on Fractional Differentiation and its Applications 2004, Bordeaux, France, July 19-20, 2004.
9. C. A. Monje, B. M. Vinagre, V. Feliu, Y.Q. Chen, "Tuning and auto-tuning of fractional order controllers for industry applications", Control Engineering Practice, vol. 16, pp.798–812, 2008.
10. Charef, A., Fergani, N. (2010). "PIλDμ Controller Tuning For Desired Closed-Loop Response Using Impulse Response", Proceedings of Fractional Differentiation and its Applications, Badajoz, Spain, October 2010
11. D.Xue and C.Zhao, 'Fractional order PID controller design for fractional order systems' [J]. Control Theory and Applications, Vol.24,No.5: 771-776, 2007.(in Chinese)
12. S. E. Hamamci, "An algorithm for stabilization of fractional-order time delay systems using fractional- order PID controllers," IEEE Trans. Automat. Control, Vol. 52, pp. 1964-1969, 2007.
13. N.Tan, O.F.Ozguvenand, M.M. Ozyetkin,"Robust stability analysis of fractional order interval polynomials," ISA Transactions, vol. 48, pp.: 166-172, 2009.
14. C. A. Monje, Design methods of fractional order controllers for industrial applications. Ph.D. thesis, University of Extremadura, Spain, 2006.
15. D. Valerio, "Ninteger v. 2.3. Fractional control toolbox for matlab," 2005.
16. Valério, D. (2001). Non-integer order robust control: an application. In: Student Forum, Porto, 25-28.
17. Valério, D. and Sá da Costa, J. (2002). Time domain implementations of non-integer order controllers. In: Controlo, Aveiro, 353-358.
18. Valério, D. and Sá da Costa, J. (2003a). Optimisation of non-integer order control parameters for a robotic arm. In: International Conference on Advanced Robotics, Coimbra.
19. YangQuan Chen and Ivo Petráš and DingyuXue, "Fractional order control – a Tutorial," Proceedings of the 2009 American Control Conference, St. Louis, MO, USA, 2009.
20. C. Yeroglu, N. Tan, "Development of a Toolbox for Frequency Response Analysis of Fractional Order Control Systems", *19th European Conference on Circuit Theory and Design*, Antalya, August 2009.
21. Tan, N., Yeroglu, C.:"Note on fractional-order proportional–integral–differential controller design" IET Control Theory Appl., 2011, Vol. 5, Issue. 17, pp. 1978–1989