Strongly Prime Ternary Semi Groups


Abstract: In this article we imported the concept of right strongly prime ternary semi group and developed some interesting properties of right strongly prime ternary semi group.

I. INTRODUCTION

The concept of a semi group is very simple and plays a large role in the development of Mathematics. The characterization of prime ideals in semi groups is developed by Anjaneyulu [1] in the year 1981. The literature of the theory of ternary operations is vast and scatters over diverse areas of mathematics. In 1932 Lehmer introduced the notion of Ternary algebraic systems. The algebraic system is said to be triplices was imported by Lehmer which turn out to be commutative-ternary groups. Kasner also developed the structures and give the ideal of n-ary algebras. Ternary semi groups are universal algebras with one associative ternary operation. The ideal theory in ternarysemigroup was imported by Sioson. Jayalalitha[3] discussed some properties of rightideals in ordered semi groups. Some previous works of ternarysemigroup may be found in [2,4]. Throughout this article T will always denoted by a ternarysemigroup with zero and T = T\{0\}. In this article we imported and study the concept of rightstronglyprime ternarysemigroup.

II. RIGHT STRONGLY PRIME TERARY SEMIGROUPS

Definition 2.1: A ternarysemigroup T is known rightstronglyprime if for every x ∈ T, ∃ finite subsets P1,P2,P3 of T s.t xP1P2P3y = {0} ⇒ y = 0 ∀ y ∈ T.

Proposition 2.2: A ternarysemigroup T is right stronglyprime if for every x ∈ T, ∃ a finite subset P of T s.t xTTTy = {0} ⇒ y = 0 ∀ y ∈ T.

Proof: Assume that T is a right stronglyprime ternarysemigroup. Let x ∈ T. Then ∃ finite subsets P1,P2,P3 of T s.t xP1P2P3y = {0} ⇒ y = 0 ∀ y ∈ T. Let P = P1 ∪ P2 ∪ P3. Then P1,P2,P3 ⊆ P and P is finite. Suppose xPPTY = {0} ∀ y ∈ T. Then xP1P2P3y ⊆ xPPTY = {0} ∀ y ∈ T. This shows that y = 0 ∀ y ∈ T. Converse part is obvious.

Theorem 2.3: Every right strongly prime ternarysemigroup is a prime ternarysemigroup.

Proof: Suppose that T is a right strongly prime ternarysemigroup. Let P,Q,R be three ideals of T such that PQR = {0}. Suppose P ≠ {0} and Q ≠ {0}. Since P ≠ {0}, there exists a(≠ 0) ∈ P. Since T is a right strongly prime ternarysemigroup, by proposition 2.2, there exists a finite subset F of T s.t aFFy = {0} implies that y = 0 ∀ y ∈ T. Now aFFF(QTR) = (aFF)(FQT)R ⊆ (PTT)(TQR) ⊆ PQR = {0}. This implies that QTR = {0}. Again, since Q ≠ {0}, ∃ q(≠ 0) ∈ Q and for this q ≠ 0, there exist a finite subset F' of T s.t q F' F' F' r ⊆ QTTR ⊆ QTR = {0} for r ∈ R. This implies that r = 0. Since r is an arbitrary element of R, we find that R = {0}. This shows that {0} is a primeideal of T and hence T is a prime ternarysemigroup.

Theorem 2.4: Let T be a ternarysemigroup with unital element ‘e’.

Proof: Assume that T is a rightstronglyprime ternarysemigroup and I be a nonzero ideal of T. Since I is a nonzero ideal of T, there exist x(≠ 0) ∈ I. Again, since T is a rightstronglyprime, ∃ a finite subset F of T such that xFFy = {0} implies that y = 0 ∀ y ∈ T. Hence I is a rightstronglyprime ideal of T.

Theorem 2.5: Let T be a ternarysemigroup with unital element ‘e’.

Proof: Let a ∈ T. Then by our assumption there exist t ∈ T and finite subsets F′, F of T s.t xTF′Fy = {0} implies that y = 0 ∀ y ∈ T. Hence T is rightstronglyprime.

Definition 2.6: Let A be a nonempty-subset of a ternarysemigroup T. Then the right annihilator of A w.r.t B(⊆ T) in T, represented by r(A,B) is defined by r(A,B) = {x ∈ T : ABx = {0}}.

Proposition 2.7: The right annihilator of a subset A with respect to subset B of a ternarysemigroup T is a right ideal of T.

Proof: We note that 0 ∈ r(A,B), since AB0 = {0}. So r(A,B) is nonempty. Let s,t ∈ r(A,B). Then ABs = ABt = {0}. Now AB(xy) = (ABs)xy = {0}xy = {0} for all x,y ∈ T implies that sxy ∈ r(A,B).
respect to a rightideal $B$ of a ternarysemigroup $T$ with unital $e$ is an ideal of $T$.

**Proof:** From Proposition 2.7, it follows that $r_s(A,B)$ is a rightideal of $T$. Now it remains to show that $r_s(A,B)$ is a left and lateralideal of $T$. Let $s \in r_s(A,B)$. Then $ABs = \{0\}$. Now since $B$ is a rightideal of $T$, we find that $AB(xy) = A(Bxy) \subseteq A(ABT)s \subseteq ABs = \{0\}$. If $x, y \in T$ implies that $xy \in r_s(A,B)$. This implies that $r_s(A,B)$ is a lateralideal of $T$. Again, since $B$ is a rightideal of $T$, we find that $AB(xy) = A(Bxy) \subseteq A(ABT)(xy) \subseteq AB(xy) = (ABs)xy = \{0\}$. If $x, y \in T$ implies that $xy \in r_s(A,B)$. This implies that $r_s(A,B)$ is a lateralideal of $T$. Hence $r_s(A,B)$ is an ideal of $T$.

**Definition 2.9:** A ternarysemigroup $T$ is called to satisfy descending chain condition (D C C) on rightideal of $T$ if for every sequence of rightideals $A_1, A_2, A_3, \ldots$ of $T$ with $A_1 \supseteq A_2 \supseteq A_3, \ldots$ any positive integer $n \ \exists A_n = A_{n+1} = \ldots$. We have shown that every rightstrongly prime ternarysemigroup is a prime ternarysemigroup. But a prime ternarysemigroup may not be a rightstrongly prime ternarysemigroup. In particular we have the following result.

**Theorem 2.10:** If $T$ is a ternarysemigroup with descending chain condition (D C C) on right annihilator ideals of $T$, then $T$ is a right strongly prime ternarysemigroup.

**Proof:** Let $I$ be a non-zero ideal of $T$ and let $C_e$ denotes the class of all right annihilators of the form $r_s(F,F')$, here $F,F'$ are finite subsets of $I$ and $T$ respectively. Since $T$ satisfies descending chain condition (D C C) on right annihilator ideal of $T$, $C_e$ contains a smallest element, $J = r_s(F_0',F_0)$, say. We claim that $J = \{0\}$ is possible. Let $J \neq \{0\}$. Since $T$ is a prime ternarysemigroup $\{0\} \subseteq F' = F_0' \cup \{x\}$ and $F'' = F_0' \cup \{t\}$. Since $F_0'dd, F''dF'' \subseteq F' \cap F'' \subseteq J$. Again $y \in J$ and $xy \neq 0$ implies that $r_s(F,F') \subseteq J$ which is a contradiction to the minimality of $J$. Hence $J = \{0\}$. Therefore, by using theorem 2.5, we find that $T$ is right strongly prime.

**Definition 2.11:** A nonempty subset $A$ of a ternarysemigroup $T$ is known as an $m$-System if for each $x, y, z \in A$ there exists elements $a_1, a_2, a_3, a_4$ of $A$ such that $x + y \in A_1, x + y \in A_2, x + y \in A_3, x + y \in A_4$.

**Theorem 2.12:** A proper ideal $A$ of a ternarysemigroup $T$ is prime $\iff$ its complement $T \setminus P$ is as $m$-System.

Now we consider the matrix ternarysemigroup $M_n(T)$ where $T$ is a ternary semi group. Let $T$ be a ternary semigroup with unital element $e$ and $M_n(T)$ be the set of all square matrices of order $n(n \in N)$ with entries from $T$.

Suppose $A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}, C = (c_{ij})_{n\times n} \in M_n(T)$ we define ternary multiplication in $M_n(T)$ as follows: $(a_{ij})(b_{kl})(c_{mn}) = (d_{ij})_{n\times n}$ where $d_{ij} = \sum_{k,l=1}^{n} a_{ik} b_{kl} c_{lj}$, $1 \leq i, j \leq n$

It can be easily verified that together with above defined multiplication $M_n(T)$ is a ternarysemigroup with unital element. We call $M_n(T)$ the matrix ternarysemigroup. Let $x \in T$. Then the notation $xE_y$ will be used to denote the $n \times n$ matrix in which the

**Theorem 2.13:** Let $T$ be a ternarysemigroup with unital element $e$ and $M_n(T)$ be the rightstrongly prime ternarysemigroup.

**Proof:** Let $T$ is a right strong prime ternarysemigroup if $M_n(T)$ is a right strong prime ternarysemigroup.

REFERENCES


Blue Eyes Intelligence Engineering & Sciences Publication