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Applications Of Anti Fuzzy Graph and Role of Domination on Anti Fuzzy Graph

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Abstract*: *In this paper, we discuss the application of anti fuzzy graph and the application of domination on anti fuzzy graph. We discuss a method to find the suitable employees in merging bank by using anti cartesian product of anti fuzzy graph and domination on anti fuzzy graph.

Index Terms*: *Anti Fuzzy Graph, Domination, Anti Cartesian product.

I. INTRODUCTION

A mathematical model can come to rescue when a critical problem arises. One of the best possible means to find a solution is by presenting it in the form of a graph. The main parts of the problem are converted as vertices. Depending upon the role of vertices, a fuzzy value is assigned to them in graph. The relation between the parts is converted as edges. Similarly, the edges are also assigned with a fuzzy value. In some situation, vagueness occurs among the relation that attains a maximum value. That is, the edge is assigned with maximum value of its incident vertices. Such a graph is called as "Anti Fuzzy Graph". There exist a vagueness in the description of objects is defined as fuzzy sets and fuzzy relations which is introduced by L.A.Zadeh [12] in 1965. Further in 1975, Rosenfield [6] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. At the same time, R.T.Yeh and S.Y.Bang[11] also introduced various connectedness concepts in fuzzy graphs. In 2012, Muhammad Akram introduced the concept of Anti fuzzy structures on graphs and characterized some basic concept of connected anti fuzzy graph[1]. In 2016, R.Seethalakshmi and R.B.Gnanajothi [8] introduced the definition of anti fuzzy graph. R.Muthuraj and A. Sasireka [2-5] defined some types of anti fuzzy graph. Also explained the concept of domination on anti fuzzy graph and anti cartesian product of anti fuzzy graph. Domination plays a vital role in graph theory. The domination concepts also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks and in land surveying. Ore and Berge began the study of domination set in graphs. A.Somasundram and S.Somasundram [9] discussed domination in Fuzzy graphs and determined the domination number for several types of fuzzy graphs. A. Somasundaram[10] studied several operations on fuzzy graphs such as union, join, composition, cartesian product and obtained their domination parameters.

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II. PRELIMINARIES

In this section, basic concepts of anti fuzzy graph are discussed. Notations and more formal definitions are followed as in [2- 5,8].

Definition 2.1 [8]

An anti fuzzy graph G = (σ, μ) is a pair of functions σ : V→ [0,1] and μ : V×V→[0,1], where for all u,v∈V, we have μ(u,v) ≥ σ(u) ∨ σ(v) and it is denoted by GA(σ, μ).

Definition 2.2 [4]

Every vertex in an anti fuzzy graph GA has unique fuzzy values then GA is said to be v-nodal anti fuzzy graph. i.e. σ(u) = c for all u∈V(GA).

Definition 2.3[4]

Every edge in an anti fuzzy graph GA has unique fuzzy values then GA is said to be e-nodal anti fuzzy graph. i.e. μ(u,v) = c for all uv∈ E (GA)

Definition 2.4 [4]

Every vertices and edges in an anti fuzzy graph GA have the unique fuzzy value then GA is called as uninodal anti fuzzy graph. If σ(u) =c1 and μ(u,v) = c2 in an anti fuzzy graph GA, then GA is called as binodal anti fuzzy graph.

Definition 2.5 [8]

An anti fuzzy graph GA= (σ,μ) is a strong anti fuzzy graph of μ(u,v) = σ(u) ∨ σ(v) for all (u,v)∈μ* and GA is a complete anti fuzzy graph if μ(u,v) = σ(u) ∨ σ(v) for all (u,v) ∈ μ* and u,v∈ σ *. Two vertices u and v are said to be neighbors if μ(u,v) > 0.

Definition 2.6 [5]

An edge e = {u, v} of an anti fuzzy graph GA is called an effective edge if μ(u,v) = σ(u) ∨ σ(v). An edge e = {u,v} of an anti fuzzy graph GA is called an weak edge if μ(u,v) ≠ σ(u) ∨ σ(v).

Definition 2.7 [3]

Let $G_A^* = G_{A_1}^* x G_{A_2}^*$ = (V, E′) be the anti cartesian product of anti fuzzy graphs where V= V1 ×V2 and E′ = {(u1,u2),(u1,v2) / u1∈V1 , (u2,v2)∈E2}∪{(u1,w2),(v1,w2) / w2∈V2 , (u1,v1)∈E1 }. Then the anti cartesian product of two anti fuzzy graphs, $G_A^* = G_{A_1}^* x G_{A_2}^*$ (σ1×σ2, μ1×μ2) is an anti fuzzy graph and is defined by

(σ1 × σ2)(u1, u2) = max {σ1 (u1), σ2(u2)} for all (u1, u2)∈V

(μ1 × μ2)((u1, u2), (u1, v2)) = max{σ1(u1), μ2(u2, v2)} for all u1∈V1 and (u2, v2)∈E2

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$(\mu 1 \times \mu 2)((u1, w2), (v1, w2)) = \max \{\sigma 2 (w2), \mu 1(u1, v1)\}$ for all $w2 \in V2$ and $(u1, v1) \in E1$,

Then the fuzzy graph GA = $(\sigma 1 \times \sigma 2, \mu 1 \times \mu 2)$ is said to be the anti cartesian product of G_{A_1}= (σ1, μ1) and G_{A_2} = (σ2, μ2).

Definition 2.8[5]

A set $D \subseteq V(GA)$ is said to be a dominating set of an anti fuzzy graph GA if for every vertex $v \in V(GA) \setminus D$ there exists u in D such that v is a strong neighborhood of u with $\mu(u,v) = \sigma(u) \vee \sigma(v)$ otherwise it dominates itself.

A dominating set D with minimum number of vertices is called a minimal dominating set if no proper subset of D is a dominating set.

The maximum fuzzy cardinality taken over all minimal dominating set in GA is called a domination number of anti fuzzy graph GA and is denoted by γ(GA) or γA. ie, $|D|f = \sum_{v \in D} \sigma(v)$.

III. Applications of Anti Fuzzy Graph

A. Representation of Anti Fuzzy Graph

Consider a group of students who have close relationship with each other. Depending upon the knowledge (k) they possess, fuzzy values are assigned in the range of $0 < k < 1$ for poor to intelligent student respectively. Assume the students as vertices and their relationship as edges. A student who possesses intrinsic knowledge has relationship with other students with less knowledge than him or her, may try to disseminate his or her entire knowledge to all of them. However, his or her friends could imbibe as much as they can and act accordingly. Such a scenario can be framed as anti fuzzy graph.

For example, consider five students A, B, C, D and E whose knowledge level is graded with fuzzy values as 0.5, 0.7, 0.2, 0.4 and 0.6 respectively as shown in Figure 3.1. Student A has relationship with all other students except D. B has relationship with A and D. C has relationship with A and D. D has relationship with C and B. E has relationship only with A. Even though B and E are intelligent and expose their knowledge to A, A can provide knowledge to his friends what he possesses. Similarly, B and D converse among them with higher knowledge level but when D converses with C, D could not share as much as that exists with B. As per human nature, the students those who have less intelligence some time get exaggerated and disguise themselves as if they possess more knowledge at a certain situation. Even in such situations anti fuzzy graph concept comes into picture. This type of circumstance is represented as anti fuzzy graph and it is shown in Figure 3.1.

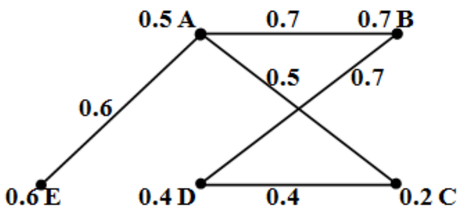

Figure 3.1 Representation of anti fuzzy graph G_A

B. Application of E-nodal Anti Fuzzy Graph

The railway department network is the best example of anti fuzzy graph. Depending upon the nature of train, it is classified as passenger, express and superfast. Passenger train stops in all stations and takes over the people. Express and superfast trains do not halt in all stations, because it may delay in engine pickup. Consider two stations where atleast 10 trains run between them. For example, consider the two stations to be Dindigul and Madurai Junction. These two stations are main junctions of southern railways in Tamil Nadu. Dindigul is the main city in Tamil Nadu which connects northern and southern part of it. In between Dindigul and Madurai, there exist five stations such as Aambathdurai, Kodai road, Vadipatti, Samayanallur and Kudal nagar. All the trains should cross over this route but need not halt in those stations. Superfast and express trains halt in kodai road station only. This scenario is converted as anti fuzzy graph as shown in Figure 3.2. Dindigul, Aambathdurai, Kodai road, Vadipatti, Samayanallur, Kudal nagar and Madurai stations are considered as vertices (say D, A, KO, V, S, KU, M). Depending upon the number of halts in a station, fuzzy value is assigned to the vertices as 0.9, 0.2, 0.5, 0.3, 0.4, 0.1 and 0.9 respectively. Since the track connects all the stations between the junctions, it is assigned a maximum fuzzy value of 0.9 to the edges of the anti fuzzy graph. This type of anti fuzzy graph is called e-nodal anti fuzzy graph.

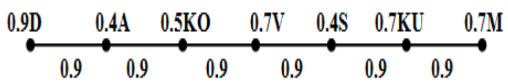

Figure 3.2 e-nodal anti fuzzy graph GA

IV. Domination on Anti Fuzzy Graph

In this section, finding the solution to the problem that arises in real life situations using domination concept on anti fuzzy graph are discussed.

A. Selection of Group leaders in class by applying Domination on Anti Fuzzy Graph

Domination on anti fuzzy graph concept can be applied for assigning group leaders in the class. Among the group of students, some could not directly interact with the teachers and many hesitate for clearing their doubts. In such situations, class can be formed into groups and a leader is assigned to each group of students who have higher knowledge than others. This will help to improve overall performance of the students. To select a leader in each group, domination concept can be applied.

Assume that there are 15 students in a class. To construct a dominating set among these students, consider a set of students who possess higher knowledge and have good relationship with other students. To get a domination number, it is necessary to find the minimal dominating set. Moreover, to construct the minimal dominating set with a student who has higher knowledge (vertex fuzzy value) as well as maximum neighborhood with effective edge value must be preferred. Such a dominating set cannot be minimized

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further. Consider a class with 15 students framed as an anti fuzzy graph GA with 15 vertices (say ‘a’, ‘b’, .., ‘o’) as shown in Figure 4.1. The students knowledge level is considered as vertex set of anti fuzzy graph V(GA) = {a = 0.5, b = 0.4, c = 0.8, d = 0.2, e = 0.3, f = 0.6, g = 0.1, h = 0.3, i = 0.1, j = 0.7, k = 0.3, l = 0.2, m = 0.5, n = 0.3, o = 0.4}. The relationship among the students with each other is considered as edge set of anti fuzzy graph.

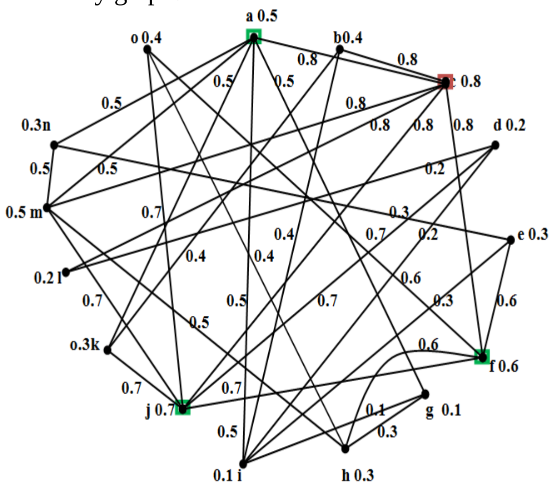

Figure 4.1 Anti fuzzy graph GA

A student with knowledge level of 0.8 is assigned as a class leader and group leader for two students who are his friends having knowledge level 0.4 and 0.2 as shown in Figure 4.1. Similarly, students possessing knowledge level of 0.6, 0.7 and 0.5 are assigned as group leaders. Therefore, the minimal dominating set for the above example is {0.8, 0.6, 0.5, 0.7}, γA = 2.6.

B. 4.2 Fixing Stations in Railways using Domination on Anti Fuzzy Graph

Dominating set plays a vital role in fixing stations in railway department. Generally in this department, a railway track is constructed by fixing the source and destination point. Between these places, they need to fix the stations. In this scenario, domination concept helps to reduce the complications which are described as follows.

Consider a source point as A and destination point as H. A railway track is constructed and stations are fixed within these places. For this purpose, the roadway and population of the people in each area from A to H must be surveyed to know how many use the roadways and sufficiency of their transport.

Consider A and H as the main cities of a state where A is the origin city and the people who frequently travel to H by any means of transport. Within these cities, so many rural areas exist. To construct a track, this problem is converted as anti fuzzy graph. The places are considered as vertices and the existing direct roadway between the places as edges. Depending upon the usage of transport among the population, the fuzzy value is assigned to each vertex. Based on the frequency of usage of road by the people between the areas, fuzzy value is provided to the edges.

For an instance, from source point A to reach H, one needs to cross 8 places (say A1, A2, A3, A4, A5, A6, A7 and A8). A track is constructed connecting via these places only. But railway cannot fix stations in all these places. However, all the trains should cross through this track. So, all the tracks (edges) should attain the same maximum fuzzy value. Depending upon the frequent use of travelling and population of the area, fuzzy value is set to the vertices in the range of 0.1 to 0.9. So fuzzy value to the cities A and H are assigned as 0.9. Consider the fuzzy value of the vertices A1 as 0.4, A2 as 0.5, A3 as 0.9, A4 as 0.4, A5 as 0.7, A6 as 0.9, A7 as 0.3 and A8 as 0.2.

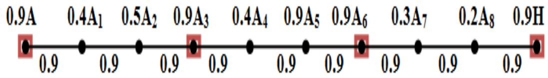

Figure 4.2 Fixing station using anti fuzzy graph GA

To select a place for fixing a station, domination concept plays an important role. In Figure 4.2, the encircled vertices cover its neighboring vertices by its maximum fuzzy value. These vertices are the best places to fix a station. The people who are living in A1 may use the nearby station A. The people who are living at A2 and A4 may use the station A3 for having best travelling whereas people living at A5 and A7 may use the station A6 and people living in A8 may use the nearby station H. The vertices A5 and A6 are assigned with same fuzzy value because the population of the area and usage of transport are equal. If both A5 and A6 stations are selected, then the dominating set is not minimal. The criterion is to fix minimal number of stations with maximum benefit. Therefore, instead of choosing both A5 and A6, vertex A6 alone is chosen. Henceforth, the dominating set contains {A, A3, A6, H}.

Similarly, tracks can be constructed to connect the nearby cities in a state. Depending upon the strong adjacency within the stations, the stations are classified as central, junction and terminus which also shown by Figure 4.3. In Figure 4.3, T represents terminus station, S1, S2, S3, … etc., represents stations in specified areas. C represents central. J represents Junction.

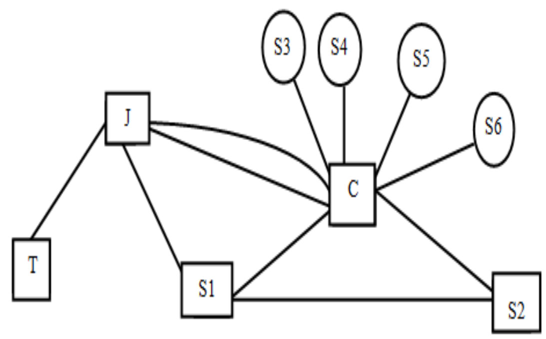

Figure 4.3 Framing names to station by their tracks(edges)

Suppose a vertex (station) of a graph is a strong neighbour with its adjacent vertices then the vertex is set as central. That is, the vertex may exist in all paths of that graph. In railway department, such station (vertex) may contain a large number of arrivals and departures of train with its nearby cities.

If a vertex (station) having atleast three strong neighbours in an anti fuzzy graph, then the vertex is set as junction. Similarly, a station which has atleast three different routes coming in and going out of a station is set as junction.

A pendent vertex has exactly one strong adjacent with its neighbour. So, the path with the pendent vertex exists with a single way to arrive and depart. That type of station is called as terminus.

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V. APPLICATION ON ANTI CARTESIAN PRODUCT OF ANTI FUZZY GRAPH

In this section, applications of anti cartesian product of anti fuzzy graphs are illustrated with a fine instance of merging banks together.

A. Merging of two Banks

The Government has made many efforts to repair the country's economic downturn and fix the financial situation of the banks. One of the most important decisions in this process is integration of the banks. The poor financial condition of the banks can be controlled by integrating the banks that face bad debts. If the plan goes through, the merged entity will become one of the largest banks in the country. The merger will also allow the weak banks to sell the assets, reduce overheads and shut money-losing branches. The merging framework is a better mechanism to maintain sound financial health of the banks. It facilitates banks from breach of risk thresholds for identified areas of monitoring such as capital and asset quality to take corrective measures so that they are protected from financial crisis.

However, some complications arise when such a merging policy is executed. One such a problem when coordinating the banks is that paying attention to the bank's workforce and customers. The bankers are classified as clerk, probationary officer, specialist officer, manager, and above. Furthermore, customers having account in a bank should be divided into two kinds as those who have account only in single bank or multiple banks. The employees and customers are represented as a graph. For finding the solution of effective employees, the subgraph of employees is needed which is constructed by applying anti cartesian product. The bank employees based on their cadre are considered as nodes and their relationship with other employees is taken as edges of anti fuzzy graph. Applying anti cartesian product on this anti fuzzy graph will help in merging banks and applying domination concept to get the solution to the problem.

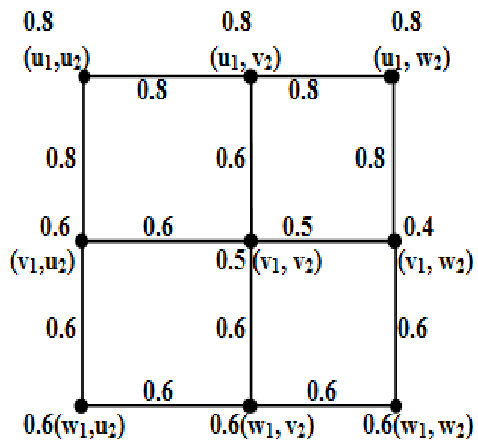
For example, consider two banks G_{A_1} and G_{A_2} for merging. The position of the employee is considered as manager, officer and the clerk. Officers and the clerk must work according to the manager's orders whereas clerk must obey the orders of manager through officer. This is defined as some graph theoretical model. The fuzzy value of the vertex is assigned based on the weightage of the person depending upon their qualification, experience, and achievements carried out every year. Generally, weightage given to the vertices of anti fuzzy graph helps to take decision at ease when merging is applied.

Suppose, there exist two banks say G_{A_1} and G_{A_2} in same area. If it is not possible to run two banks in that area, then they can be merged into single bank. But it is hard to serve all customers by the staff at a branch after merging. The interrelationship between the working people is considered more important when they are placed in merged entity. The concept of anti cartesian product is suitable in such case. Also, there is a situation where employees are selected only based on their efficient working from both the branches for effective running of the bank. Other employees can be transferred to any other branch or any other decisions can be taken by the bank. Domination concept comes into picture make this selection.

If the customers have an account only in G_{A_1} then they can operate their account in G_{A_2} by transferring the details to G_{A_2} and get served. Suppose the customers who have account on both the banks, when the integration of the accounts is done, their deposit in account, loan amount and others are considered. Suppose a customer could maintain account in a single bank but having account on both the banks. In such case, the bank insists the customer to maintain as a single account and the deposit amount can be merged. To find the solution to this problem, the graph is subdivided as two parts. Firstly, a graph (H_{A_1}) contains only the employees as vertices. Secondly, a graph contains only customers as vertices. In this subgraph (H_{A_1}), the anti cartesian product is applied.

Consider a working category of H_{A_1} and H_{A_2} as a vertex set of anti fuzzy graph (say {u1,v1,w1} and {u2,v2,w2}). Consider the mingling of those people as (u1,u2),(u1,v2), (u1,w2), (v1,u2),(v1,v2), (v1 ,w2), (w1,u2), (w1,v2), (w1,w2). That is, the vertex u1 (manager cadre of a bank) joins with the employees of H_{A_2} then the efficiency of the employee can be considered as fuzzy value. This can be set by anti cartesian product. Similarly, consider the remaining employees of the bank who are more efficient. This can be done by considering the vertex fuzzy value of who have maximum neighbours with effective edge. First choose such a vertex for constructing a dominating set. In some cases when the job cannot be dealt with same number of persons, it is necessary to appoint some more employees in the merging branch. This can be done by domination concept on anti fuzzy graph. The above example is illustrated as Figure 5.1.

$H_{A_1} \bar{\times} H_{A_2}$:



VI. RESULTS

In Figure 5.1, the dominating sets are D1 = {(u1,v1), (w1,u2) ,(w1 ,w2)}, D2 = {(w1,v2),(v1 ,u2), (u1,w2)}, D3 = {(u1,u2),(v1,w2),(w1,v2)}. If the dominating set is selected as D1, there is no employee in officer cadre. Further, all the employees who are officers cannot be ignored and clerk cannot be promoted as officer. So this is not possible to choose D1 as dominating set and the set of employees also. Moreover, in the set D2 and D3 all the cadres of employees are present. The maximum fuzzy value of set D2 is greater than the fuzzy value of set D3. Therefore, the set D2 is chosen as dominating set and employees can serve in the merging bank to make better profit. Hence the dominating set is D = {(w1,v2),(v1 ,u2), (u1,w2)}.

VII. CONCLUSION

The application of anti fuzzy graph and the application of domination on anti fuzzy graph are discussed with real life situations. A method is introduced to find the suitable employees in merging bank by using anti cartesian product of anti fuzzy graph and domination on anti fuzzy graph is discussed.

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