Pseudo Forces in Yarn Unwinding from Packages

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Abstract: In a recent paper we have shown the origin of the pseudo forces in equations of motion in a non inertial observation frame. In addition to Coriolis and centrifugal force, there is another pseudo force in rotating frames with time dependant angular velocity. This force affects the yarn dynamics. Using experimental data as an input, we will calculate oscillations of yarn tension for different package geometries and winding angles. We show how the winding angle and the package radius influence the angular velocity of the yarn during the unwinding. Since the pseudo forces on the yarn in the balloon depend on the angular velocity, this velocity has a large influence on the oscillations of yarn tension that we wish to reduce.

Index terms: pseudo forces, dynamics of yarn, balloon theory, non inertial systems.

I. INTRODUCTION

In production of garments thread unwinding exists in a sewing process. In order to achieve low and constant tension of thread or yarn it is necessary to optimize the process of unwinding. Computer simulation are now in use for this purpose, so it is important to obtain a mathematical description of yarn motion [1,2,3,4,5,6]. Building on this foundation we have simplified the problem even further. In paper [10] we have shown the origin of the pseudo forces in the equations of motion in a non-inertial reference frame. In a recent paper [11] we measured how the tension at the eyelet depends on the angular velocity of unwinding and we developed a mathematical model:

\[ f(t) = \text{sign}(\sin t)\sin t \left(\frac{1}{40}\right), \]

which would permit to simulate the process of unwinding. We measured the unwinding speed, package radius and tension of yarn as we unwound yarn from parallel cylindrical packages. We used a balloon limiter, that limits the balloon radius.

\[ F_c = 2m\omega \times v \times (m\omega \times r) - m\omega \times r = ma' \]

(2) In a non-inertial reference frame, the body is accelerated not only by the true external forces \( \mathbf{F} \), but also by the pseudo forces. These are the Coriolis, centrifugal and Euler's forces. There are numerous situations where the use of non-inertial reference frames is more convenient that that of inertial frames, in spite of the addition of the pseudo forces. In weather prediction it is, for example, necessary to solve a complex system of differential equations in a reference frame which is fixed to the Earth. Since the Earth is not an inertial reference frame (due to its rotation), it is crucial to take into account the Coriolis and centrifugal forces.

Figure-1. Dependence of tension \( T \) on angular velocity \( \omega \) of the yarn [11].

Figure-2. Unwinding yarn from packages. At the lift-off point \( L_p \) the yarn lifts from the package and forms a balloon. At the unwinding point \( U_p \) the yarn starts to slide on the surface of the package. Angle \( \phi \) is the winding angle of the yarn on the package [10].

The Newton's second equation for a point-like particle in a rotating reference frame takes the form [7,8,9]:

\[ F - 2m\omega \times v - m\omega \times (m\omega \times r) = m\ddot{\mathbf{r}} - \mathbf{F}_c \]
Particularly important is the Coriolis force which makes the low-pressure systems spin in the counter-clockwise direction in the Northern hemisphere [16,17,18,19]. Non-inertial reference frames are also useful in studying the yarn unwinding. It may be used, for example, when working within the quasi-stationary approximation to describe the yarn unwinding from packages where layers have a large number of yarn loops.

The equations of motion which govern the motion of the yarn are known:

\[ \rho D^2 \mathbf{r} = \frac{\partial}{\partial s} \left[ (T(t)) + f - 2\rho \mathbf{\omega} \times D\mathbf{r} - \rho \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}) - \rho \mathbf{\ddot{r}} \times \mathbf{r}, \right] \]

we have established them in our previous paper [10, 12,13,14,15]. They can be partially analytically solved, as we show in the following. Here \( \rho \) is the mass per unit length of the yarn, \( D \) is the comoving time derivative operator, \( T \) is the yarn tension and \( f \) is the density of external forces acting on the yarn. We identify three pseudo forces that act on an infinitesimal segment of yarn in a non-uniformly rotating frame:

1. \(-2\rho \mathbf{\omega} \times D\mathbf{r}\) the Coriolis force
2. \(-\rho \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r})\) the centrifugal force
3. \(-\rho \mathbf{\ddot{r}} \times \mathbf{r}\) an additional force due to changing rotational velocity of the non-inertial observation frame. It is the Euler's forces.

Figure-3. Pseudo forces on a short yarn segment.

III. RESULTS OF SIMULATION AND DISCUSSION:

In Fig. 4 we show the results for the oscillations of tension for a range of four winding angles \( \phi_0 \sim 0^\circ, \phi_0 = 10^\circ, \phi_0 = 20^\circ \) and \( \phi_0 = 30^\circ \) for a very small package radius of \( c=70 \) mm and for unwinding velocities, \( V=1400 \) m/min, respectively. We observe very high tension in the yarn for all enumerated winding angles. The tension oscillates from 11.2 cN to 182 cN.

Figure-5 shows the time dependence for three winding angles 0°, 5° and 10° for package radius \( c=500 \) mm and unwinding velocity \( V=2000 \) m/min. For packages with such large radius the tension and the tension oscillations are very small for all winding angles.

The effect of pseudo forces can be estimated by comparing them to each other. Let us compute the ratio between the Euler and the centrifugal force:

\[ \frac{m\ddot{r}}{m\omega^2 r} = \frac{\dot{\omega}}{\omega^2}. \]

The angular acceleration is written as

\[ \frac{\dot{\omega}}{\omega^2} \frac{2\pi}{n} \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\omega_0}. \]

This velocity can be related to the period of the motion (the time during which one loop of yarn is unwound) \( t=n2\pi/\omega_0 \). We have

\[ \frac{\dot{\omega}}{\omega^2} \frac{2\pi}{n} \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\omega_0}. \]

The factor \( 2\pi/n \) can be assumed to 1. Hence,
\[ \frac{\dot{\omega}}{\omega^2} = \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\omega_0}. \]  

(7)

In Figures 6, 7 and 8 we plot the amplitude of yarn tension oscillations as a function of unwinding velocity \( V = 1000-2000 \text{ m/min} \) and winding angles from 0° to 20° for three different package radii, namely \( c = 500 \text{ mm}, c = 200 \text{ mm} \) and \( c = 70 \text{ mm} \). For all the packages we notice that the tension oscillations are larger at high unwinding velocity and for larger winding angles. The amplitudes become particularly high for velocities above \( V = 1600 \text{ m/min} \) and for angles larger than \( \phi = 5^\circ \). For large radius of \( c = 500 \text{ mm} \) (Fig. 8) the tensions oscillations are very small in the full range of parameters, thus from such packages it should be possible to unwind yarn at all velocities and for all winding angles. For smaller package radius of \( c = 70 \text{ mm} \), however, the unwinding is only possible for sufficiently small winding angle below 5°.

\[ \Delta \omega = \omega_{\text{max}} - \omega_{\text{min}} = 2V \tan \phi_0. \]  

(8)

In the calculation we have used the value for \( \phi < 25^\circ \), we have \( \tan \phi \approx \phi \).

We get

\[ \Delta \omega \approx \frac{2V \phi_0}{c}. \]  

(9)

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\[ \Delta \omega = \omega_{\text{max}} - \omega_{\text{min}} = \frac{2V}{c} \tan \phi_0. \]

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We can compute the average angular velocity during unwinding as follows

\[ \omega_0 = \frac{\omega_{\text{max}} + \omega_{\text{min}}}{2} = \frac{V}{c} \cos \phi_0 \approx \frac{V}{c}. \]  

(10)

Here we made use of the small-angle approximation \( \cos(\phi_0) \approx 1 \).

In Fig. 9 we plot the comparison of the amplitude of the tension oscillation as a function of the unwinding velocity \( V \) and the package radius \( c \). The winding angle is constant, \( \phi = 5^\circ \).

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IV. CONCLUSION

We have demonstrated the analogy between the Newton’s equation for a point particle and the equation of motion for yarn in a non-inertial reference frame. We have emphasized the commonly neglected Euler’s force. We thus conclude, that the importance of Euler’s velocity is equal to the ration between the difference of the angular velocity for unwinding in either direction and their average. We have shown that its affect by increasing the angle of winding has doubled. By increasing the winding angle the yarn tension oscillations are also increased. This is particularly noticeable for packages with small radius. Furthermore, we find that it is advisable for the winding angle to be below 5°.

REFERENCES


