Controlling Industrial Processes Using Multivariate Exponential Weighted Moving Average (Mewma)

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Abstract: This paper discusses the Multivariate Exponential Weighed Moving Average (MEWMA) in controlling industrial processes. MEWMA is a multivariate control diagram that can be used to detect autocorrelation and detect the mean vector shift. But the success of the measurement with the MEWMA control diagram depends on the smoothing parameter$\lambda$and the comparative weightω. So far, it has never been specifically seen whether the MEWMA control chart can meet certain assumptions, but can provide appropriate information. Another multivariate control diagram that can be used to control the process is the T^2Hotelling control chart . Therefore, this initial phase will be simulated with the MEWMA control chart, and the results will be compared with the T^2Hotellingas a comparison. The results of the study showed that the MEWMA control chart yielded more sensitive results to detect the shift in the mean vector process compared to the multivariate T^2-Hotelling control chart .

Index terms: T^2 Hotelling, parameter λ, comparison weightω, mean vector

I.INTRODUCTION

Quality control is a technique that needs to be done so that a company can know the quality of its products before being marketed to consumers.The quality control process is carried out from the stage of product planning, process planning to the product offering process to consumers. The main purpose of quality control in a process is in the framework of product evaluation, with the hope of reducing production costs, repetitive work does not occur, reducing delays and increasing machine usage. Thus, quality control is important because it will result in increased productivity, increased market share, increased number of products, guaranteed business continuity and business development, [19].

The control chart is one tool that can be used in statistical quality control. Shewhart's multivariate control chart was first introduced in 1947 and is known as the T^2Hotellingdiagram [19]. According to Montgomery (2009) the main drawback of this Shewhart control chart is that this chart only uses the last sample information and ignores the information provided by the previous sample. This causes the Shewhart control chart to be less sensitive to small process shifts.

Research on multivariate control processes involving periodic series data is interesting to develop. Periodic Series Analysis relates to data collected over time. The objective of this analysis depends on the application, but one of the main outputs is to predict the future value of the data set. This value depends, usually on stochastic behavior, on the observations currently available. This dependence must be taken into account when forecasting future data based on past data, taking into account trends, seasonal and other features of the data. One of the most successful methods to overcome this problem is to use the concept of exponential smoothing. For this reason, a quality control process will be studied using a multivariate exponential weighted moving average (MEWMA). Furthermore, the results of this study will be compared with the Hotelling$\boldsymbol{T}^2$control diagram[1]-[7].

II.METHODOLOGY

A.T^2Hotelling Multivariate Control Chart

The T^2Hotellingstatistics on the multivariate control chart are used to detect signs that are out of control, whether the process is controlled statistically or not. In the T^2 Hotelling diagram, the scattering points that are still below the upper control limit can we assume that the production process is controlled, and the function of the T^2Hotellingstatistic is that it can show an optimal statistical test to detect a general shift in the average vector.

However, the weakness of the T^2Hotellingdiagram is that it cannot detect and explain which characteristics or variables cause uncontrolled processes.If product quality consists of several characteristics, random variables*(p > 2)*for example$X = (X_1, X_2, \dots, X_p)$, then quality control through the establishment of a multivariate control chart . The complete data structure can be seen in the following table.

Table 1. Observation Data Structure for Multivariate Data

No observation	Variable				
	X_1	X_2	X_3	.....	X_p
1	X_{11}	X_{12}	X_{13}	.....	X_{1p}
2	X_{21}	X_{22}	X_{23}	.....	X_{2p}
3	X_{31}	X_{32}	X_{33}	.....	X_{3p}
⋮	⋮	⋮	⋮		⋮
M	X_{m1}	X_{m2}	X_{m3}	.....	X_{mp}

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The average vector is :

$$\bar{\boldsymbol{X}} = (\bar{\boldsymbol{X}}_1 \quad \bar{\boldsymbol{X}}_2 \cdots \quad \bar{\boldsymbol{X}}_p)$$

with $\bar{\boldsymbol{X}}_j = \frac{1}{m}\sum_{i=1}^{m} X_{ij}$, $i = 1,2,...,m$, $j = 1,2,...,p$

And has a covariance matrix

$$\boldsymbol{S} = \begin{pmatrix} s_{11} & s_{12} & s_{13} \cdots s_{1p} \\ s_{21} & s_{22} & s_{23} \cdots s_{2p} \\ s_{31} & s_{32} & s_{33} \cdots s_{3p} \\ \vdots & \vdots & \vdots \cdots \vdots \\ s_{p1} & s_{p2} & s_{p3} \cdots s_{pp} \end{pmatrix}$$

With

$$\boldsymbol{S}_{ij} = \frac{1}{m-1}\sum_{i=1}^{m}(\boldsymbol{X}_{ij} - \bar{\boldsymbol{X}}_j)(\boldsymbol{X}_{ij} - \bar{\boldsymbol{X}}_j)' , i = 1,2,...,m , j = 1,2,...,p$$

In observing the multivariate process where the data has individual observations, form the T^2Hotellingstatistic to form a multivariate control diagram, the equation used is

$$\boldsymbol{T}^2 = (\boldsymbol{X}_{ij} - \bar{\boldsymbol{X}}_j)'\boldsymbol{S}^{-1}(\boldsymbol{X}_{ij} - \bar{\boldsymbol{X}}_j), \quad i = 1,2,...,m , j = 1,2,...,p$$

with $\boldsymbol{T}_i{}^2$is a distribution of data from observed characteristics. T^2 Hotellinghas only one control limit, namely the Upper Control Limit (UCL) while the Lower Control Limit (LCL) is definitely 0 because T^2Hotellingis a quadratic form so that its value cannot be negative. The upper control limit value given is

$$T^2 \sim \frac{p(m-1)(m+1)}{m(m-p)} F_{1-\alpha,p,m-p}$$

with p indicating the number of characteristics studied and the number of samples. observed characteristics

B.Control Chart Generalized Variance

Generalized Variance (GV) control chart is used to determine whether the variance in a production process has been controlled or not. The method used for GV control charts GV`|S| uses E(|S|) as follows , [19].

$$E(|S|) = b_1|\Sigma|,$$
$$Var(|S|) = b_2|\Sigma|^2,$$

where:

$$b_1 = \frac{1}{(n-1)^p}\prod_{i=1}^{p}(n-i),$$

$$b_2 = \frac{1}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)x\left\{\prod_{j=1}^{p}(n-j+2) - \prod_{j=1}^{p}(n-j)\right\}$$

with control limits The GV control chart is :

$$UCL = |\Sigma|(b_1 + 3b_2^{1/2})$$
$$CL = |\Sigma|b_1$$
$$LCL = |\Sigma|(b_1 - 3b_2^{1/2}$$

C.Control Chart of Exponential Weighted Moving Average (EWMA) Multivariate.

The Exponential Weighted Moving Average (EWMA) control chart was introduced by Roberts in 1959. The EWMA control chart is a good alternative to *Shewhart* control charts when used to detect small process shifts. According to Montgomery (2009) EWMA control chart statistics can be written as follows:[19]

$$Z_i = \lambda X_i + (1-\lambda)Z_{i-1}$$

With 0<λ<1 was the*smoothing*parameter, $X_i$was the observation the-*i* and$Z_0$the initial value obtained from the average observation (μ_0).

Multivariate Exponential Weighted Moving Average (MEWMA) control chart is a control diagram used to detect changes in the mean process. MEWMA is the development of EWMA control diagrams that are used to control the quality of multivariate data. One of the advantages of the MEWMA control diagram is that it is robust against the assumption of normal distribution, meaning that if the data does not meet the assumption of a normal multivariate distribution then the MEWMA control chart can still be made. [19] Following is the MEWMA vector to be explained in the equation:

$$\boldsymbol{Z}_i = \lambda\boldsymbol{X}_i + (1-\lambda)\boldsymbol{Z}_{i-1}$$

with value 0<λ<1 , *i= 1,2,3,…,n*, $\mathbf{Z_0=0}$

n =number of observations made (observation time).

P = the number of quality characteristics variables observed.

Λ= the amount of weight

MEWMA control chart can be used with the same weighting value or not. Calculation of observation points in the MEWMA control diagram can be explained in the equation below:

$$\boldsymbol{T}_i^2 = \boldsymbol{Z}_i' \sum\nolimits_{Z_i}^{-1} \boldsymbol{Z}_i$$

After obtaining the control diagram with the calculation of the data above, then the data can be said*out of control* if$\boldsymbol{T}_i^2$>H. The value of H >0, this value will be chosen to get the ARL determination which is controlled by a covariance matrix**Σ**as follows, Khoo, 2015:

$$\sum\nolimits_{Z_i} = \frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]\Sigma$$

information:

i=1,2,…,*n*

j=1,2,…,*p*

p=the number of quality characteristics variables observed

h_4=MEWMA control chart limits

The Upper Control Limit Value (UCL) for the MEWMA control diagram is stated in the value of *H*. The calculation of the value of *H* relates to the specified value of$\lambda$and the number of characteristic variables to be

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measured. The Lower Control Limit (LCL) for the MEWMA control diagram is equal to 0 because the T_i^2 value is always positive so that the most minimum Lower Control Limit of a positive value is 0. Making MEWMA control maps by making plots of T_i^2 which are obtained from the formula in the above equation with UCL = H and LCL=0. The process is said to be uncontrolled if there is a value of T_i^2 > UCL (Xie,Y, Xie, M , 2011).with 0<λ<1 is a smoothing parameter, Z_i is the graph of the MEWMA control chart for the-i, and W_0 is the initial value obtained from the average observation (μ_0). The midline and control limits of the multivariate MEWMA graph are as follows:

$$UCL_{MEWMA} = \mu_0 + L\sigma$$

$$CL = \mu_0$$

$$LCL_{MEWMA} = \mu_0 - L\sigma$$

where λ is the *smoothing* parameter, μ_0 is the process average which is assumed by $\bar{x}$, σ is the standard deviation from observation (X_i), and L is the multiplier σ for the width of the control boundary[8]-[10].

III.RESULS AND DISCUSSION

A.Diagram Kendali T² Hotelling dan Indeks Kapabilitas Proses

The study results related to quality control have been and will be presented in several seminar activities. The title of the paper submitted is as follows:

a). Industrial Process Control Using Control Charts T^2 *Hotelling*, and was presented at the National Mathematics and Applied Seminar on November 28-29, 2017.

b). Controlling Industrial Processes using Multivariate Exponential Weighted Moving Average (MEWMA), will be presented at the Malikussaleh International Conference on Social Science and Humanities Development (MICoSH) on September18-19, 2018.

From the previous study in point a) above it was concluded that by using the Hotelling T^2 control chart it can be said that the mean vector was controlled statistically, all observation points were within the predetermined control chart limit (*in of control*).

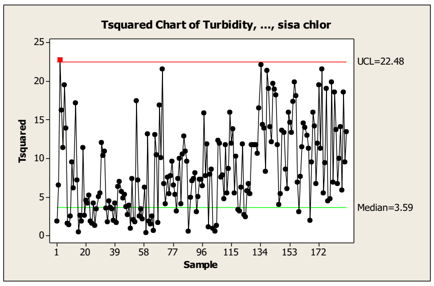

Figure 1. Hotelling T^2 control diagram

The process capability index produced by the quality characteristics of turbidity, pH and residual chlorine turns out to give different results with the Hotelling T^2 control diagram, namely monitoring for measurement of data accuracy is still very low (MCpk =0,62 dan MCpm =0,58).

B.Multivariate EWMA (MEWMA) Control Chart

MEWMA control diagram is used to observe whether the data observation data is in the status of control or out of control. This diagram is theoretically more sensitive to small shifts, so by continuing to this stage, it is hoped that a more stringent level of supervision will be obtained. EWMA Multivariate (MEWMA) control chart making analysis in this study uses various kinds of weighting values λ for each variable. The results can be seen in Figure below. It can be seen that, with the EWMA multivariate (MEWMA) method of observation points that were previously in control using the Hotelling T^2 control diagram, it turns out that most of the monitoring points are out of control for various lambda values λ.

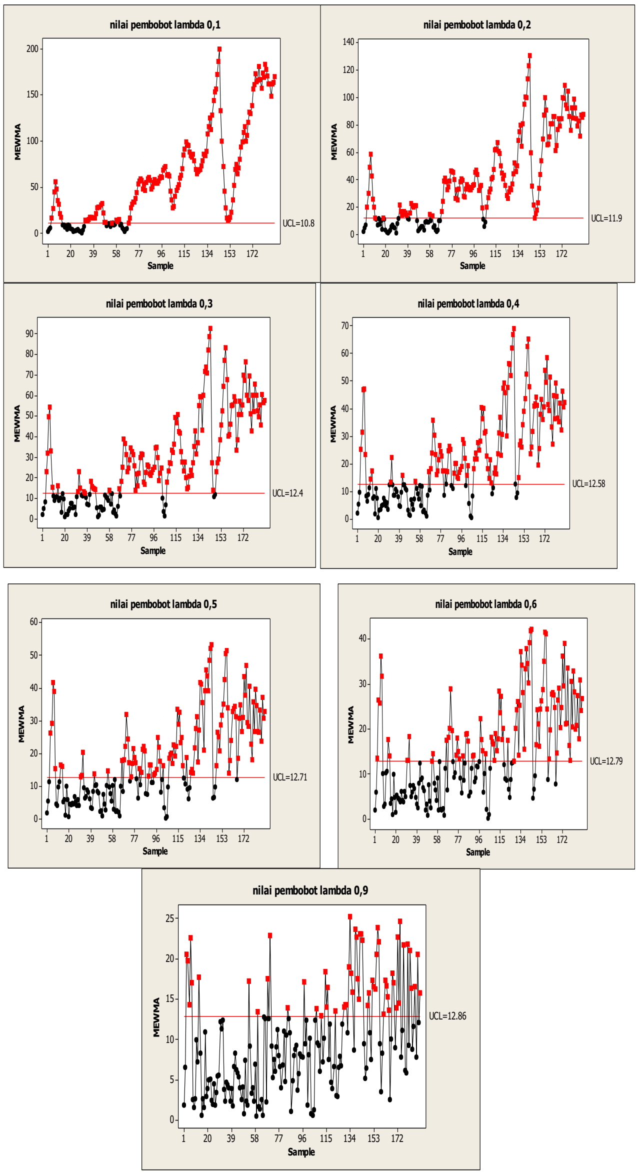

Figure 2. MEWMA control diagram for various weighting values λ

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IV. CONCLUSION

In line with the results of the study above, the purpose of this study is to develop alternative methods of product quality control using the Multivariate Exponential Weighted Moving Average (MEWMA) approach. The results of the research shown in the form of control diagrams show that the periodic data approach with regard to past, trend, seasonal data and several other features of the data is a more successful method to overcome the process shift problem in the control diagram making compared to the HotellingT^2control diagram method.

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