Abstract: The study deals the dynamic expression for motion of a multilink manipulator with mechanical flexibility. The hybrid composite material manipulator links are connected in series and actuated at the joints of multilink. A power transmits through double links to complete the task. The Lagrangian technique of equations is obtained for simulation study. The equations are transformed into state space model and the matrix is changed into transfer function which is utilized for the system control optimization. Using a time domain and frequency graphical interpolation analysis to investigate stability of the system through linear time invariant viewer and controller part for the unsteady structure is to be organized. A variety of control techniques are added to control the tip position of serial flexible links through MATLAB algorithms. The quadratic optimal regulator is used to find hub angular displacement, flexible deflection, transient period and response of steady time of a multilink robotic manipulator.

Key wards: Multilink, polymer composite, stability, control.

1. INTRODUCTION

The flexibility in industrial service robot connectors are to perform a critical task in a dangerous environment, an easier way to carry out the work is called work flexibility; and the other is mechanical part flexibility of a robot manipulator which has wider range of applications. The roles of flexible manipulators are in space shuttle [1], solar panel and mobile service robot etc. Further the elastic flexibility has two meanings named as structural flexibility and joint flexibility [2]. The implementation of structural flexibility using light weight composite material for link, leads to weightless manipulator designed for faster movement and quick completion of task with low energy consumption [3]. A lightweight structure always subjected to oscillation or vibration while in motion then the controller take care the vibration of the mechanical link adequately damped [4]. Methods of modeling and motion equations [5] and control of lightweight links were found in the reviews [6]. The elastic link vibration control of link [7] and LQR robust control design for a flexible link have been proposed [8]. An article by Rahimi presented a review about dynamic analysis and intelligent control [9]. Reduction of vibration using minimum time optimal controller has been described [10].

In this paper, the dynamic equations of motions are studied based on Lagrange approach, and state space matrix of a serial link robotic manipulator by adding mechanical flexibility are obtained. The composite material parameters are used for simulation case study. The tip deflections of a links and numerical analysis are carryout using software of MATLAB computational language. To check the stability of a system a linear time invariant viewer techniques are adopted and different control methods implemented to control the system.

II. MATERIALS AND MODELING METHODS

A. Modeling Scheme

A flexible manipulator consists of two links connected in serial joints. The base ends of the each links are attached by an actuating mechanism to rotate the links. The second link end is designated as tip of the link. The joints in between the links which are numbered serially from first to last as indicated in the figures (1) and (2). The dynamic model of multi links rotational motion is derived by using Lagrange approach. To determine the Lagrange function, the system associated energies such as kinetic (K_e) and potential (P_e) are calculated.

![Figure 1. Schematic Diagram of 2 Flexible Links](image-url)

**Figure 1. Schematic Diagram of 2 Flexible Links**
The representation values of θ used for control the system.

This state space model is written as a mathematical equation:

\[ X = AX + Bu \]  

(3)

where A, B, and C are matrices of the system parameters. The matrix A defines the system dynamics, B represents the input, and C relates the output to the state variables.

B. Composite Materials

Lightweight hybrid polymer composite material parameters are given in Table 1. The composite material consists of carbon fiber of 204gsm, which is a thickness of 0.22mm, Glass fiber of 202gsm and its thickness is 0.18mm, Kevlar fiber of 200gsm, Kevlar fiber thickness is 0.22mm. Using volume and weight ratio relationship, the weight and thickness of composite materials are calculated. The composite material consists of carbon fiber + E glass fiber + graphite particulate + epoxy. For a case study, the parameters [14] of double flexible links are given in the table 1.

Table 1. Data for two links

<table>
<thead>
<tr>
<th>Parameter of flexible link manipulator</th>
<th>Symbol</th>
<th>Units</th>
<th>Flexible link-1</th>
<th>Flexible link-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link length</td>
<td>L₀</td>
<td>m</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Mass of flexible link</td>
<td>L₁</td>
<td>kg</td>
<td>0.065</td>
<td>0.070</td>
</tr>
<tr>
<td>Tip deflection</td>
<td>d</td>
<td>m</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Link MOI</td>
<td>J₀</td>
<td>m²</td>
<td>0.00195</td>
<td>0.00093</td>
</tr>
<tr>
<td>Eq. MOI at Load</td>
<td>J₁</td>
<td>m²</td>
<td>0.099</td>
<td>0.092</td>
</tr>
<tr>
<td>Eq. viscous damping</td>
<td>B₀</td>
<td>-</td>
<td>0.199</td>
<td>0.199</td>
</tr>
<tr>
<td>Rotating motor efficiency</td>
<td>ηₘ₂</td>
<td>-</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>Back e.m.f torque const.</td>
<td>kₘ₀</td>
<td>-</td>
<td>0.0078</td>
<td>0.0077</td>
</tr>
<tr>
<td>Constant of motor torque</td>
<td>kₜ₀</td>
<td>-</td>
<td>0.0078</td>
<td>0.0077</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>k₅₀</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Gear box efficiency</td>
<td>η₅₀</td>
<td>-</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Link natural frequency</td>
<td>fₙ₀</td>
<td>Hz</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Manipulator link stiffness</td>
<td>kₕ₀</td>
<td>N/m</td>
<td>0.7883</td>
<td>0.7883</td>
</tr>
<tr>
<td>Armature resistance</td>
<td>Rₐ₀</td>
<td>Ω</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>Armature input voltage</td>
<td>Vₐ₀</td>
<td>V</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

C. Matrix Form

The system state matrix values are calculated as follows:

\[ X = AX + Bu \]  

(4)

\[ X = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.9627 & 0 & -2.7056 & 0 \\ 0 & 0 & 0 & 4.0997 & 0 & -2.9114 & 0 \\ 0 & 0 & -412.2217 & 0 & 2.7056 & 0 & 0 \\ 0 & 0 & 0 & -408.3587 & 0 & 2.9114 & 0 \end{bmatrix} \]  

(5)

\[ B = \begin{bmatrix} 0 & 0 & 0 & 1.2953 & 1.3939 & -1.2953 & -1.3939 \end{bmatrix}^T \]  

(6)

\[ C = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}^T \]  

(7)

III. SYSTEM STABILITY CHECK ANALYSIS

A. Root Locus

The root locus is a picture of a graphical method, which is used to determine the system characteristic equation roots.
Hybrid Polymer Composite Flexible Multi-Link Robotic Manipulator Simulation Analysis

\[ G(s) = \frac{\theta(s)}{V_m} \]  

(9)

\[ 0.4525s^3 + 120.5s^2 + 6.984s^4 + 1131s^3 + 2472s^4 + 63.11s^2 - 60.7s + 28.1 \]

\[ s^2 = 0.5475s^3 + 815.2s^2 - 418.7s^3 + 22.2s^4 - 86.6s^3 - 132.6s^2 + 0s + 0 \]

The fig.3, exhibits that the root locus graphs respect to the double link robot manipulator. From the graphical method, noticed that the system is unstable.

B. Nyquist Graph

The fig.4 shows the Nyquist plots for a double link manipulator and noticed that the system is unstable.

The fig.5 is the Bode diagram for a two link arm. From the diagram noticed that the system is braking at frequency of 20 rad./sec, which indicates that the link system is unstable. The conclusion from the fig (3), fig (4) and fig (5) of the above link system is unstable and to overcome these issues, a controller technique design is to be expected to control the system.

IV. IMPLEMENTATION OF CONTROLLER

A. PID Controller

Using the MATLAB algorithm and command to obtained the system transfer function from the state space equation (3) given above for the double soft link system is,

\[ Y(s) = U(s) \times \frac{G(s)}{1 + G(s)} \]

(10)

The m-file code of MATLAB is implemented to open-loop transfer function equation. The above transfer function utilizes all the three such as derivative, integral control and proportional control. This open-loop transfer function modeled through Mat lab. The polyadd functions are not in the MATLAB tools. It is copied to a new m-file and simulated. The step input given to the control system and response graphs shown in fig 6.
B. Design of State Feed Back Controller

In pole-placement approach the closed-loop poles are arbitrarily placed, by considering complete state controllable [16]. In design of pole placement approach variety of desired closed-loop poles are considered, compared the characteristics response and best suited is selected. The state space of a control general equation of the system and the output are calculated as,

\[ x = A x + B u, y = C x + D u \] (11)

Where, \( A \) is \( n \times n \) constant matrix, \( x \) is state vector (\( n \)-vector), \( B \) is \( n \times 1 \) constant matrix, \( y \) is output signal (scalar), \( C \) is \( 1 \times n \) constant matrix, \( D \) is constant (scalar) and \( u \) is control signal (scalar). The control circuit for the systems has shown. To select the control signal \( u = -K x \) and the control signal \( u \) is determined by an instantaneous state. This type of scheme is called state feedback. The stability and transient response are determined using eigen values of matrix by considering the examples of case studies.

C. Case Study 1

Choose the closed-loop poles as \( \mu_1 = -3 + j2, \mu_2 = -3 - j2, \mu_3 = -4 + j2, \mu_4 = -4 - j2, \mu_5 = -3, \mu_6 = -2, \mu_7 = -1, \mu_8 = -4 \).

For the second case to find out the state feedback-gain matrix of \([K]\) through MATLAB code, and algorithms that generate gain matrix \([K]\) given as

\[ K = [3.3349 \ 171.8775 \ -0.0898 \ -10.3945] \]

D. Case Study 2

Choose the closed-loop poles as \( \mu_1 = -3 + j2, \mu_2 = -3 - j2, \mu_3 = -4 + j2, \mu_4 = -4 - j2, \mu_5 = -3, \mu_6 = -2, \mu_7 = -1, \mu_8 = -4 \).

The value of \( P \) and \( K \) are determined from the equations given below,

\[ A^*P + PA - PBR^{-1}B^*P + Q = 0 \] (7)

\[ K = T^*(T^*B^*P + R)^{-1}B^*P \] (8)

The reduced-matrix in the equation \((7)\) is called as Riccati-Equation. The design ladder for the control systems is (a)
To solve the equation (7) for reduced matrix $P$ and gear angle is settling to the set point very quickly without any overshoot. (b) The value of $P$ substitutes into the equation (8). The result obtained is optimal gain matrix $[K]$. The step response to the double link system through LQR techniques is given in the fig(11) and noticed from that the gear angle is very quickly settle to the set point without overshoot. The linear quadratic optimal regulator responses diagram as shown in fig (11). It is noticed that the values $x_1$ to $x_6$ and the $\theta$ is gear angle which represents $x_7$, and arm deflection $\alpha$ is $x_8$. The values of $x_1$ and $x_4$ indicates to $\theta$ and $\alpha$ for the link one and two respectively.

![Figure10. Response of Double Link](image)

**VI. CONCLUSION**

The dynamic model has been presented. The modeling equations are used to control two links manipulator and investigated with control techniques. These models are controlled using classical PID and modern control techniques like linear quadratic optimal controller and state feedback method. The time domain and frequency domain analysis are carried out. The studies show that flexible manipulator systems are unstable due to lightweight composite material structure. The different controllers have been attached to the system and controlled the tip deflection. The LQR yield the better result among the other type of controllers.

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