

Developing an Optimization Model for Vehicle Routing Problem with Time Windows Considering Delivery and Pick-Up Simultaneously

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Abstract: Vehicle Routing Problem (VRP) is a key element of many distribution systems which involve routing and selecting of vehicles through a set of nodes. Wireless modes of communication have received attention to such an extent that some of the cities in the world cannot operate without digital control systems. Notably, developments have been implemented beyond operations such as WLAN and mobile phones, witnessing the arrival of time windows in cities such as London and Stockholm. The paper investigates a variant of VRP which incorporates time windows, fleet and driver scheduling, pick-up and delivery in the planning horizon.

1. INTRODUCTION

Despite massive expansions in roads and related infrastructure, increasing traffic congestion continues to be attributed to factors such as the increase in private vehicles and potential adversities accruing from poor planning. According to Devi, Kesavan and Gayathri (2015)[15], there is a density of 5409 cars in every 1,000 persons and this outcome makes it one of the highest in the world and leads in the region. The last eight years have also witnessed the number of vehicles in this zone double. Towards 2014, this number was averaged at 1.4 million, a significant increase when compared to 2006's 740,000 [16]. The eventuality is that there is an increase of 8.2 percent (annual) in the number of vehicles. From other emirates, it has been established that about 450,000 vehicles enter major cities daily, with the morning peak hours witnessing the city welcome about 40,000 vehicles. The northern emirates dominate this entry and contribute 370,000 vehicles on average weekdays, 33,500 coming in the morning [15]. As such, there is a direct correlation between the increase in the number of vehicles and traffic congestion.

A growth in daytime population forms another major cause of congestion. Notably, the daytime population exceeds this number by one million; dominated by professionals working during the day and commuting from and to the workplace [17]. The trickle-down effect is that traffic congestion accounts for a Dh4.6 million losses linked to

fuel and time. For individuals, the loss varies considerably based on the reasons prompting their journeys. With stress levels increasing when such persons run out of time, anti-social habits have been reported and include speeding when one is clear of congestion and queue jumping, decisions that pose danger to the lives of residents.

A. Scholarly Insights

According to Dargie and Poellabauer (2010)[19], time windows are established through communication nodes that are autonomous. The communication processes are realized via radio, excluding backbone infrastructures. Therefore, two nodes that are not within a range of mutual transmission can communicate using intermediate nodes to relay messages [21]. Time windows can be applied in fields such as community mesh networks, disaster relief, data gathering, monitoring, and surveillance. Notably, technological demands in the field of time windows have led to further research to address the current challenges that smart cities face while providing services to the citizens. In vehicular networking, time windows aid in establishing routing systems, medium access control, deployment strategies, topology control, and the design of energy efficient systems. The following figure shows a probable outlook of an ideal smart city (in which time windows are effective).

A study by Azimi, Bhatia, Rajkumar and Mudalige (2011) focused on the role of sensor networks in minimizing collisions among vehicles. Findings indicated that the evolution of time windows accounts for the significant reduction of collisions at the intersections. In a similar study, Dargie and Poellabauer (2010) [19] affirmed that there is a direct correlation between technology incorporation and safety among vehicles. Specifically, the improvement in safety was attributed to the real-time form of communication that the sensors offer. According to Kemal and Mohamed (2005), time windows play a critical role in routing in which communication modes and signal provision may alert the drivers on possible routes that may be deemed safe and, with little traffic.

Kemal and Mohamed (2005) asserted that highly integrated forms of wireless sensing have led to improved forms of communication and safety, with the assertion suggesting that vehicular networking is critical because it

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strives to alleviate adversities on the road.

II. MATHEMATICAL FORMULATION OF VRFPDTWP

The mathematical formulation for this problem is presented as follows:

$$\min K_1 \cdot \sum_{l \in D_l} \sum_{t \in T_l} s_l^t + K_2 \cdot \sum_{l \in D_E} \sum_{t \in T_l} r_l^t + Z'$$

Subject to:

$$\sum_{k \in K} \sum_{j \in N_0} x_{ijk}^t = 1$$

$$\sum_{k \in K \setminus K_i} \sum_{j \in N_0} x_{ijk}^t = w_i^t$$

$$\sum_{i \in N} \sum_{j \in N_0} q_i x_{ijk}^t \leq Q_k$$

$$\sum_{i \in N} \sum_{j \in N_0} p_i x_{ijk}^t \leq P_k$$

$$u_{jk}^t \geq u_{ik}^t + c_{ij} - M(1 - x_{ijk}^t) - Mz_{ik}^t$$

$$u_{jk}^t \geq c_{ij} - M(1 - x_{ijk}^t)$$

$$u_{ik}^t + \sum_{j \in N_0} c_{ij} x_{ijk}^t - F \leq Mz_{ik}^t$$

$$v_{jk}^t \geq v_{ik}^t + d_i^t + e \cdot p_i^t \cdot w_j^t + c_{ij} + G \cdot z_{ik}^t - M(1 - x_{ijk}^t)$$

$$b_i \geq v_{ik}^t \geq a_i$$

$$v_{0k}^t \geq \sum_{l \in D} (g_l^t \cdot y_{lk}^t)$$

$$v_{n+1,k}^t \leq \sum_{l \in D} (h_l^t \cdot y_{lk}^t)$$

$$s_l^t \geq \sum_{i \in N_0} \sum_{j \in N_0} c_{ij} x_{ijk}^t - M(1 - y_{lk}^t)$$

$$r_l^t \geq v_{n+1,k}^t - g_l^t - M(1 - y_{lk}^t)$$

$$\sum_{t \in T_l} r_l^t \leq H$$

$$\sum_{k \in K} \theta_{jk}^t = \alpha_j^t$$

$$\sum_{k \in K} \sigma_{jk}^t = \beta_j^t$$

$$x_{ijk}^t, w_i^t, z_{ik}^t, y_{lk}^t \in \{0, 1\} \forall i, j \in N_0, l \in D, k \in K, t \in T$$

$$v_{ik}^t, u_{ik}^t, r_l^t, s_l^t \geq 0 \forall i, j \in N_0, l \in D, k \in K, t \in T$$

$$\theta_{jk}^t, \sigma_{jk}^t \in \{0, 1, 2, \dots\} \forall j \in N, k \in K, t \in T$$

The objective function (1) minimizes weighted sum of the travel time of the internal drivers and the working duration of the external drivers over the planning horizon. Constraints (2) state that each customer must be visited by one vehicle on each of its delivery days. Constraints (3) define whether each customer is visited by a preferable vehicle. Constraints (4-5) guarantee that the vehicle capacities are respected in both weight and volume.

Constraints (6-7) define the elapsed driving time. More specifically, for the vehicle (k) travelling from

customer *i* to *j* on day *t*, the elapsed driving time at *j* equals the elapsed driving time at *i* plus the driving time from *i* to *j* (i.e., $u_{jk}^t \geq u_{ik}^t + c_{ij}$) if the vehicle does not take a break at customer *i* (i.e., $z_{ik}^t = 0$); Otherwise, if the vehicle takes a break at customer *i* (i.e., $z_{ik}^t = 1$), the elapsed driving time at *j* will be constrained by (1) which make sure it is greater than or equal to the travel time between *i* and *j* (i.e., $u_{jk}^t \geq c_{ij}$). Constraints (8) guarantee that the elapsed driving time never exceeds an upper limit *F* by imposing a break at customer *i* (i.e., $z_{ik}^t = 1$) if driving from customer *i* to its successor results in a elapsed driving time greater than *F*.

Constraints (9) determine the time to start the service at each customer. If *j* is visited immediately after *i*, the time v_{jk}^t to start the service at *j* should be greater than or equal to the service starting time v_{ik}^t at *i* plus its service duration d_i^t , the extra service time $e \cdot p_i^t$ if *i* is visited by an inappropriate vehicle (i.e., $w_i^t = 1$), the travel time between the two customers c_{ij} , and the break time *G* if the driver takes a break after serving *i* (i.e., $z_{ik}^t = 1$). Constraints (10) make sure the service starts within the customers' time window.

Constraints (11-12) ensure that the starting time and ending time of each route must lie between the start working time and latest ending time of the assigned driver. Constraints (13) calculate the total travel time for each internal driver. Constraints (14) define the working duration for each driver on every working day, which equals the time the driver returns to the depot minus the time he/she starts work. Constraints (15) make sure that the internal drivers work for no more than a maximum weekly working duration, referred to as 37 week-hour constraints. Constraints (16-17) define the pick up and delivery for each customer. Constraints (18-20) define the binary and positive variables used in this formulation.

III. NEIGHBOURHOOD SEARCH

Let $[\beta]_k$ be an integer point belongs to a finite set of neighbourhood $N([\beta]_k)$. We define a neighbourhood system associated with $[\beta]_k$, that is, if such an integer point satisfies the following two requirements

1. if $[\beta]_j \in N([\beta]_k)$ then $[\beta]_k \in [\beta]_j, j \neq k$.
2. $N([\beta]_k) = [\beta]_k + N(0)$

With respect to the neighbourhood system mentioned above, the proposed integerizing strategy can be described as follows.

Various solutions have been recommended. For instance, Keilo and Montagne (2012)[16] observed that traffic congestion can be solved by getting more vehicles out of the roads. The argument behind this recommendation was that even in situations where expansions were done to the road networks, the step is unlikely to be adequate due to the increasing demand for cars, a tertiary impact associated

with increasing population. Indeed, mixed outcomes arise in relation to this recommendation. On the one hand, a reduction in the number of vehicles on the road could ease congestion while enhancing smooth movement. On the other hand, the recommendation fails to consider complexities in terms of the travelers' reasons of arranging the journeys. For instance, if a traveler intends to head to an airport while another traveler intends to access a shopping mall in the same direction but at a shorter distance, eliminating a vehicle that accesses the airport could prove detrimental to the former traveler. The situation becomes more complex when logistical operations are considered whereby public vehicles may be permitted on specific routes while private cars are highly preferable beyond certain zones when public means is less applicable. In such a case, eliminating the private cars tends to be highly inconvenient.

Another recommendation as made by Tai, Nghah, Shah and Al Ali (2016). In the study, it was proposed that heavy taxes may be imposed on car drivers and use the money for purposes of improving public transport. Indeed, the benefit of such a step is that it is likely to discourage a majority of car owners because this option will become expensive. Hence, the groups are likely to opt for public transport, a step that will not only reduce pollution but also curb the transport problem of congestion. An additional benefit of the taxation option lies in the increasing number of people opting for public transport not only due to discouragements arising from high cost but also improved public transport. Notably, the recommended solution states that heavy taxes are imposed on car owners and the money used to improve the public transport system, implying that high taxes are attributable for the generation of money to be used in making necessary changes. However, drawbacks are linked to the use of taxes as a solution to traffic congestion. For instance, present taxes are already high to most of the people and further taxation might be detrimental to their economic outcomes, discouraging potential investors. Similarly, the solution or option might prompt fixed amounts of tax and individuals with lower income might be hit harder, implying that it is unlikely to be a fair tax.

Devi, Kesavan and Gayathri (2015)[15] advocated for the concept of autonomy as that which could play a leading role in addressing the problem of traffic congestion. It was noted that humans are unlikely to drive safely if faced with a two-second distance separating their vehicles. However, autonomous vehicles were noted as those constituting advanced sensors. In the wake of growing technology, autonomous cars exhibit the capacity of following at one or less of a second, an outcome that was poised to double capacity. The recommendation suggested further that autonomous cars do not require wide lanes when compared to those required by humans, an additional merit that was noted to double the capacity because of close proximities along with the cars travel in relation to those in the neighbouring lanes. Hence, the option of adopting autonomous vehicles is seen to yield significant reductions in traffic congestion, extending the benefit to non-freeways in which a majority of travelers could switch to freeways; perceived to be less congested.

The criticality of ad hoc sensor networks and the promotion of vehicular networking mechanisms arise in the case of traffic congestion and impact monitoring. Traffic noise pollution, air quality pollution, and the emission of greenhouse gases result from the urban traffic. Thus, smart cities have devised mechanisms for minimizing the adverse effects of traffic congestion in a quest to reduce socio-economic losses. Specifically, ad hoc networks have enabled the realization of online monitoring of the times of travel and the behavior of drivers from the points of origin to the destinations. Other benefits that have resulted from ad hoc sensor networks and vehicular networking mechanisms include the reductions of air pollution and the reduction of the length of queues among city traffic systems.

IV. THE ALGORITHM

After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let

$$x = [x] + f, \quad 0 \leq f \leq 1$$

be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable x and f is the fractional component.

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T \alpha_j$

With j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased from LB or decreased from UB. If none go to next i^* .

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay

feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasis by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

In the field of ad hoc sensor networks, the aspect of routing messages from one source to another or from one source to multiple destinations is crucial. Information within smart cities is disseminated by routing systems, either as anycast, broadcast, multicast, or unicast. It is worth noting that standard approaches have been established to achieve the routing processes — through the broadcast technique. The implication is that the ad hoc sensor networks are geared towards achieving efficiency in routing systems while seeking to serve the optimal amounts of overhead — generated by logarithms. However, the manner in which information is disseminated in smart cities in unicast, multicast, or anycast modes remains challenging (Kemal & Mohamed, 2005).

According to by Magno, Boyle and Brunelli et al. (2014)[21], future transportations are perceived to gain from vehicular networking. Some of the key features of vehicular networking include the efficient management of traffic systems, standardization, infotainment, and road safety (Kemal & Mohamed, 2005). Particularly, the process of installing communication devices in roadside infrastructure components and cars has the moving vehicles communicate with other vessels in the network — because of the establishment of ad hoc networks that are ephemeral and rapidly changing. Furthermore, it is projected that moving vehicles will directly access network infrastructure, which will be fixed on the roadside, with smart cities perceived to be unexceptional.

In a study by Dargie and Poellabauer (2010)[19], it was asserted that smart modern cities have numerous luxury cars in which central computers are embedded to serve the purpose of connecting various networks and systems. Furthermore, the cars are equipped with communication devices of wireless nature; including cell phones that provide telematics and Internet connection services when needed. In the end, ad hoc sensors and the concept of vehicular networking play a crucial role in enhancing safety by promoting real-time communication between vehicles. The following photograph illustrates how vehicular networking is applied in smart cities to minimize destruction during dangers such as fire. In the photo, cars that surround the region of disaster are fed with information from a central point of dissemination, upon which alternative routes are sought without necessarily causing traffic jams. Notably, the drivers establish alternative routes upon receiving information about the appropriate routes on which smooth traffic flows have been observed.

V.CONCLUSIONS

In conclusion, smart cities are characterized by a good economy, smart environments, smart mobility, smart living, smart people, and a smart system of governance. The aspect of information communications and technology (ICT) has emerged towards the future realization of perfect smart cities. The paper has examined the application of ad hoc sensor networks in smart cities. The manner in which ad hoc sensor networks can function has been highlighted, focusing on probable future applications of the sensor

systems. The field of vehicular networks has received attention because of the need to reduce traffic congestion. Additionally, sensor application has been associated with impact monitoring. Ad hoc sensor networks have also led to the realization of online monitoring of the times of travel and the behavior of drivers from the points of origin to the destinations, accounting for significant reductions in traffic noise pollution, air quality pollution, and emissions of greenhouse gases in the urban traffic.

Overall, a wide-scale adoption of autonomous cars (to solve traffic congestion) can be seen to be highly valid and reliable. With complex algorithms in these vehicles, the vehicles are capable of determining distance from another vehicle and appropriate stopping distance, with the practice of platooning leading to significant improvements in traffic conditions while alleviating congestion. Ad hoc sensor networks governing the operations of these autonomous cars are also poised to predict the nature of the traffic on roads and determine the most appropriate routes, having gained data regarding the nature of the traffic ahead and detecting regions with congestion due to causes such as accidents. Hence, the autonomous vehicles are a way to go for the dire state of traffic congestion because these vehicles are expected to determine the most appropriate route perceived to be less congested and yield the overall merit of increased capacity.

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