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Some Applications of Engineering Mathematics in Mechanical Engineering

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Abstract: Mathematics or particularly engineering or applied mathematics is widely used in mechanical engineering. This paper will discuss some examples of applications of mathematics in mechanicalengineering. The applications are those found in real life that is in industry where many mechanical engineers are working.While engineering mathematics is a mandatory subject taught in the mechanical engineering departments, not many students understand it very well. Applications shown in the class tend to be very simple; even many engineering mathematics textbooks do not touch real applications well. Most students who have studied mathematics a lot cannot relate mathematics to other subjects which have a lot of mathematics in their contents. It is hoped that through examples given, mechanical engineering students couldappreciate the use of engineering mathematics in real life and be motivated to understand their engineering problems better. It is also expected that mathematics lecturers could be encouraged to find and provide mathematics problems which are more related to engineering fields rather than give examples which are dull and dry.

Index terms:engineering mathematics, applications, mechanical engineering.

I.INTRODUCTION

Strangely enough, the definitions of engineering mathematics are not found in many engineering mathematics text books (see for example, [1]−[5])or even not found in various encyclopedia of mathematics and dictionary of mathematics. Thoseauthors may assume that the readers (students and lecturers) understand the definition well. Fortunately, Wikipedia provides a definition as follows: *Engineering Mathematics*is a branch of mathematics concerning mathematical methods and techniques that are typically used in engineering and industry [6]. Now, it is easy to see scopes which are offered by engineering mathematics textbooks that fit this definition. Most texbooks discuss about calculus, vector and vector calculus, ordinary differential equations, partial differential equations, numerical methods, fourier and laplace transforms. In addition, optimization, probability and statistics are also included in [2].

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Like many engineering fields, mechanical engineering has many subjects which have a lot of mathematics in their contents, which should be properly called as engineering mathematics. Mechanics of Materials, Thermodynamics, Heat Transfer and Internal combustion engines are examples of those subjects. However, while students have studied engineering mathematics a lot, most of them cannot relate it to engineering subjects. After studying differential equations in calculus, they are expected to be able to apply them to solve deflection problems in mechanics of materials, for example. However, not only students, even engineering lecturers often find it difficult to apply mathematics to engineering problems. There are many examples of engineering applications in the textbooks but they are often differently applied in mechanical engineering. As an example is on Newton's first law about cooling of an object in a room. Theoretically, it will take a very long time (infinity!) for the object to have the same temperature as the room. However, in practice it will be achieved in less than one hour or even less. In the following, we will discuss some applications of engineering mathematics in mechanical engineering which are important in industry.It ishoped that real problems taken from engineering subjects will encourage students to study engineering mathematics better.Mathematics lecturers are encouraged to find and provide mathematics problems which are more related to engineering fields rather than give examples which are dull and dry.

A.Mechanics of Materials

There are many applications of the subject such as in buildings and structures of cars and heavy equipment. An example of the application is in the determination of the deflection when a beam is given a partial triangle load; see Fig. 1.

We will find the curve of deflection along the beam and determine the maximum deflection that occurs. The relationships between the bending moment $M_x$for the distance x from the left support and the deflection Y is given by

$M_x = EId^2y/dx^2$ (1)

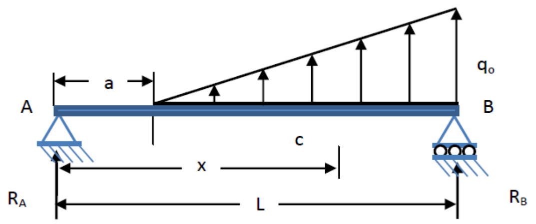

Figure 1 Partial triangular load on a simply supported beam

Here, E is Young modulus of elasticity and I is moment of inertia of the beam. Collectively, EI is called the flexeural ridity of the beam. Most textbooks take the downward deflection to be positive but it is taken to be negative in [7], [8]. The moment of inertia is positive if the direction is clockwise. Integrating Eq. (1) with respect to x gives slope of the deflection curve and integrating it produces the curve of deflection.

For the partial triangular load, M_x can be derived as

$$M_x = \frac{q_0 b^2 x}{6L} - \frac{q_0[x-a]^3}{6b} \quad (2)$$

Integrating Eq. (2) twice produces

$$EI\frac{dy}{dx} = \frac{q_0 b^2 x^2}{12L} - \frac{q_0[x-a]^4}{24b} + C_1 \quad (3)$$

$$EIy = \frac{q_0 b^2 x^3}{36L} - \frac{q_0[x-a]^5}{120b} + C_1 x + C_2 \quad (4)$$

The square bracket means that a value inside it which is less than zero will be taken to be zero.
Boundary conditions:
Deflection at the supports will be zero, so y(0) = 0 and y(L) = 0. Without any derivation the constant C_2 will be found to be zero while C_1 is given by

$$C_1 = \frac{q_0 b^2(3b^2-10L^2)}{360L} \quad (5)$$

If a = 0, b = L, we then have

$$C_1 = \frac{7q_0L^3}{360L} \quad (6)$$

which is the same as the value of C1 for a tringular load; for example, see [7] and [8] for the derivation of C_1.Here we only need calculus to derive the curve of deflection.When the cross section varies, the analysis of the deflection becomes more difficult; see [9] for the case of a cantilever with variable cross section

B.Kinematics

Kinematics can be defined as the study of motion without regard to forces [10]. Kinematics has many applications such as in bicycles, automobiles, tractors and cranes. The basic of kinematics is a four-bar linkage as shown in Fig. 2.

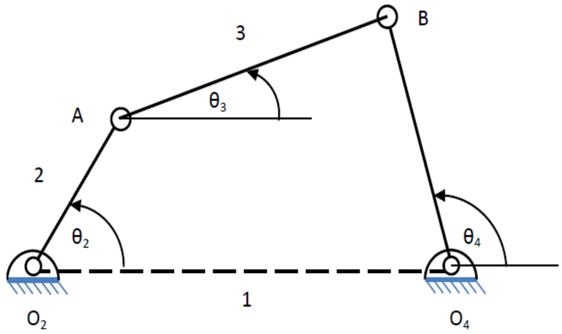

Fig. 2 A Four-bar linkage

It is assumed that link 2 (AO_2) rotates counter clockwise, dragging link 3 (AB) and link 4 (BO_4). Link 1 (O_2O_4) is fixed. Suppose that at after t seconds after it rotates, the position of each link is as shown in Fig. 2. Given the angle of link 2 (θ_2) and the lengths of all links, the angles of link 4 and 3 ($\theta_4$and θ_3, respectively) can be found; they are given in Eqs. (7) and (8). See [10] for the derivation for the formulas.

Let

$$K_1 = \frac{d}{a};\ K_2 = \frac{d}{c}; K_3 = \frac{a^2-b^2+c^2+d^2}{2ac}; K_4 = \frac{d}{b}; K_5 = \frac{c^2-d^2-a^2-b^2}{2ab}$$

$$Au^2 + Bu + C = 0$$

where

$u = \tan\theta_4/2$ and $A = (1-K_2)\cos\theta_2 - K_1 + K_3$, $B = -2\sin\theta_2$, $C = K_1-(1+K_2)\cos\theta_2+K_3$

So,

$$\theta_4 = 2\tan^{-1}\frac{B\pm\sqrt{B^2-4AC}}{2A} \quad (7)$$

Also,

$$Dv^2 + Ev + F = 0$$

where

$v = \tan\theta_3/2$ and $D = (1+K_4)\cos\theta_2\ K_1+K_5$, $E = -2\sin\theta_2$, $F = K_1+(K_1-1)\cos\theta_2\ K_5$

So,

$$\theta_3 = 2\tan^{-1}\frac{E\pm\sqrt{E^2-4EF}}{2D} \quad (8)$$

As θ_2 changes, θ_3 and θ4 will also change. However, when link 2 continuously rotates CCW, link 4 may rotate in the *opposite* direction, not necessarily in the same direction as link 2. Complete analysis of a four-bar linkage is quite complicated; see [10].

II.THERMODYNAMICS

Thermodynamics is a very important subject which has many applications in industry and our everyday life.

Here we will show where a lot of mathematics is needed in the Equation of State (EOS) which is a part of thermodynamics. Students without enough background in mathematics will be very difficult to understand it.

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EOS is a relationship among pressure (P), volume (V) or specific volume (v) and temperature (T) of a fluid. There are many EOS that have been developed; the simplest one is for an ideal gas,

$$Pv = RT \tag{9}$$

which is the simplest one but unfortunately is the least accurate. In SI unit, P is in kPa, V in m^3, v in m^3/kg and T in K.

Better EOS are in the form of cubic equation, see for example [11]−[13]; there are many of them but we will only show four equations that is Van der Waals (VdW), Redlich-Kwong (RK), Soave (SRK) and Peng-Robinson (PR). All cubic EOS can be written

$$P = \frac{RT}{V-b} + \frac{a}{V^2+ubV+wb^2} \tag{10}$$

where u and w are integers while a and b arereal numbers [11], [13]. Parameters a and b can be computed from critical condition:

$$\left(\frac{\partial P}{\partial V}\right)_{T_c} = 0; \left(\frac{\partial^2 P}{\partial V^2}\right)_{T_c} = 0 \tag{11}$$

Constants for those four EOS are given in Table 1, which is taken from [13]. P_c and T_c are critical pressure and temperature, respectively. Their values for many compounds are given in [13].

Table 1 Constants for four common cubic equations of state*

Equation	u	w	b	a
van der Waals	0	0	$\frac{RT_c}{8P_c}$	$\frac{27}{64}\frac{R^2T_c^2}{P_c}$
Redlich-Kwong	1	0	$\frac{0.08664RT_c}{P_c}$	$\frac{0.42748R^2T_c^{2.5}}{P_cT^{1/2}}$
Soave	1	0	$\frac{0.08664RT_c}{P_c}$	$\frac{0.42748R^2T_c^2}{P_c}[1+f\omega(1-T_r^{1/2})]^2$ where $f\omega = 0.48 + 1.574\omega - 0.176\omega^2$
Peng-Robinson	2	−1	$\frac{0.07780RT_c}{P_c}$	$\frac{0.45724R^2T_c^2}{P_c}[1+f\omega(1-T_r^{1/2}]^2$ where $f\omega = 0.37464 + 1.54226\omega - 0.26992\omega^2$

RK and PR EOS have another parameter called acentric factor ω; their values are given in [13]. As an example, for steam, P_c = 22060 kPa, T_c = 647.1 K, ω = 0.344. At P = 5000 kPa and T = 300°C, the value of v from a steam table is 0.4535 m^3/kg [12]. Using the ideal gas and each cubic EOS, each value of v and its deviation from the actual one is shown in Table 2.

Table 2 Specific volume of steam computed using different EOS

EOS	v, m^3/kg	Deviation, %
Ideal	0.05291	16.66
VdW	0.04770	5.18
RK	0.04658	2.71
SRK	0.04609	1.62
PR	0.04558	0.51

For this example, the ideal EOS is the worst while the PR EOS is the best.

III.CONCLUSION

Mechanical Engineering has many subjects with a lot of mathematics content in them. However, many students cannot relate mathematics to those subjects. Many examples of engineering applications can be found in the textbooks but they are often differently applied in mechanical engineering. Examples of applications of engineering mathematics in this paper are important in the industry.Hopefully, real problems taken from engineering subjects will encourage students to study engineering mathematics better.

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