A Cram on the Prolonged Network of Star Grid

K. Thiagarajan, P. Mansoor

***Abstract*: Nowadays graph theory has indispensable role in many applications of networking. The graph structures are very useful in the modelling of pair wise relations between various nodes of a network. The star graph is a network graph for which various results have been already established. The introduction of semi nodes to each edge in a non trivial star graph S_n on $n \geq 2$ nodes; will result in a new network graph, denoted by $S_n^{'}$. This paper attempts to study some properties of the graph $S_n^{'}$ in terms of graph theoretic parameters, namely maximum degree Δ, chromatic number χ, harmonious chromatic number h, domination number γ.**

***Index Terms*: Expanded Star Graph, Chromatic Number, Harmonious Chromatic Number, Domination Number and Semi Node.**

I. INTRODUCTION

In this paper, assume that all the graphs to be considered are connected, loop less and undirected. We begin with basic definitions.

A graph G depicts of a non-empty set V of points, called nodes, and a set E of two point subsets of V, called edges connecting pairs of nodes. The number of edges incident with a node is known as the degree of the node. A walk in a graph G is an alternating sequence of distinct nodes and edges which starts and terminates with a node. If no edges are repeating, then the walk is called a trail. In a trail if the starting node and the terminating node are same, then the trail is a circuit. If no vertices are repeating in a walk, then it is called a path. In a path if the starting node and the terminating node are same, then the path is a cycle. A graph G is said to be connected if there exists a path between every pairs of nodes of G. A node of degree 1 is called a pendant node. A circuit which contains all the edges exactly once is called an Euler circuit and a graph with an Euler circuit is said to be Eulerian. A cycle which contains all the nodes exactly once is called a Hamiltonian cycle and a graph with a Hamilton cycle is said to be Hamiltonian.

Let G = (V, E) be a graph and A be a subset of V. If every node which is not an element in A is adjacent to at least one element of A, then A is said to be a dominating set for the graph G. The cardinality of a minimal dominating set of a graph G is known as the domination number of G and it is denoted by $\gamma(G)$. Note that in a graph G, if v is a node, then v dominates itself and all of the adjacent nodes of v.

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A proper colouring of a graph G is a labelling of nodes of G such that no two adjacent nodes receive the same colour. A colouring of a graph G using at most k colours is called a k-colouring. The chromatic number $\chi(G)$ of a graph G is the minimum number of colours required for colouring G properly.

A harmonious colouring of a graph G is a special type of proper colouring of G such that the colour pairs never repeats. The minimum number of colours required for a harmonious colouring of G is called the harmonious chromatic number of G, which is denoted by $h(G)$.

A semi graph G is a pair (V, E) where V is a non-empty set whose members are called nodes of G and E is the set of edges whose elements are the n-tuples of distinct nodes for various $n \geq 2$, satisfying the following two conditions:

(i) Any two edges have at most one node in common.

(ii) Two edges $(u_1, u_2, \ldots, u_n)$ and $(v_1, v_2, \ldots, v_k)$ are considered to be equal if and only if $k = n$ and either $u_i = v_i$ or $u_i = v_{n-i+1}$ for $i = 1, 2, 3, \ldots, n$.

Thus, the edges $(u_1, u_2, \ldots, u_n)$ and $(u_n, u_{n-1}, \ldots, u_1)$ are the same.

When splicing a graph G, the new nodes obtained are called semi nodes and the new edges formed by the decomposition of edges are called semi edges.

II. HARMONIOUS CHROMATIC NUMBER OF A STAR GRAPH $S_n, n \geq 2$

Definition: A graph G having $n-1$ pendant nodes and exactly one node of degree $n-1$ is said to be a star graph. It is denoted by S_n, where $n > 0$. Here we consider only the non trivial star graphs $S_n, n \geq 2$.

For a star graph S_n, we have the following:

The number of nodes $|V| = n$.
The number of edges $|E| = n-1$,
Maximum vertex degree $\Delta = n-1$.
Minimum vertex degree $\delta = 1$.
The domination number $\gamma = 1$
The chromatic number $\chi = 2$.

By the definition of a star graph S_n, since $n-1$ vertices are adjacent to a single vertex, it requires $n-1+1 =$ colours for the harmonious colouring. It shows that the harmonious chromatic number $h = n$ for a star graph $S_n, n \geq 2$.

Example: Consider the star graph S_4.

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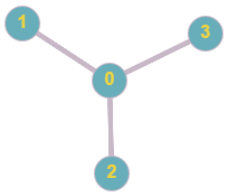

Here $|V| = 4$, $|E| = 4-1 = 3$, $|\Delta| = 4-1 = 3$, $\delta = 1$, $\gamma = 1$, $\chi = 2$ and $h = 4$.

III. EXPANDED STAR GRAPH

Definition: An expanded star graph denoted by $S_n' = (V, E)$ is a graph obtained from a star graph S_n on $n \geq 2$ vertices by introducing semi nodes to each of the edges.

The number of nodes $|V| = n + (n-1) = 2n - 1$, since we introduce a node for each of the $n-1$ edges and $|E| = n - 1 + 2(n-1) = 3n - 3$, as $n-1$ edges gives two more new edges.

In an expanded star graph S_n', all the vertices are adjacent to a single node. Therefore the domination number $\gamma(S_n') = 1$.

Each edge of S_n will form a cycle of length three in S_n' and hence the chromatic number $\chi(S_n') = 3$. Since $\chi(S_n) = 2$ for the star graph S_n, we have $\chi(S_n') = 3 = 2 + 1 = \chi(S_n) + 1$. That is, $\chi(S_n') = \chi(S_n) + 1$.

Proposition 1: For any expanded star graph S_n' on $n \geq 2$ nodes, the maximum degree is $\Delta(S_n') = 2(n-1)$ and the minimum degree is $\delta(S_n') = 2$.

Proof: Let S_n be a star graph on $n \geq 2$ nodes. Suppose that u is the maximum degree node with $\Delta(S_n) = \deg(u) = n - 1$ and $e = (u, v)$ is the edge connecting u and v, a pendant node of S_n. Let w be a semi node introduced on the edge e. Then e splits into two semi edges $e_1 = (u, w)$ and $e_2 = (w, v)$ and this will contribute an additional count of one into the degree of the node u. Similarly, for each of the remaining edges we obtain additional count of one in to the degree of u.

Thus,

$$\deg(u) = n - 1 + (1 + 1 + \ldots + 1)$$
$$= n - 1 + (n-1) = 2(n-1)$$

That is, $\deg(u)$ increases by $n-1$ on the introduction of semi nodes to each edge of S_n. But each of the pendant nodes of S_n will become a node of degree two of S_n'. Therefore we can conclude that u is the maximum degree node of the expanded star graph $S_n', n \geq 2$.

Hence the maximum degree is $\Delta(S_n') = \deg(u) = 2(n-1)$ and the minimum degree is $\delta(S_n') = 2$.

Proposition 2: For any expanded star graph S_n', $n \geq 2$, the harmonious chromatic number $h(S_n') = 2n - 1$.

Proof: For any graph G, we have $\Delta(G) + 1 \leq h(G) \leq |V(G)|$ which implies that

$\Delta(S_n') + 1 \leq h(S_n') \leq |V(S_n')|$

i.e., $2n - 2 + 1 \leq h(S_n') \leq 2n - 1$

i.e., $2n - 1 \leq h(S_n') \leq 2n - 1$

i.e., $h(S_n') = 2n - 1$.

Remarks: From the above result, we have

(i) $h(S_n') = 2h(S_n) - 1$, since $h(S_n) = n \Rightarrow 2h(S_n) - 1 = 2n - 1$.

(ii) $h(S_n') = h(S_n) + m(S_n)$, since $h(S_n) = n$ and $m(S_n) = |E(S_n)| = n - 1$

$\Rightarrow h(S_n) + m(S_n) = 2n - 1 = h(S_n')$.

(iii) $h(S_n') = \Delta(S_n') + \gamma(S_n')$, since $\Delta(S_n') = 2(n-1) = 2n - 2$ and $\gamma(S_n')$.

$\Rightarrow \Delta(S_n') + \gamma(S_n') = 2n - 2 + 1 = 2n - 1 = h(S_n$

Example: Consider the expanded star graph S_4'.

Here $|V| = 2 \times 4 - 1 = 7$, $|E| = 3 \times 4 - 3 = 9$, $\gamma = 1$, $\chi = 3$, $\Delta = 2(4-1) = 6$, $\delta = 2$ and $h = 2 \times 4 - 1 = 7$.

Theorem: Every expanded star graph $S_n', n \geq 2$ is Eulerian.

Proof: Let $S_n', n \geq 2$ be an expanded star graph with u as the maximum degree vertex. Then $\deg(u) = 2(n-1)$, which is even. All the other vertices are of degree 2. That is, all the vertices of $S_n', n \geq 2$ are of even degree and hence it is Eulerian.

Theorem: The expanded star graph $S_n', n \geq 2$ is not Hamiltonian.

Proof: In the expanded star graph $S_n', n \geq 2$, there exists a common node which is adjacent to all the other nodes. All the cycles formed in $S_n', n \geq 2$ will intersect at this node and therefore we cannot find a circuit without the repetition of this common node. Hence is not Hamiltonian.

IV. CONCLUSION

In this paper we introduced the expanded star graph $S_n', n \geq 2$, which is a network graph obtained from the star graph by introducing semi-nodes in each edges and discussed some properties of expanded star graph.

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