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A Study On Different Approaches For Solving Definite Integrals

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Abstract: ***This paper studies different approaches of solving definite integrals. Contour integration is used in integral transforms which is used to solve differential equations. Cauchy's residue theorem is a consequence of Cauchy's integral formula***

$$f(a)=\frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz$$

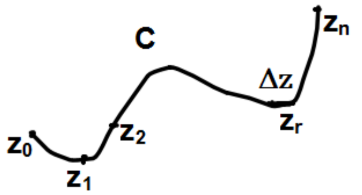
where f is an analytic function and C is the simple closed contour enclosing the point a over the positive orientation. Fourier transform is a mathematical tool frequently used in a number of technical fields, as diverse as applied mechanics, image and sound analysis and solving partial differential equations.The Fourier transform is used to introduce the relationship between the signal waveform and the signal frequency. It is a generalization of the Fourier series by changing the sum to an integral. The Fourier transform translates between convolution and multiplication of functions. Applying the transform techniques and methodology of contour integration, the solution of evaluating $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+a^2)(x^2+b^2)}$ ***has been discussed.and Roman Numbering (I, II, III…etc), Before Spacing 12, After Spacing 6, with single line spacing. All Sub Heading must be in Title Case, Left 0.25 cm, Italic, and Alphabet Numbering (A, B, C…etc), Before Spacing 6, After Spacing 4, with Single Line Spacing. Manuscript Details must be in Font Size 8, in the Bottom, First Page, and Left Side with Single Line Spacing. References must be in Font Size 8, Hanging 0.25 with single line spacing. Author Profile must be in Font Size 8, with single line spacing. Fore more details, please download TEMPLATE HELP FILE from the website.***

Index Terms*: *Contour integral, Residues, Definite integrals.

I. INTRODUCTION

In the real domain, the concept of residues helps us to use the mathematical technology called the contour integration to evaluate some interesting definite integrals. The concept of contour integration employs transforming the definite integrals into a contour integral, then applying residue theorem to solve the definite integrals.

In defining the function of a complex variable $z=x+iy$, let C be a curve in the complex plane, subdividing the curve into subintervals denoting $z_0, z_1, \ldots, z_n$as the points on the boundaries of these subdivisions. Let $\Delta z = z_{r+1}-z_r$ where $r=0,1,2,3,\ldots,$



For all k, the product $f(z_r)\Delta z$ is a complex number and the sum of all these complex numbers is also a complex number.

$$(i.e)\sum_{r=1}^{n} f(z_r)\Delta z$$

Approaching the limit of the above sum to infinity, it remains a complex number. The limit of the above sum is called a contour integral.

$$\lim_{n\to\infty}\sum_{r=1}^{n} f(z_r)\Delta z = \int_C f(z)\,dz$$

Recalling the definition of Riemann sum, the concept of definite integrals relates the area under a curve. Since f(z) is a function in complex variable, the contour integral $\int_C f(z)\,dz$ contains no such area interpretation.

Contour integration is used in integral transforms used to solve differential equations. Contours are the group of curves on which we define contour integration. A contour is a directed curve which is made up of a finite sequence of directed smooth curves whose endpoints are matched to give a single direction.

Applying the idea of Cauchy's integral formula and Cauchy's residue theorem, the evaluation of definite integrals around the whole of the contour is much simple.

In measuring the signal amplitude as a function of time, the Fourier transforms converts it to function of frequency. By the fact that any signal that changes with respect to time has at least one non – zero frequency.

The Fourier transform is used to introduce the relationship between the signal waveform and the signal frequency. It is a generalization of the Fourier series by changing the sum to an integral. On the whole, Fourier transform expresses signals in terms of summation of complex exponentials.

Consider a signal $f(t)\in\mathbb{R}$ contains components of different frequencies, then the Fourier Transform of $f(t)$ is defined as

$$F(s)=\int_{-\infty}^{\infty} f(t)\,e^{-ist}dt$$

Its inversion formula is

$$f(t)=\int_{-\infty}^{\infty} F(s)\,e^{ist}d$$

II. SOLUTION OF $\int_{-}^{\infty}\frac{t^2}{(t^2+a^2)(t^2+b^2)}dt$ BY

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USING FOURIER TRANSFORM TECHNIQUES:

Consider

$$\int_{-\infty}^{\infty} \frac{t^2}{(t^2+a^2)(t^2+b^2)}\,dt = \frac{1}{2}\int_{0}^{\infty} \frac{t^2}{(t^2+a^2)(t^2+b^2)}\,dt$$

Let $f(t) = e^{-at}$ and $g(t) = e^{-bt}$

By Parseval's Identity

$$\int_0^{\infty} f(t)\ g(t)\ dt = \int_0^{\infty} F_s(s)\ G_s(s)\ ds \text{ ------ } (1)$$

where $F_s(s)$ and $G_s(s)$ are the Fourier sine transforms of $f(t)$ and $g(t)$.

We know that,

$$F_s[e^{-at}] = \sqrt{\frac{2}{\pi}}\left(\frac{s}{a^2+s^2}\right)$$

$$G_s[e^{-bt}] = \sqrt{\frac{2}{\pi}}\left(\frac{s}{b^2+s^2}\right)$$

$$\Rightarrow \int_0^{\infty} e^{-at}e^{-bt}dt = \int_0^{\infty}\sqrt{\frac{2}{\pi}}\left(\frac{s}{a^2+s^2}\right)\sqrt{\frac{2}{\pi}}\left(\frac{s}{b^2+s^2}\right)ds$$

$$\Rightarrow \int_0^{\infty}\frac{2}{\pi}\left(\frac{s^2}{(a^2+s^2)\ (b^2+s^2)}\right)ds = \left(\frac{e^{-(a+b)t}}{-(a+b)}\right)_0^{\infty}$$

$$\Rightarrow \int_0^{\infty}\frac{2}{\pi}\left(\frac{s^2}{(a^2+s^2)\ (b^2+s^2)}\right)ds = \frac{1}{a+b}$$

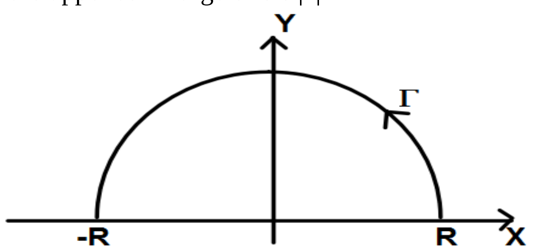
$$\int_0^{\infty}\left(\frac{s^2}{(a^2+s^2)(b^2+s^2)}\right)ds = \frac{\pi}{2(a+b)}$$

$$\int_0^{\infty}\left(\frac{s^2}{(a^2+t^2)(b^2+t^2)}\right)ds = \frac{\pi}{2(a+b)}.$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{t^2}{(t^2+a^2)(t^2+b^2)}dt = \frac{\pi}{(a+b)}$$

III. SOLUTION OF $\int_{-}^{\infty}\frac{t^2}{(t^2+a^2)(t^2+b^2)}dt$ BY USING CONTOUR INTEGRATION:

Consider $\int_C \frac{z^2}{(z^2+a^2)(z^2+b^2)}$, where C is the contour consisting of Γ, the segment of the real axis from –R to +R and the upper semi large circle |z| = R



$$\therefore \int_{\Gamma} f(z)dz + \int_{-R}^{R} f(t)dt = \int_C f(z)dz$$

When R→ ∞, $\int_{\Gamma} f(z)dz = 0$.

Hence, $\int_{-\infty}^{\infty} f(t)\ dt = \int_C f(z)dz$

$\Rightarrow \int_{-\infty}^{\infty} f(t)\ dt = 2\pi i(\text{Sum of the residues})$, by Cauchy's residue theorem.

Let $f(z) = \frac{z^2}{(z^2+a^2)(z^2+b^2)}$

The singularities of the integral are given by $(z^2+a^2)(z^2+b^2) = 0$.

By solving the above equation, we get the factors as $z = \pm ia$ and $z = \pm ib$, which are simple poles of f (z). Of these poles, $z = ia$ and $z = ib$ lie inside C.

Residue of $f(z)$ at $z = a$ = $\lim_{z\to a}(z-a)f(z)$

Residue of $f(z)$ at $z = ia$ = $\lim_{z\to ia}(z-ia)\frac{z^2}{(z^2+a^2)(z^2+b^2)}$

$$= \lim_{z\to ia}(z-ia)\frac{z^2}{(z-ia)(z+ia)(z^2+b^2)}$$

$$= \frac{a}{2i(a^2-b^2)}$$

Similarly, Residue of $f(z)$ at $z = ib$ = $\lim_{z\to ib}(z-ib)\frac{z^2}{(z^2+a^2)(z^2+b^2)}$

$$= \lim_{z\to ib}(z-ib)\frac{z^2}{(z-ib)(z+ib)(z^2+a^2)}$$

$$= -\frac{b}{2i(a^2-b^2)}$$

$$\int_C \frac{z^2}{(z^2+a^2)(z^2+b^2)}dz = 2\pi i(R_1+R_2) = \frac{\cdot}{a}.$$

IV. RESULT

Definite integrals have wide range of application in science and engineering. The solution of the definite integral $\int_{-\infty}^{\infty}\frac{t^2}{(t^2+a^2)(t^2+b^2)}dt$ have been determined using the transform technique and the methodology of contour integration.

V. CONCLUSION

The study of infinite series has wide range of application in science and engineering. The solution of the infinite series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ by different methods have been determined.

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