

An Interactive Method for Solving a lass of Stochastic Multi Objective Integer Linear Programming Problem

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Abstract: Decision problems of stochastic or probabilistic optimization arise when certain coefficient of an optimization model are not fixed or known but are instead, to some extent, stochastic (or random or probabilistic) quantities. This paper focused on multiobjective stochastic optimization. We propose a method for solving a multiobjective chance constraints integer programming problem based on interactive approach. We assume that there is randomness in the right-hand sides of the constraints only and that the random variables are normally distributed. Some examples are presented.

Index terms: Multiobjective integer linear programming; chance-constrained technique; interactive approach

I. INTRODUCTION

Decision problems of stochastic or probabilistic optimization arise when certain coefficient of an optimization model are not fixed or known but are instead, to some extent, stochastic (or random or probabilistic) quantities.

In recent years methods of multiobjective stochastic optimization have become increasingly important in scientifically based decision-making involved in practical problems arising in economic, industry, health care, transportation, agriculture, military purposes and technology. We refer the Stochastic programming Web Site (2002)[10] for links to software as well as test problem collections for stochastic programming. In literature there are many papers that deal with stability of solutions for stochastic multiobjective optimization problems. Among the many suggested approaches for treating stability for these problems [3, 4, 8, 17].

Since its inception, stochastic multiobjective integer linear programming continues to receive increasing interest. In particular, the interest has been associated with enterprises. Similarly, virtualization forms one of the core technologies supporting the use of stochastic multiobjective integer linear programming. According to Sahu and Tiwari (2012), virtualization emerges as a deviation from the traditional hosting method that has been associated with the adversity of inefficient cloud. Indeed, the traditional hosting method emerged to link several servers via Virtual LAN. Whereas some studies contend that this core technology supporting the

use of stochastic multiobjective integer linear programming is advantageous because it is secure (Grobauer, Walloschek & Stocker, 2011), others hold that the method is inefficient in the long-term (Kevin, 2012). For opponents of the traditional hosting method as a stochastic multiobjective integer linear programming platform, the argument is that the majority of the available physical hardware remains unused. In a quest to address this challenge of inefficiency, virtualization has emerged. In the study by Mishra, Mathur, Jain and Singh (2013), it was observed that the use of virtual machine monitors implies that a given physical server could host several instances of operating systems. Thus, virtualization implies that one server uses the hardware power in a more efficient way. This paper focuses on the current literature survey regarding virtualization as a core technology supporting stochastic multiobjective integer linear programming – and deviating from the traditional hosting method that has been, perceivably, marred by inefficiency.

II. PROBLEM STATEMENT AND THE SOLUTION CONCEPT

The chance-constrained multiobjective integer linear programming problem with random parameters in the right-hand side of the constraints can be stated as follows:

$$\begin{aligned} \text{(CHMOILP):} \quad & \max F(x), \\ & \text{subject to} \\ & x \in X, \end{aligned}$$

where

$$X = \left\{ x \in \mathbb{R}^n \mid P\left\{g_i(x) \sum_{j=1}^n a_{ij}x_j \leq b_i\right\} \geq \alpha_i, i=1, 2, \dots, m, x_j \geq 0 \text{ and integer, } j=1, 2, \dots, n \right\}$$

According to Moreno-Vozmediano, Montero and Llorente (2013), stochastic multiobjective integer linear programming environments that rely heavily on virtualization provide room for the transfer of virtual machines. The transfer occurs between physical systems. Noor, Sheng, Zeadally and Yu (2013) observed that the competitive nature of the field has prompted cloud providers to establish new approaches through which optimal virtual machine placement could be acquired. Pearce, Zeadally and Hunt (2013) concurred that the majority of cloud providers have responded to the need to achieve optimal virtual machine placement by ensuring that they address challenges such as SLA violation, performance

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degradation, high cost, and energy consumption. In another study, Pék, Buttyán and Bencsáth (2013) documented that virtualization offers an opportunity for reducing hardware and energy costs via the consolidation of servers. This assertion concurred with the findings established by Roy, Sarkar, Ganesan and Goel (2015). In the latter study, it was documented that virtualization meets the resource needs of various applications that virtual machines host (by optimizing resource sharing). The eventuality is that stochastic multiobjective integer linear programming can be achieved in environments entailing shared resource pools that play the role of public and private clouds.

In the study by Tari (2014), the main aim was to determine different types of virtualization and their application to cloud-based environments. In the findings, it was noted that three min techniques aid in the virtualization of guest operating systems. The techniques were documented to include paravirtualization, hardware-assisted virtualization, and full virtualization. Toosi, Calheiros and Buyya (2014) observed that fully virtualized environments provide guest operating systems with a quantity and quality of services similar to those that physical systems offer. The eventuality is that full virtualization offers services such as virtualized memory, virtual devices, and virtual BIOS. Varadharajan and Tupakula (2014) observed further that stochastic multiobjective integer linear programming environments that implement the full virtualization technique imply that the operating systems is unlikely to detect that it has been virtualized, implying further that it (the operating system) does not require modification.

Definition 1.

A point $x^* \in X$ is said to be an efficient solution for problem (CHMOILP) if there does not exist another $x \in X$ such that $F(x) \geq F(x^*)$ and $F(x) \neq F(x^*)$ with

$$P\{g_i(x^*) \sum_{j=1}^n a_{ij}x_j^* \leq b_i\} \geq \alpha_i, i=1, 2, \dots, m$$

In this case, the set of constraints X of problem (CHMOILP) can be rewritten in the deterministic form as:

$$X' = \left\{ x \in R^n \mid \sum_{j=1}^n a_{ij}x_j \leq E\{b_i\} + K_{\alpha_i} \sqrt{\text{Var}\{b_i\}}, i=1,2,\dots,m, x_j \geq 0 \text{ and integer}, j=1,2,\dots,n \right\}$$

Thus, problem (CHMOILP) can be understood as the following deterministic version of a multiobjective integer linear programming problem:

$$\begin{aligned} \text{(MOILP):} \quad & \max [f_1(x), f_2(x), \dots, f_k(x)], \\ & \text{subject to} \\ & x \in X'. \end{aligned}$$

Regarding paravirtualization as one of the techniques of virtualization, Xiao and Xiao (2013) indicated that the method yields better performance than hardware-assisted virtualization or full virtualization. In particular, this type of virtualization provides room for communication between the hypervisor and the guest operating system. The implication is that paravirtualization modifies the guest operating system to steer the communication mentioned above. In particular, paravirtualization achieves guest operating system modification via the translation of non-virtualizable instruction with hypercalls. Sahu and Tiwari (2012) documented that hypercalls constitute instructions responsible

for direct communication with virtualization layers. Lastly, hardware-assisted virtualization has emerged with the aim of overcoming the demerits associated with paravirtualization; especially those that accrue from hardware modifications seeking to enhance communication between the hypervisor and the guest operating system.

For this purpose, we consider the following integer linear programming problem with a single-objective function as:

$$\begin{aligned} \text{Ps}(\varepsilon): \quad & \max f_s(x), \\ & \text{Subject to} \end{aligned}$$

$$X(\varepsilon) = \{x \in R^n \mid f_r(x) \geq \varepsilon_r, r \in K - \{S\}, x \in X'\}$$

where $s \in K = \{1, 2, \dots, k\}$ which can be taken arbitrary. Notably, virtualization as a core technology supporting stochastic multiobjective integer linear programming implies that the cloud service providers do not create actual versions of network resources, operating systems, storage devices, desktops, or servers. Instead, virtualized versions are created (Grobauer, Walloschek & Stocker, 2011). In relation to stochastic multiobjective integer linear programming, most of the past scholarly studies avow that virtualization plays a crucial role. In particular, Kevin (2012) observed that stochastic multiobjective integer linear programming enables

users to share cloud-based data such as applications while virtualization ensures that these users share the infrastructure. Therefore, virtualization has emerged to complement applications by offering standard versions to the respective cloud users. Should a next version of the application be released, virtualization demands that the cloud providers offer latest versions to the respective cloud users (Mishra, Mathur, Jain & Singh, 2013). In relation to this pattern, Moreno-Vozmediano, Montero and Llorente (2013) cautioned that the implementation of virtualized environments in stochastic multiobjective integer linear programming poses the merit of responsiveness but the downside of the core technology is that it is proves costly in small enterprises.

In the study by Noor, Sheng, Zeadally and Yu (2013), the main aim as to unearth some of the benefits and drawbacks accruing form virtualization in cloud-based environments. From the positive perspective, it was noted that virtualization poses merits such as application isolations, improved disaster recovery, increased uptime, and cost-efficiency. However, the study highlighted further that virtualization as a core technology of stochastic multiobjective integer linear programming has also come with demerits. For instance, the hypervisor plays the role of separating the physical hardware from virtual machines' operating systems (Pearce, Zeadally & Hunt, 2013). As such, the introduction of virtual machines on the same physical machines implies that the latest security updates need to be installed and accompany the operating system; besides proper



implementation accounts for the success of running virtualized machines in stochastic multiobjective integer linear programming. According to Grobauer, Walloschek and Stocker (2011), regular updates of the hypervisor promise to prevent attacks. In particular, the study acknowledged that the hypervisor poses a low number of vulnerabilities. As such, the number of times that cloud-based service providers need to update it (the hypervisor) remains low. Whereas Kevin (2012) concurred that the regular update of the hypervisor does not guarantee total protection against zero-day vulnerabilities, it protects the virtual world from vulnerabilities that are known. In another study, Mishra, Mathur, Jain and Singh (2013) sought to highlight some of the mitigation approaches to resource allocation as a risk associated with virtualization in stochastic multiobjective integer linear programming. Indeed, it was asserted that before proceeding with resource allocation to new machines, proper protection needs to be assured. As such, it was noted that memories ought to be filled with zeros when service providers decide to assign physical memories to new machines. In so doing, Moreno-Vozmediano, Montero and Llorente (2013) concurred that cloud service providers ensure that the data does not leak, especially that which has been used by initial virtual machines. Other studies indicate that the challenge of resource allocation in virtualization could be addressed by ensuring that new virtual machines employ different forensic tools while reading the hard drive's unstructured information into files, proceeding further to analyze for the possibility of leaked data. Noor, Sheng, Zeadally and Yu (2013) advocated for the need to overwrite hard drives (that old partitions may have used) with zeros while they are assigned to new virtual machines.

In the study by Pearce, Zeadally and Hunt (2013), the main aim was to examine some of the current solutions to virtual machine attacks in stochastic multiobjective integer linear programming. Indeed, the study suggested that cloud-based service providers ought to ensure that virtualization software are capable of differentiating against traffic that goes to and come from various virtual machines. It was also noted that the service providers ought to analyze the traffic to detect potential and known attacks. To achieve this trend, Pék, Buttyán and Bencsáth (2013) highlighted that the service providers ought to embrace port mirroring to ensure that traffic is copied on specific switch ports to other ports (via the mirroring procedure) before allowing data analysis via IDS/IPS. Lastly, Roy, Sarkar, Ganesan and Goel (2015) focused on migrant attacks and how they can be curbed in virtualization. The study indicated that cloud-based service providers ought to embrace proper mechanisms responsible for MITM attack detection and prevention. It was also noted that the service providers could enhance security by ensuring that migrations take place over communication channels that are deemed secure; inclusive of TLS. However, the problem can be started as the following equivalent mixed integer linear programming problem.

$$\max y \tag{5}$$

Subject to:

$$f_k(x) - (\bar{f}_k - f_k)y \geq f_k, k \in H, \tag{6}$$

$$f_k(x) - \bar{f}_k + \alpha(\bar{f}_k - f_k), k \in H, \tag{7}$$

$$f_k(x) = \bar{f}_k, k \in E \tag{8}$$

$$x \in X \tag{9}$$

$$y \geq 0 \tag{10}$$

The solution of (2) (or equivalently (B)) is a weak efficient solution for (1). However, if it is desired to obtain an efficient solution, then we may solve the following single objective surrogate problem:

$$\max_{x \in X} T(x) = \max_{x \in X} \left\{ \min_{k \in H} \frac{f_k(x) - f_k}{\bar{f}_k - f_k} + \beta \sum_{k \in K} f_k(x) - f_k \right\} \tag{11}$$

Subject to:

$$f_k(x) \geq \bar{f}_k + \alpha(\bar{f}_k - f_k), k \in L \tag{12}$$

$$E \quad f_k(x) = \bar{f}_k, k \in \tag{13}$$

where β is an arbitrary small positive number. Problem (11) can be reduced to the following equivalent mixed integer linear programming problem.

$$\max(y + \beta \sum_{k \in K} y_k) \tag{14}$$

Subject to

$$f_k(x) - \bar{f}_k = y_k, k \in H \tag{15}$$

$$f_k(x) - \bar{f}_k = -y_k, k \in L \tag{16}$$

$$f_k(x) - (\bar{f}_k - f_k)y \geq f_k, k \in H \tag{17}$$

$$f_k(x) = \bar{f}_k, k \in E \tag{18}$$

$$f_k(x) = \bar{f}_k, k \in E \tag{19}$$

$$x \in X \tag{20}$$

$$y, y_k \geq 0, k \in K \tag{21}$$

V. PROPOSED ALGORITHM

The proposed algorithm consists of the following three steps.



- Steps 1. Determine an initial (weak) efficient solution.
Steps 2. Show the solution to the DM. if DM is satisfied with the solution,
Stop: otherwise, ask the DM to specify a new reference point \bar{f}_k , using AHP and go to step 3.
Steps 3. Based on the values of \bar{f}_k and f_k (the last solution), solve (5) (or (11)) and find a new intermediate weak efficient (or efficient) solution $f_k(x)$; go to Step 2.

There are two ways to avoid this problem. One way is to require the DM to state the aspiration levels such that $\bar{f}_k < f_k$ for at least one $k \in K$. However, this puts an extra constraint on the DM. The second way to avoid the problem is to solve (22) (or (26)) instead of (5) (or (14)).

That is,

$$\max(z_1 - z_2) \quad (22)$$

Subject to

$$f_k(x) - (\bar{f}_k - f'_k)(z_1 - z_2) \geq f'_k + \alpha(\bar{f}_k - f_k), k \in K \quad (23)$$

$$x \in X \quad (24)$$

$$z_1, z_2 \geq 0 \quad (25)$$

Or

$$\max(z_1 - z_2 + \beta \sum_{k \in K} y_k) \quad (26)$$

$$f_k(x) - f_k = y_k, k \in K \quad (27)$$

$$f_k(x) - (\bar{f}_k - f'_k)(z_1 - z_2) \geq f'_k + \alpha(\bar{f}_k - f_k), k \in K \quad (28)$$

$$x \in X, \quad (29)$$

$$y_k \geq 0, k \in K \quad (30)$$

$$z_1, z_2 \geq 0, \quad (31)$$

where

$$k_1 = \arg \max((\bar{f}_k - f_k))$$

and

$$f'_k = \begin{cases} f_k + \frac{(\bar{f}_k - f_k)}{2}, & k = k_1 \\ f_k, & k = k_1 \end{cases}$$

In summary, this paper has focused on some of the past scholarly studies examining the use of virtualized environments in stochastic multiobjective integer linear programming or cloud-based applications. Major virtualization techniques include paravirtualization, hardware-assisted virtualization, and full virtualization. From the positive perspective, the deviation of virtualized environments from traditional environments comes with merits such as application isolations, improved disaster recovery, increased uptime, and cost-efficiency. On the other hand, major drawbacks surrounding virtualized environments (relative to

cloud-based applications) include data leakage during resource allocation, hypervisor attacks, and compromised virtual machines that threaten other machines sharing the physical host. The implication for the future of stochastic multiobjective integer linear programming is that virtualization, as highlighted by the current literature, poses a two-fold outcome entailing benefits and drawbacks. Hence, cloud-based service providers seeking to survive the changing user demands, needs, and preferences in the near and far future are expected to maintain the positive side of the pattern while transforming the perceived drawbacks into opportunities for improvement.

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