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A Distributed Delay Model With a Prey, Predator and Competitor

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Abstract: ***In this paper we tend to study the stability analysis of prey(N_1) ,predator(N_2) and competitor(N_3) model. Here competitor is vying with prey and is neutral with the predator. Besides that, the carrying capacities and the death rates relating to all the three species are also considered for investigation. The delay arguments are introduced in the interaction between prey and competitor species. The model is studied by a couple of integro-differential equations. The axial equilibrium point is known and local stability is studied at this point. Global stability analysis is carried out at this point by constructing appropriate lyapunov's function. Further the system with delay and without delay arguments are compared by choosing suitable parameters and delay arguments further stabilize or destabilize the system is shown by aid of MATLAB simulation.***

Index Terms: prey, predator, competitor equilibrium point, local and global stability, numerical simulation
Mathematical classification: 34DXX

I.INTRODUCTION:

Ecological modeling gained importance in recent decades. Mathematical analysis in ecology gained lots of importance due to the interdisciplinary knowledge. The primary population models are studied by Lokta [1] and Volterra [2]. Kapur [3, 4] mentioned mathematical models resembling biology, ecology and medicine. Paparao [5, 6] studied the three species models with the interactions among prey, predator and competitor. Completely different interactions in population dynamics with elaborated analysis was studied by Freedman [7] and also the mathematical aspects of ecological models were studied by Paul Colinvaux [8]. The Differential equations are accustomed represent the models in ecology. Braun [9] and Simon's [10] wide studied the applications of differential equations. In this paper, we tend to study the dynamics of a prey, a predator and a competitor model. The delay arguments are introduced within the interaction between prey and competitor species. We applied a system of integro differential equations to characterize the model and derived axial equilibrium point. Local stability is analyzed at this point. By constructing a suitable Lyapunov's function, the global stability is studied. Numerical simulation is carried out with the aid of MAT LAB simulation in support of stability analysis. A rich dynamics is observed with different parameters involved in this model.

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II. MATHEMATICAL MODEL

We considered a model with Prey (N_1), predator (N_2) and a competitor (N_3) species is taken for investigation. The competitor (N_3) is vying with a prey (N_1) and neutral with the predator (N_2). Besides that, the carrying capacities and also the death rates are associated to all the three species are considered for investigation. The delay is introduced within the interaction of prey and competitor species. The model is built by a system of integro differential equations shown as below.

$$\frac{dN_1}{dt} = a_1N_1\left[1-\frac{N_1}{c_1}\right] - \alpha_{12}N_1N_2 - \alpha_{13}N_1\int_{-\infty}^{t} k_1(t-u)N_3(u)du - d_1N_1$$
$$\frac{dN_2}{dt} = a_2N_2\left[1-\frac{N_2}{c_2}\right] + \alpha_{21}N_1N_2 - d_2N_2$$
$$\frac{dN_3}{dt} = a_3N_3\left[1-\frac{N_3}{c_3}\right] - \alpha_{31}N_3\int_{-\infty}^{t} k_2(t-u)N_1(u)du - d_3N_3 \quad (2.1)$$

A. Nomenclature

S.No	Parameter	Description
1	N_1, N_2 & N_3	Prey, predator and competitor populations respectively
2	a_1, a_2, a_3	Growth rates of prey, predator and competitor respectively.

3	c_1,c_2,c_3	Carrying capacities of prey, predator and competitor respectively
4	α_{12}	Interaction coefficient of prey and predator
5	α_{21}	Interaction coefficient of predator and prey
6	α_{13}	Interaction coefficient of prey and competitor
7	α_{31}	Interaction coefficient of competitor and prey
8	d_1,d_2,d_3	Death rates of prey, predator and competitor respectively.
9	$k_1(t-u)\&k_2(t-u)$	Kernel strengths of prey and competitor

Notations: $\dfrac{a_1}{c_1}=k_1^*,\dfrac{a_2}{c_2}=k_2^*,\dfrac{a_3}{c_3}=k_3^*.$

Assume throughout the analysis $(a_i-d_i)>0\ (i=1,2,3)$

Rewrite the system of equations by assuming t-u = z and convert in to the following system of equations as given below

$$\frac{dN_1}{dt}=a_1N_1\left[1-\frac{N_1}{c_1}\right]-\alpha_{12}N_1N_2-\alpha_{13}N_1\int_0^\infty k_1(z)N_3(t-z)dz-d_1N_1$$
$$\frac{dN_2}{dt}=a_2N_2\left[1-\frac{N_2}{c_2}\right]+\alpha_{21}N_1N_2-d_2N_2$$
$$\frac{dN_3}{dt}=a_3N_3\left[1-\frac{N_3}{c_3}\right]-\alpha_{31}N_3\int_0^\infty k_2(z)N_1(t-z)dz-d_3N_3$$

(2.2)

Here the kernels $k_1(z)$ and $k_2(z)$ are satisfies the subsequent conditions

$$\int_0^\infty k_1(z)dz=1,\int_0^\infty k_2(z)dz=1$$
$$\int_0^\infty zk_1(z)dz<\infty,\int_0^\infty zk_2(z)dz<\infty$$

(2.3)

III. EQUILIBRIUM STATES

The axial equilibrium points is given by

VIII. E_1 : Co-existing state :

$$\overline{N_1}=\frac{(a_1-d_1)k_2^*k_3^*-\alpha_{12}k_3^*(a_2-d_2)-\alpha_{13}k_2^*(a_3-d_3)}{k_2^*(k_1^*k_3^*-\alpha_{13}\alpha_{31})+k_3^*\alpha_{12}\alpha_{21}},$$
$$\overline{N_2}=\frac{k_2^*\left[(a_2-d_2)k_3^*-\alpha_{13}\alpha_{31}\right]+\alpha_{21}\left[(a_1-d_1)k_3^*-(a_3-d_3)\alpha_{13}\right]}{k_2^*(k_1^*k_3^*-\alpha_{13}\alpha_{31})+k_3^*\alpha_{12}\alpha_{21}},$$
$$\overline{N_3}=\frac{(a_3-d_3)k_1^*k_2^*+\alpha_{21}\left[(a_3-d_3)\alpha_{21}+(a_2-d_2)\alpha_{31}\right]-(a_1-d_1)\alpha_{31}k_2^*}{k_2^*(k_1^*k_3^*-\alpha_{13}\alpha_{31})+k_3^*\alpha_{12}\alpha_{21}}.$$

(3.1)

The equilibrium state exists only when

$$\overline{N_1}>0\overline{N_2}>0,\overline{N_3}>0.$$

(3.1.a)

IV. LOCAL STABILITY OF THE EQUILIBRIUM POINT E8

Theorem: The axial equilibrium point $E_1\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$ is locally asymptotically stable

If $k_1^*k_3^*>\alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)$

Proof: The community matrix for the system is given by

$$J=\begin{bmatrix} a_1-2k_1^*\overline{N_1}-\alpha_{12}\overline{N_2}-d_1 & -\alpha_{12}\overline{N_1} & -\alpha_{13}\overline{N_1}k_1(\lambda) \\ \alpha_{21}\overline{N_2} & a_2-2k_2^*\overline{N_2}+\alpha_{21}\overline{N_1}-d_2 & 0 \\ -\alpha_{31}\overline{N_3}k_2(\lambda) & 0 & a_3-2k_3^*\overline{N_3}-\alpha_{32}\overline{N_3}-d_3 \end{bmatrix}$$

(4.1)

Where $k_1(\lambda)\&k_2(\lambda)$ is laplace transforms of delay kernels

$$=\begin{bmatrix} -k_1^*\overline{N_1} & -\alpha_{12}\overline{N_1} & -\alpha_{13}\overline{N_1}k_1(\lambda) \\ \alpha_{21}\overline{N_2} & -k_2^*\overline{N_2} & 0 \\ -\alpha_{31}\overline{N_3}k_2(\lambda) & 0 & -k_3^*\overline{N_3} \end{bmatrix}$$

(4.2)

The characteristic equation of (4.2) be

$$\lambda^3+b_1\lambda^2+b_2\lambda+b_3=0$$

(4.3)

Where

$$b_1=k_1^*\overline{N_1}+k_2^*\overline{N_2}+k_3^*\overline{N_3}$$

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$$b_2 = k_2^* k_3^* \overline{N_2}\,\overline{N_3} + k_1^* k_2^* \overline{N_1}\,\overline{N_2} + k_1^* k_3^* \overline{N_1}\,\overline{N_3} + \alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2} - \alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3} k_1(\lambda) k_2(\lambda)$$

$$\begin{aligned} b_3 &= \overline{N_1}\,\overline{N_2}\,\overline{N_3}\left[k_1^* k_2^* k_3^* + k_3^*\alpha_{12}\alpha_{21} - k_2^*\alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\right] \\ &= \overline{N_1}\,\overline{N_2}\,\overline{N_3}\left[k_2^*\left(k_1^* k_3^* - \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\right) + k_3^*\alpha_{12}\alpha_{21}\right] \end{aligned} \quad (4.4)$$

Using Routh-Hurwitz criteria one will show that the system is stable if

$b_1 > 0$, $(b_1 b_2 - b_3) > 0$ and $b_3(b_1 b_2 - b_3) > 0$.

Clearly here $b_1 > 0$

The algebraic calculations of $(b_1 b_2 - b_3)$ gives that

$$\begin{aligned} (b_1 b_2 - b_3) &= 2k_1^* k_2^* k_3^* \overline{N_1}\,\overline{N_2}\,\overline{N_3} + k_1^* \overline{N_1}^2\left(k_1^* k_2^* \overline{N_2} + k_1^* k_3^* \overline{N_3} + \alpha_{12}\alpha_{21}\overline{N_2}\right) \\ &\quad + k_2^* \overline{N_2}^2\left(k_2^* k_3^* \overline{N_3} + k_1^* k_2^* \overline{N_1} + \alpha_{12}\alpha_{21}\overline{N_1}\right) + k_3^* \overline{N_3}^2\left(k_1^* k_3^* \overline{N_1} + k_2^* k_3^* \overline{N_2} - \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\overline{N_1}\right) \\ &= 2k_1^* k_2^* k_3^* \overline{N_1}\,\overline{N_2}\,\overline{N_3} + k_1^* \overline{N_1}^2\left(\overline{N_2}\left(k_1^* k_2^* + \alpha_{12}\alpha_{21}\right) + k_1^* k_3^* \overline{N_3}\right) \\ &\quad + k_2^* \overline{N_2}^2\left(k_2^* k_3^* \overline{N_3} + \overline{N_1}\left(k_1^* k_2^* + \alpha_{12}\alpha_{21}\right)\right) + k_3^* \overline{N_3}^2\left(\overline{N_1}\left(k_1^* k_3^* - \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\right) + k_2^* k_3^* \overline{N_2}\right) \end{aligned}$$

(4.5)

From the equation (4.5) $(b_1 b_2 - b_3) > 0$ if $k_1^* k_3^* > \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)$

And from equation (4.4) , $b_3 > 0$ if $k_1^* k_3^* > \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)$

Therefore $b_3(b_1 b_2 - b_3) > 0$ if $k_1^* k_3^* > \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)$

Therefore the axial equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is locally asymptotically stable if $k_1^* k_3^* > \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)$

V. GLOBAL STABILITY

Theorem: The axial equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is globally asymptotically stable.

Proof: Choose the proper lyapunov function

$$V(N_1, N_2, N_3) = \sum_{i=1}^{3} N_i - \overline{N_i} - \overline{N_i}\log\left(\frac{N_i}{\overline{N_i}}\right) + \frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\int_{t-z}^{t}[N_3 - \overline{N_3}]^2 du dz + \frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\int_{t-z}^{t}[N_1 - \overline{N_1}]^2 du dz$$

(5.1)

Calculate the time derivate along the solutions of equation (2.1)

$$\begin{aligned} V^1(t) &= \sum_{i=1}^{3}\frac{\left[N_i - \overline{N_i}\right]}{N_i}N_i^1 + \frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3 - \overline{N_3}\right]^2 dz - \frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3(t-z) - \overline{N_3}\right. \\ &\quad + \frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1 - \overline{N_1}\right]^2 dz - \frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1(t-z) - \overline{N_1}\right]^2 dz \end{aligned}$$

(5.2)

from the equations (2.2) and (2.3) we have

$$\begin{aligned} V^1(t) &= \left[N_1 - \overline{N_1}\right]\left(a_1 - k_1^* N_1 - \alpha_{13}\int_0^\infty k_1(z)N_3(t-z)dz - d_1\right) \\ &\quad + \left[N_2 - \overline{N_2}\right]\left(a_2 - k_2^* N_2 + \alpha_{21}N_1 - d_2\right) + \left[N_3 - \overline{N_3}\right]\left(a_3 - k_3^* N_3 - \alpha_{31}\int_0^\infty k_2(z)N_1(t-z)dz - d_3\right) \\ &\quad + \frac{1}{2}\alpha_{13}\left[N_3 - \overline{N_3}\right]^2 + \frac{1}{2}\alpha_{31}\left[N_1 - \overline{N_1}\right]^2 - \frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_2(t-z) - \overline{N_2}\right]^2 dz - \frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1(t-z) - \overline{N_1}\right]^2 dz \end{aligned}$$

Choose the values a_1, a_2 & a_3 from the equation (2.2) we can write

$$\left(a_1 = k_1^* \overline{N_1} + \alpha_{13}\int_0^\infty k_1(z)N_3(t-z)dz + d_1\right)$$

$$\left(a_2 = k_2^* \overline{N_2} - \alpha_{21}\overline{N}_1 + d_2\right) \& \left(a_3 = \alpha_{31}\int_0^\infty k_2(z)\overline{N}_1(t-z)dz + k_3\overline{N_3} + d_3\right)$$

$$\begin{aligned} V^1(t) &= -k_1^*\left(N_1 - \overline{N_1}\right)^2 - k_2^*\left(N_2 - \overline{N_2}\right)^2 - k_3^*\left(N_3 - \overline{N_3}\right)^2 + \alpha_{21}\left(N_1 - \overline{N_1}\right)\left(N_2 - \overline{N_2}\right) + \frac{1}{2}\alpha_{13}\left[N_3 - \overline{N_3}\right]^2 + \frac{1}{2}\alpha_{31}\left[N_1 - \overline{N_1}\right]^2 \\ &\quad - \frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3(t-z) - \overline{N_3}\right]^2 dz + \frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1 - \overline{N_1}\right]^2 dz - \frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1(t-z) - \overline{N_1}\right]^2 dz \end{aligned}$$

(5.3)

Using the inequality $ab \le \frac{a^2 + b^2}{2}$, $\int_0^\infty k_1(z)\left[N_3(t-z) - \overline{N_3}\right]^2 \le \int_0^\infty k_1(z)dz = 1$

& $\int_0^\infty k_2(z)\left[N_1(t-z) - \overline{N_1}\right]^2 \le \int_0^\infty k_2(z)dz = 1,$

$$\begin{aligned} V^1(t) &= -k_1^*\left(N_1 - \overline{N_1}\right)^2 - k_2^*\left(N_2 - \overline{N_2}\right)^2 - k_3^*\left(N_3 - \overline{N_3}\right)^2 + \frac{\alpha_{21}}{2}\left[\left(N_1 - \overline{N_1}\right)^2 + \left(N_2 - \overline{N_2}\right)^2\right] \\ &\quad + \frac{1}{2}\alpha_{13}\left[N_3 - \overline{N_3}\right]^2 + \frac{1}{2}\alpha_{31}\left[N_1 - \overline{N_1}\right]^2 - \frac{1}{2}\alpha_{13} - \frac{1}{2}\alpha_{31} \end{aligned}$$

$$V^1(t) \le -\left\|\left(k_1^* - \frac{1}{2}\alpha_{13} - \frac{1}{2}\alpha_{21}\right)\right\|\left(N_1 - \overline{N_1}\right)^2 - \left\|\left(k_2^* - \frac{1}{2}\alpha_{21}\right)\right\|\left(N_2 - \overline{N_2}\right)^2 - \left\|\left(k_3^* - \frac{1}{2}\alpha_{31}\right)\right\|\left(N_3 - \overline{N_3}\right)^2 - \frac{1}{2}\left\|\left(\alpha_{13} + \alpha_{31}\right)\right\|$$

$$V^1(t) \le -\mu \sum_{i=1}^{3} \left[N_i - \overline{N_i}\right]^2 < 0$$

Where

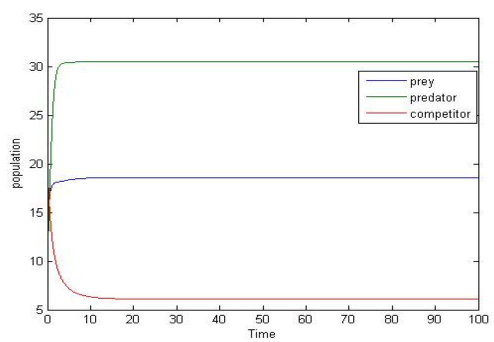
$$\mu = \min\left(k_1^* + k_2^* + k_2^* - \frac{1}{2}\alpha_{13} - \frac{1}{2}\alpha_{31} - \frac{1}{2}\alpha_{21} - (\alpha_{13} + \alpha_{31})\right)$$

Therefore the system is globally stable at the axial equilibrium point

VI.NUMERICAL SIMULATION

Fig A: denotes Time series analysis of system
Fig B: denotes phase portrait of system
Example 1: a_1=1.5; a_2=1.6; a_3=1.5; α_{12}=0.01; α_{13}=0.01; α_{21}=0.02; α_{31}=0.06; c_1=25, c_2 =25, c_3=25, d_1 =0.02, d_2 = 0.02, d_3 =0.02, N_1=15; N_2 =10; N_3=20.

The system exhibit neutral stability and connection the fastened equilibrium point E(19,31,6) that are shown with in the following graphs wherever the delay argument t is absent for the system of equations (2.2)



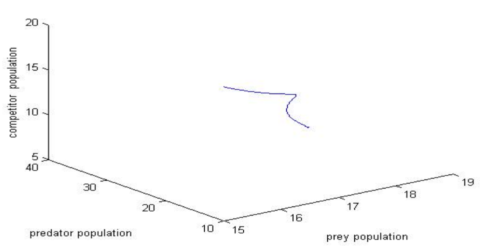
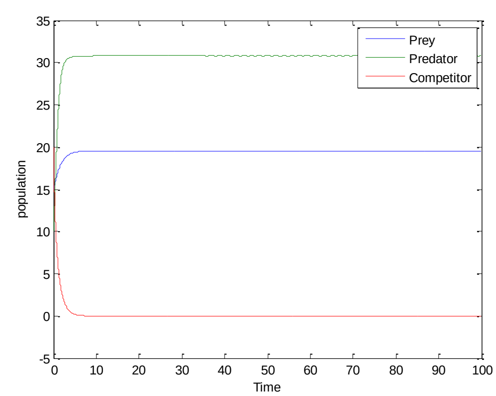
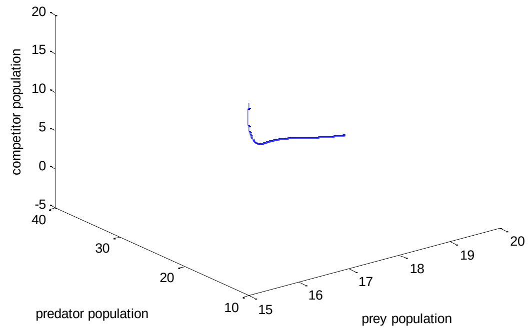


Fig1. (A) Fig1.
(B)

A. Nature of the System with Different Delay Kernels and carrying capacities:

S.No	Kernel strengths and equilibrium positions	Nature of system
1	α=0. 5, β=0.5 , c_1=25, c_2 =25, c_3=25 E (20, 31, 0). Graph (2A &2B)	The competitor population is extinct and prey, predator populations are converging to fixed equilibrium point. The system exhibits neutrally stable behavior.
2	α=1. 5, β=1.5, c_1=25, c_2 =25, c_3=25 E (18, 30, 13). Graph (3A &3B)	As on the delay kernels are increasing, the competitor population is also increasing and stabilizes at a fixed point. Hence the system is Neutrally stable.
3	α=0. 5, β=0.5, c_1=50, c_2 =50, c_3=50 E (28, 66, 0).	The competitor population is extinct and the prey and predator populations are increasing from its initial population strength. Hence the system becomes unstable.
4	α=0. 5, β=0.5, c_1=125, c_2 =125, c_3=125 E (0, 123, 123).	The system becomes unstable because the prey population is extinct.





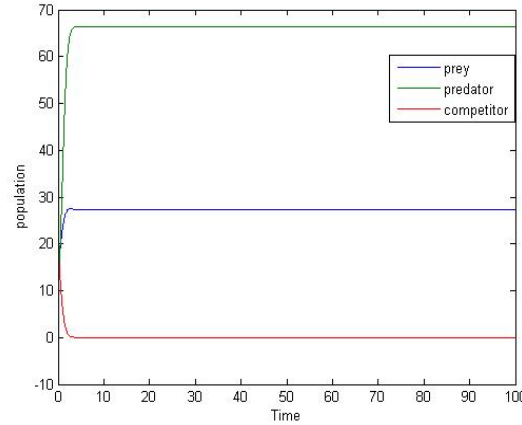
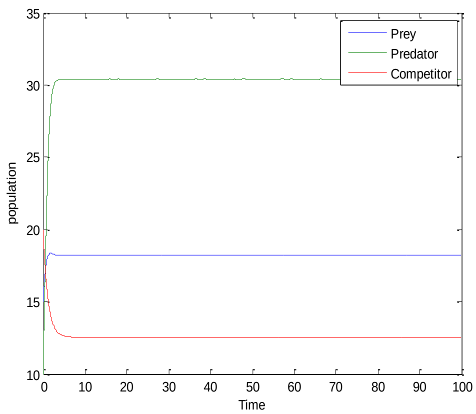


Fig2. (A)
Fig2. (B)



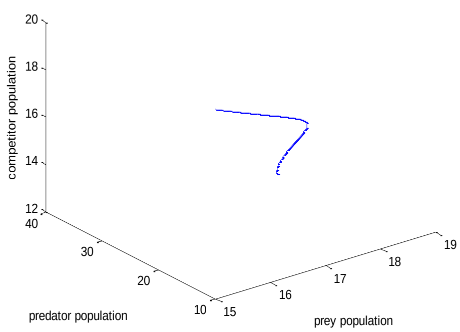


Fig3. (A)
Fig3. (B)

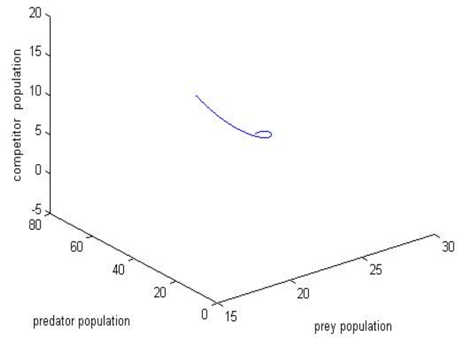
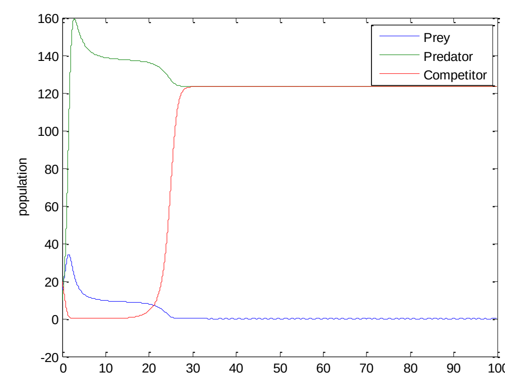


Fig4. (A)
Fig4. (B)



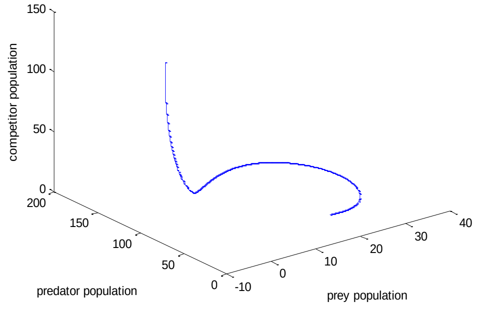
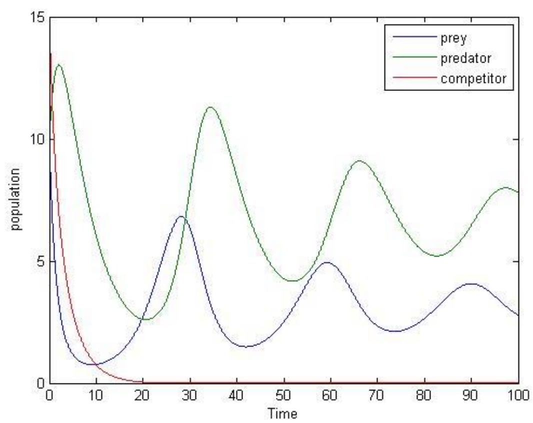


Fig5. (A)

Fig5. (B)

Example 2: $a_1=0.5$; $a_2=0.05$; $a_3=0.05$; $\alpha_{12}=0.04$; $\alpha_{13}=0.03$; $\alpha_{21}=0.05$; $\alpha_{31}=0.02$; $c_1=50$, $c_2=50$, $c_3=50$, $d_1=0.2$, $d_2=0.2$, $d_3=0.3$, $N_1=10$; $N_2=10$; $N_3=15$.

The system exhibits oscillating behavior and competitor population is extinct and eventually converging to a equilibrium point E (3, 8, 0), hence the system is asymptotically stable, the time series plot and phase portrait are shown in the graphs 6(A) and 6(B) respectively



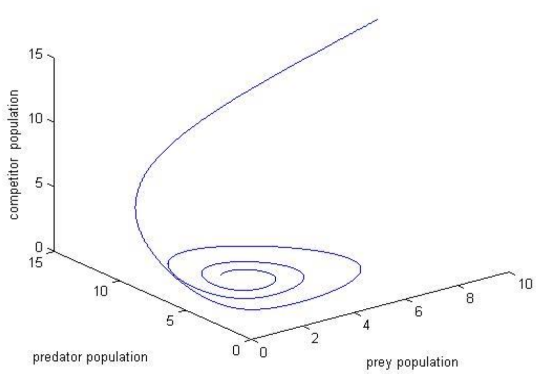


Fig6. (A)

Fig6. (B)

B. Nature of the System with Different Delay Kernels and carrying capacities:

S.No	Kernel strengths and equilibrium positions	Nature of system
1	$\alpha=0.5$, $\beta=0.5$, $c_1=50$, $c_2=50$, $c_3=50$ E (3, 9, 0). Graph (7A &7B)	The competitor population is extinct and prey, predator populations are converging to fixed equilibrium point, Hence the system isasymptoticallystable.
2	$\alpha=1.5$, $\beta=1.5$, $c_1=50$, $c_2=50$, $c_3=50$ E (3, 8, 0). Graph (8A &8B)	The competitor population is extinct and prey, predator populations are converging to a fixed equilibrium point. Hence the system is asymptotically stable.
3	$\alpha=0.05$, $\beta=0.05$, $c_1=50$, $c_2=50$, $c_3=50$ E (1, 8, 0). Graph (9A &9B)	Initially all three populations are extinct as on the time scale reaches to t =25. Later on after t = 60, there is a growth in prey population and after t = 90, the population of the prey converges to a fixed equilibrium point. The predator population undergoes fluctuations between t = 80 to 100 thereafter its population converging to a fixed equilibrium point .Hence the system is Asymptotically stable.
4	$\alpha=0.01$, $\beta=0.1$, $c_1=25$, $c_2=25$, $c_3=25$ E (13, 0, 0). Graph (10A &10B)	Initially all three populations are extinct as on the time scale reaches to t =30. Later on after t = 80, there is a sudden growth in prey population, makes system unstable.

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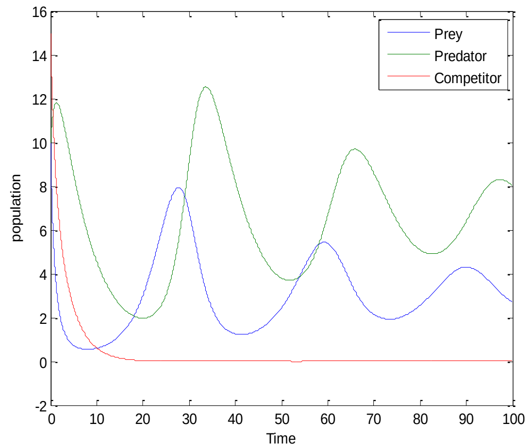


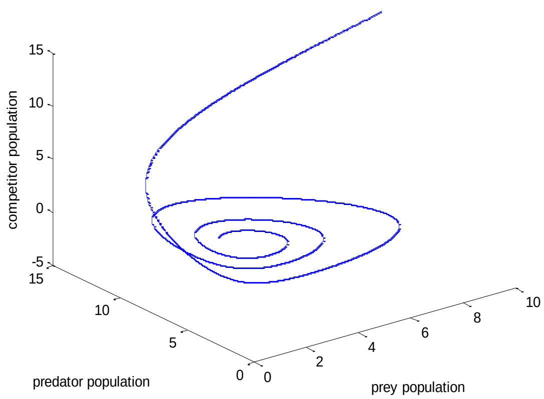
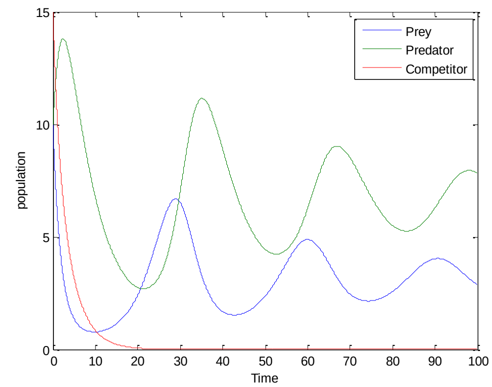


Fig7. (A)
Fig7. (B)



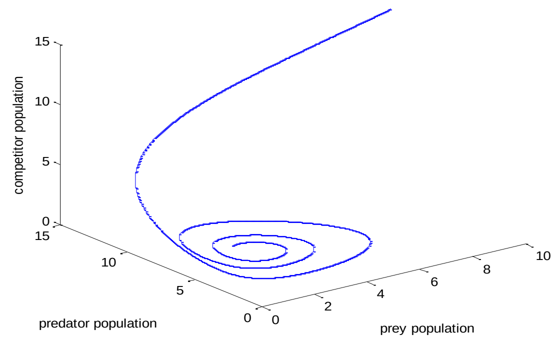
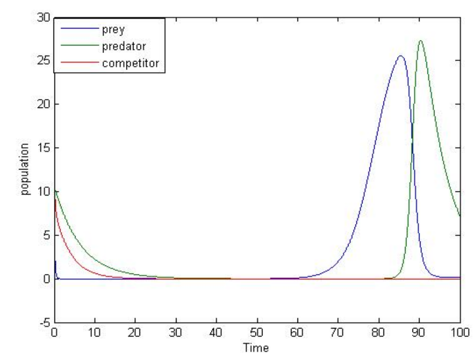


Fig8. (A)
Fig8. (B)



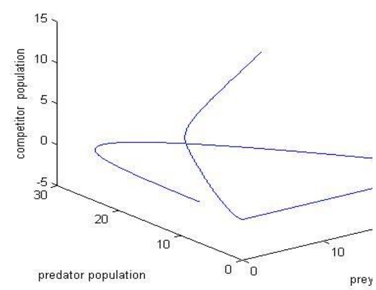
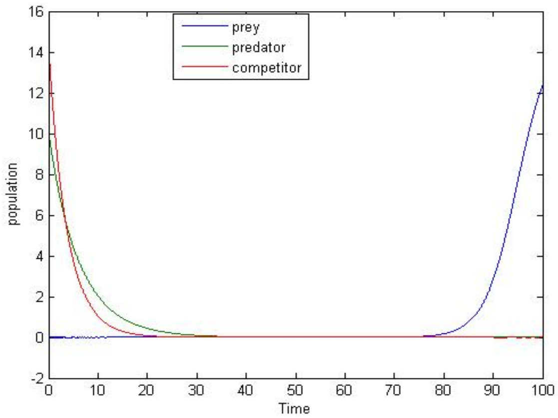


Fig9. (A)
Fig9. (B)



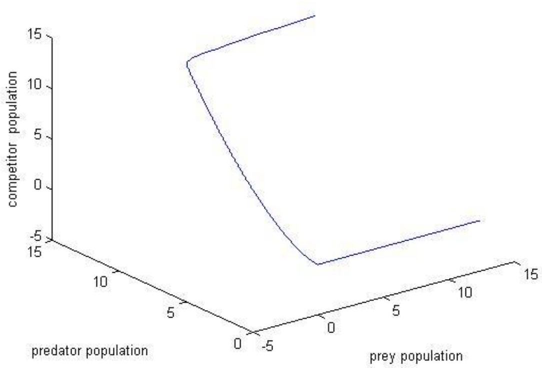


Fig10. (A)
Fig10. (B)

VII. CONCLUSION

The stability analysis of a Prey (N_1), predator (N_2) and a competitor (N_3) species is taken for investigation. We find the axial equilibrium points of the model and the system is locally stable if $k_1^* k_3^* > \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)$. By constructing the suitable Lyapunov's function the global stability is studied extensively.

The numerical simulation is carried out for two set of the parametric values of the system shown in equation (2.1). The simulation shows that for the specified parameters in examples from 1 and 2 with different kernels and carrying capacities of the three species the system exhibits

(i) Neutrally stable behavior [Example 1] without applying delay arguments

(ii) Asymptotically stable [Example 2] without applying delay arguments

We employee the different delays and carrying capacities on example 1 which is shown in table 6.1. We identified the kernels and carrying capacities for which the prey, competitor populations are extinct and direct the system to become unstable. The same procedure is adopted on example 2 and derives the suitable kernels which make the system unstable as well as asymptotically stable.

The delay kernels and carrying capacities are sensible in drawing the conditions for stability of the proposed model. The delay arguments can make stable system to unstable or vice versa.

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