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Strongly Prime Ternary Semi Groups

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Abstract: In this article we imported the concept of right strongly prime ternary semi group and developed some interesting properties of right strongly prime ternary semi group.

I. INTRODUCTION

The concept of a semi group is very simple and plays a large role in the development of Mathematics. The characterization of prime ideals in semi groups is developed by Anjaneyulu [1] in the year 1981. The literature of the theory of ternary operations is vast and scatters over diverse areas of mathematics. In 1932 Lehmer introduced the notion of Ternary algebraic systems. The algebraic system is said to be triplexes was imported by Lehmer which turn out to be commutative-ternary groups. Kasner also developed the structures and give the ideal of *n*-ary algebras. Ternary semi groups are universal algebras with one associative ternary operation. The ideal theory in ternarysemigroup was imported by Sioson. Jayalalitha[3] discussed some properties of rightideals in ordered semi groups. Some previous works of ternarysemigroup may be found in [2,4]. Throughout this article T will always denoted by a ternarysemigroup with zero and $T^* = T \backslash \{0\}$. In this article we imported and study the concept of rightstronglyprime ternarysemigroup.

II. RIGHT STRONGLY PRIME TERNARY SEMIGROUPS

Definition 2.1: A ternarysemigroup T is known as a rightstronglyprime if for every $x \in T^*$, $\exists$ finite subsets P_1, P_2, P_3 of T $\ni xP_1P_2P_3y = \{0\} \Rightarrow y = 0 \ \forall y \in T$.

Proposition 2.2: A ternarysemigroup T is right stronglyprime iff for every $x \in T^*$, $\exists$ a finite subset P of T $\ni xTTTy = \{0\} \Rightarrow y = 0 \ \forall$ y∈T.

Proof: Assume that T is a right stronglyprime ternarysemigroup. Let $x \in T^*$. Then $\exists$ finite subsets P_1, P_2, P_3 of $T \ni xP_1P_2P_3y = \{0\} \Rightarrow y = 0 \ \forall y \in T$. Let $P = P_1 \cup P_2 \cup P_3$. Then $P_1, P_2, P_3 \subseteq P$ and P is finite. Suppose $xPPPy = \{0\} \ \forall y \in T$. Then $xP_1P_2P_3y \subseteq xPPPy = \{0\} \ \forall y \in T$. This shows that $y = 0 \ \forall y \in T$.

Converse part is obvious.

Theorem 2.3: Every right strongly prime ternarysemigroup is a prime ternarysemigroup.

Proof: Suppose that T is a right strongly prime ternarysemigroup. Let P, Q, R be three ideals of T such that $PQR = \{0\}$. Supppose that $P \neq \{0\}$ and $Q \neq \{0\}$. Since $P \neq \{0\}$, there exists $a(\neq 0) \in P$. Since T is a right strongly prime ternarysemigroup, by proposition 2.2, there exists a finite subset F of $T \ni aFFFy = \{0\}$ implies that $y = 0$ $\forall y \in T$. Now $aFFF(QTR) = (aFF)(FQT)R \subseteq (PTT)(TQR) \subseteq PQR = \{0\}$. This implies that $QTR = \{0\}$. Again, since $Q \neq \{0\}$, $\exists$ $q(\neq 0) \in Q$ and for this $q \neq 0$, there exist a finite subset F' of T $\ni$ $qF'F'F'r \subseteq QTTTR \subseteq QTR = \{0\}$ for $r \in R$. This implies that $r = 0$. Since r is an arbitrary element of R, we find that $R = \{0\}$. This shows that $\{0\}$ is a primeideal of T and hence T is a prime ternarysemigroup.

Theorem 2.4: Let T be a ternarysemigroup with unital element 'e'. If T is right strongly prime then if I is a nonzero ideal of T, $\exists$ finite subsets F' of I and F of $T \ni F''Fy = \{0\}$ implies that $y = 0 \ \forall y \in T$.

Proof: Assume that T is a rightstronglyprime ternarysemigroup and I be a nonzero ideal of T. Since I is a nonzero ideal of T, there exist $x(\neq 0) \in I$. Again, since T is a rightstronglyprime, $\exists$ a finite subset F of $T \ni xFFFy = 0 \Longrightarrow$ y=0 ∀ y∈T. Let $F' = xF'$ F. Then $F' = xFF \subseteq IFF \subseteq$ I. i.e., F' is a finite subset of I. Thus there exist finite subsets F' of I and F of T $\ni$ $F'Fy = \{0\}$ implies that $y = 0 \ \forall y \in$ T.

Theorem 2.5: Let T be a ternarysemigroup with unital element 'e'. If $x \in T^*$ there exist $t \in T$ and finite subsets F', F of $T \ni$ $xtF'Fy = \{0\}$ implies that $y = 0 \ \forall \ y \in T$ then T is rightstronglyprime.

Proof: Let $a \in T^*$. Then by our assumption there exist $t \in T$ and finite subsets F', F of $T \ni$ $xtF'Fy = \{0\}$ implies that $y = 0 \ \forall y \in T$. Now taking $F_1 = \{t\}$, $F_2 = F'$ and $F_3 = F$ we find that there exists finite subsets F_1, F_2, F_3 of T such that $aF_1F_2F_3y = \{0\}$ implies that $y = 0$. Hence T is rightstronglyprime.

Definition 2.6: Let A be a nonempty-subset of a ternarysemigroup T. Then the right-annihilator of A w.r.t $B(\subseteq T)$ in T, represented by $r_a(A, B)$ is defined by $r_a(A, B) = \{x \in T : ABx = \{0\}\}$.

Proposition 2.7: The right annihilator of a subset A with respect to a subset B of a ternarysemigroup T is a right ideal of T.

Proof: We note that $0 \in r_a(A,B)$, since $AB0 = \{0\}$. So $r_a(A,B)$ is nonempty. Let $s,t \in r_a(A,B)$. Then $ABs = ABt = \{0\}$. Now $AB(sxy) = (ABs)xy = \{0\}xy = \{0\}$ for all $x,y \in T$ implies that $sxy \in r_a(A,B)$.

Hence $r_a(A,B)$ is a right ideal of T.

Proposition 2.8: The right annihilator of a subset A with

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respect to a rightideal B of a ternarysemigroups T with unital element ${}^{`}e^{'}$ is an ideal of T .

Proof: From Proposition 2.7, it follows that $r_a(A,B)$ is a rightideal of T. Now it remains to show that $r_a(A,B)$ is a left and lateralideal of T. Let $s \in r_a(A,B)$. Then $ABs = \{0\}$. Now since B is a rightideal of T, we find that $AB(xys) = A(Bxy)s \subseteq A(BTT)s \subseteq ABs = \{0\}\ \forall x,y \in T$ implies that $xys \in r_a(A,B)$. This implies that $r_a(A,B)$ is a leftideal of T. Again, since B is a rightideal of T, we find that $AB(xsy) = AB(exe)(ese)y = A(Bex)(eesey) \subseteq A(BTT)(eesey) \subseteq AB(eesey) = A(Bee)(sey) \subseteq A(BTT)(sey) \subseteq AB(sey) = (ABs)ey = \{0\}ey = \{0\}\ \forall x,y \in T$ implies that $xsy \in r_a(A,B)$. This implies that $r_a(A,B)$ is a lateralideal of T. Hence $r_a(A,B)$ is an ideal of T.

Definition 2.9: A ternarysemigroup T is called to satisfy descending chain condition (D C C) on rightideal of T if for each sequence of rightideals $A_1,A_2,A_3,...$ of T with $A_1 \supseteq A_2 \supseteq A_3,... \exists$ a positive integer $n \ni A_n = A_{n+1} = ...$ We have shown that every right stronglyprime ternarysemigroup is a prime ternarysemigroup. But a prime ternarysemigroup may not be a right stronglyprime ternarysemigroup. In particular we have the following result.

Theorem 2.10: If T is a prime ternarysemigroup with descending chain condition (D C C) on right annihilator ideals of T then T is a right stronglyprime ternarysemigroup.

Proof: Let I be a non zero ideal of T and let C_{ra} denotes the class of all right annihilators of the form $r_a(F^{'},F)$, here $F^{'}, F$ are finite subsets of I and T respectively. Since T satisfies descending chain condition (D C C) on right annihilator ideal of T, C_{ra} contains a small element, $J = r_a(F_0', F_0)$, say. We claim that $J = \{0\}$. If possible, Let $J \neq \{0\}$. Since T is a prime ternarysemigroup $ITJ \neq \{0\}$. Then there exist $x \in I$, $t \in T$ and $y \in J$ such that $xty \neq 0$. Let $F'' = F_0' \cup \{x\}$ and $F''' = F_0 \cup \{t\}$. Since $F^{'}_0 F_0 t \subseteq F^{''}F^{'''}t, r_a(F^{''},F^{'''}) \subseteq J$. Again $y \in J$ and $xty \neq 0$ implies that $r_a(F^{''},F^{'''}) \subset J$ which is a contradiction to the minimality of J. Hence $J = \{0\}$. Therefore, by using theorem 2.5, we find that T is right strongly prime.

Definition 2.11: A nonempty subset A of a ternarysemigroup T is known as an m- System if for each $x,y,z \in A$ thereexists elements a_1,a_2,a_3,a_4 of T such that $xa_1ya_2z \in A$ or $xa_1a_2ya_3a_4z \in A$ or $xa_1a_2ya_3za_4 \in A$ or $a_1xa_2ya_3a_4z \in A$.

Theorem 2.12: A proper ideal A of a ternarysemigroup T is prime $\Leftrightarrow$ its complement $T \backslash P$ is as m-System.

Now we consider the matrix ternarysemigroup $M_n(T)$ where T is a ternary semi group. Let T be a ternarysemigroup with unital element ${}^{`}e^{'}$ and $M_n(T)$ be the set of all square matrices of order $n(n \in N)$ with entries from T.

Suppose $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $C = (c_{ij})_{n\times n} \in M_n(T)$ we define ternary multiplication in $M_n(T)$ as follows: $(a_{ij})_{n\times n}(b_{ij})_{n\times n}(c_{ij})_{n\times n} = (d_{ij})_{n\times n}$ where $d_{ij} = \sum_{k,l=1}^{n} a_{ik}b_{kl}c_{lj};\ 1 \le i, j \le n$

It can be easily verified that together with above defined multiplication $M_n(T)$ is a ternarysemigroup with unital element. We call $M_n(T)$ the matrix ternarysemigroup. Let $x \in T$. Then the notation xE_{ij} will be used to denote the $n \times n$ matrix in which the $(i,j)-th$ entry is x and all other entries are zero. Then we can write A= $(a_{ij})_{nxn}$= $d_{ij} = \sum_{i,j=1}^{n} a_{ij}E_{ij}$ and it can be easily verified that

$$(xE_{pq})(yE_{rs})(zE_{uv}) = \begin{cases} (xyz)E_{pv} \text{ if } q = r \text{ and } s = u \\ 0 \text{ if } q \neq r \text{ or } s \neq u \text{ for all } x,y,z \in T \end{cases}$$

Theorem 2.13: Let T be a ternarysemigroup with unital element ${}^{`}e^{'}$. Then T is a right stronglyprime ternarysemigroup iff $M_n(T)$ is a right stronglyprime ternarysemigroup.

Proof: Let T is a right stronglyprime ternarysemigroup. We shall show that $M_n(T)$ is a right stronglyprime ternarysemigroup. Let $(a_{ij})_{n\times n} \neq (0)_{n\times n}$, $(b_{ij})_{n\times n} \neq (0)_{n\times n} \in M_n(T)$. Then there exists $a_{pq} \neq 0 \in T$ and $b_{rs} \neq 0 \in T$ for some integers p,q,r,s such that $1 \le p,q,r,s \le n$. Since T is right stronglyprime, there exist finite subsets $F = \{f_1,f_2,...,f_k\}$, $G = \{g_1,g_2,...,g_l\}$ and $H = \{h_1,h_2,...,h_m\}$ of T such that $a_{pq}f_tg_\mu h_\nu b_{rs} \neq 0$ for some $f_t \in F$, $g_\mu \in G$, $h_\nu \in H$. Now $(ps)th$ entry of the matrix $(a_{ij})(f_tE_{q1})(g_\mu E_{11})(h_\nu E_{1r})(b_{ij})$is $a_{pq}f_tg_\mu h_\nu b_{rs}$ which is nonzero. Therefore $F^{'} = \{f_tE_{i1} : 1 \le i \le n\}$, $G^{'} = \{g_\mu, E_{11}\}$ and $H^{'} = \{h_\nu E_{1j} : 1 \le j \le n\}$ and finite subsets of $M_n(T)$ such that for any $(a_{i,j})_{n\times n} \neq (0)_{n\times n}$; $(b_{i,j})_{n\times n} \neq (0)_{n\times n} \in m_n(T)$ we have $(a_{i,j})_n\times_n(f_tE_{i1})\ (g_\mu E_{11})(h_\nu E_{1j})(b_{ij})_n\times_n \neq (0)_n\times_n$ and hence $m_n(T)$ is a right strongly prime ternarysemigroup.

Conversely suppose that $M_n(T)$ is a right stronglyprime ternarysemigroup and $a(\neq 0) \in T$, $b(\neq 0) \in T$. Then aE_{11} and bE_{11} are nonzero elements in $M_n(T)$. Since $M_n(T)$ is right stronglyprime there exist finite subsets $A = \{(a_{i,j})_{n\times n}\}$, $B = \{(b_{i,j})_{n\times n}\}$ and $C = \{(c_{i,j})_{n\times n}\}$ of $M_n(T)$ such that $(aE_{11})(a_{ij})_{n\times n}(b_{ij})_{n\times n}(c_{ij})_{n\times n}\ (bE_{11}) \neq (0)_{n\times n}$. Now $(aE_{11})(a_{ij})_{n\times n}(b_{ij})_{n\times n}(c_{ij})_{n\times n}\ (bE_{11})$=$(d_{ij})_{nxn}$ where

$$d_{ij} = \begin{cases} \sum_{s,l=1}^{n} aa_{1s}b_{sl}c_{l1}b \text{ if } i = j = 1 \\ 0, \text{ otherwise} \end{cases}$$

Since $(d_{ij})_{n\times n} \neq (0)_{n\times n}$ we have $\sum_{s,l=1}^{n} aa_{1s}b_{sl}c_{l1}b \neq 0$

Hence $aa_{1s}b_{sl}c_{l1}b \neq 0$ forsome s and l; $1 \le s, l \le n$. Now choose $A^{'} = \{a_{1j} : 1 \le j \le n\} \subseteq T$, $B^{'} = \{b_{ij} : 1 \le i;j \le n\} \subseteq T$ and $C^{''} = \{c_{i1} : 1 \le i \le n\} \subseteq T$. Then $a\alpha\beta\gamma b \neq 0$ for some $\alpha \in A^{'}$, $\beta \in B^{'}$ and $\gamma \in C^{'}$ and hence T is a right stronglyprime ternarysemigroup.

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