A Special Study on Homo Cordial Labeling of Triangular Belt Graph

S.Sriram, R.Govindarajan

Abstract: ***Let G= (V, E) be a graph with p vertices and q edges. A Homo Cordial Labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1 if f(u)=f(v) or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. The graph that admits a Homo- Cordial labelling is called Homo Cordial graph. In this paper we prove that triangular belt is Homo-Cordial labelling graph and further study on the generalization of labelling a triangular belt graph.***

Key words: Homo Cordial graphs, Homo Cordial labelling

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa. A Path related Homo Cordial graph was introduced by Dr.A.Nellai Murugan and A.Mathubala[2,3,4]. Motivated towards the labelling of homo cordial labelling of graphs In this paper we prove that triangular belt graph is Homo Cordial labelling graph. Further to generalise the concept of homo cordial labelling of triangular belt graph we have ascertained the ways in which the number of labels assigned with 0 and number of labels assigned with 1 so as to identify the phenomena of triangular belt graph to be called a homo cordial labelling graph.

II. PRELIMINARIES

Definition 2.1: Let $L_n = P_n \times P_2$ ($n \geq 2$) be the ladder graph with vertex set u_i and v_i , i=1,2…n. The Triangular Belt is obtained from the ladder by adding the edges $u_i v_{i+1}$ for all $1 \leq i \leq n-1$. This graph is denoted by $TB(n)(\downarrow^n)$.

III. MAIN RESULTS

Theorem .3.1 : The Triangular Belt graph $TB(n)(\downarrow^n)$ is a homo cordial labelling graph

Proof: Let $G = TB(n)$ be the triangular belt. Let $L_n = P_n \times P_2$ ($n \geq 2$) be the ladder graph with vertex set u_i and v_i , i=1,2…n. The Triangular Belt is obtained from the ladder by adding the edges $u_i v_{i+1}$ for all $1 \leq i \leq n-1$. This graph is denoted by $TB(n)(\downarrow^n)$. The vertex set is $V = \{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n\}$ and the edge set is $E = \{u_i u_{i+1}, v_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_i, 1 \leq i \leq n\}$

.Now to label the vertices let us consider the bijective function $f : V \to \{0,1\}$ such that such that each edge uv is assigned the label 1 if f(u)=f(v) or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. We define the labelling of vertices $u_1, u_2, \ldots u_n$ and for $v_1, v_2, \ldots v_n$ as follows

$f(u_i) = 1$ for $1 \leq i \leq n$

$f(v_i) = 0$ for $1 \leq i \leq n$

Then the induced edge labelling for the triangular belt $TB(n)(\downarrow^n)$ are

$f^*(u_i u_{i+1}) = 1$ for $1 \leq i \leq n-1$

$f^*(v_i v_{i+1}) = 1$ for $1 \leq i \leq n-1$

$f^*(u_i v_i) = 0$ for $1 \leq i \leq n$

$f^*(u_i v_{i+1}) = 0$ for $1 \leq i \leq n-1$

Noticing the induced edge labelling we find that the number of vertices labelled with 0 is n and the number of vertices labelled with 1 is n and that the number of edges labelled with 0 is n+1 and the number of edges labelled with 1 is n. Hence $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.Therefore the triangular belt $TB(n)(\downarrow^n)$ graph is a homo cordial labelling graph.

Definition 3.3 : One part in Triangular Belt graph $TB(n)(\downarrow^n)$: For a triangular belt graph $TB(n)(\downarrow^n)$ we define one part denoted by $T(F)$ as shown below where each part consists of 3

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S.Sriram, Assistant Professor, Department of Mathematics, Patrician College of Arts and Science, Adyar, Chennai, India.

Dr. R. Govindarajan, Associate Professor and Head (Retd), U.G and P.G Department of Mathematics, D.G.Vaishnav College, Arumbakkam Chennai, India.

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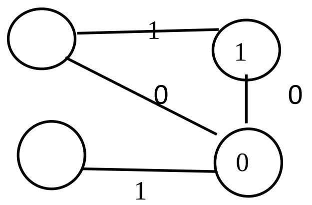
0's and 3 1's which further signifies that number of vertices and edges labelled with 0 is denoted by $T_0(TB(n)(\downarrow^n))$ and number of vertices and edges labelled with 1 is denoted by $T_1(TB(n)(\downarrow^n))$

Figure:1 : One Part of Triangular Belt Graph



It is further significant that from the basic triangular belt graph $TB(2)(\downarrow^2)$ it is possible to obtain $TB(3)(\downarrow^3)$ by adding one part $T(F)$ and so on to construct any order triangular belt graph $TB(n)(\downarrow^n)$. Which we state as a result in the following theorem.

Theorem 3.4 : If Triangular Belt graph $TB(n)(\downarrow^n)$ is homocordial graph then $T_0\left(TB(n)\right)=T_0\left(TB(2)\right)+\left(n-2\right)T_0\left(F\right)$ and $T_1\left(TB(n)\right)=T_1\left(TB(2)\right)+\left(n-2\right)T_1\left(F\right)$, $n\geq 3$ where F is the one part of $TB(n)(\downarrow^n)$

Proof : Consider Triangular Belt graph $TB(n)(\downarrow^n)$ which is homo cordial as proved in Theorem 3.1 . Since by the labelling process suggested for a triangular belt graph $TB(n)(\downarrow^n)$ we claim the above result by applying principle of mathematical induction on n.

Let us now claim for n=3

As we know from the basic triangular belt graph $TB(2)(\downarrow^2)$ which has 4 labels (both vertices and edges) labelled with 1 and 5 labels (both vertices and edges) labelled with 0 and each part has 3 labels(vertices and edges) labelled with 0 and 3 labels(vertices and edges) labelled with 1 we have

$$T_0\left(TB(3)\right)=T_0\left(TB(2)\right)+T_0\left(F\right)=8$$

$$T_1\left(TB(3)\right)=T_1\left(TB(2)\right)+T_1\left(F\right)=7$$

Which is true

Now let us assume for n =k

i.e $T_0\left(TB(k)\right)=T_0\left(TB(2)\right)+(k-2)T_0\left(F\right)$ and

$$T_1\left(TB(k)\right)=T_1\left(TB(2)\right)+(k-2)T_1\left(F\right)$$

Now let us prove for n =k+1

i.e To prove

$T_0\left(TB(k+1)\right)=T_0\left(TB(2)\right)+(k-1)T_0\left(F\right)$ and

$$T_1\left(TB(k+1)\right)=T_1\left(TB(2)\right)+(k-1)T_1\left(F\right)$$

Consider $T_0\left(TB(k)\right)=T_0\left(TB(2)\right)+(k-2)T_0\left(F\right)$

adding one part $T_0\left(F\right)$ we have

$$T_0\left(TB(k)\right)=T_0\left(TB(2)\right)+(k-2)T_0\left(F\right)+T_0\left(F\right)$$

On simplifying we have $T_0\left(TB(k+1)\right)=T_0\left(TB(2)\right)+(k-1)T_0\left(F\right)$. Similarly we can prove $T_1\left(TB(k+1)\right)=T_1\left(TB(2)\right)+(k-1)T_1\left(F\right)$. Hence the proof by induction.

Corollary 3.5 : If for a triangular belt graph $TB(n)(\downarrow^n)$ which is homo cordial labelling graph removal of each part $T(F)$ reduces the total number of vertices and edges labelled with 0 by 3 ad total number of vertices and edges labelled with 1 by 3 and reduces to the basic graph $TB(2)(\downarrow^2)$

Proof: From the above theorem 3.4 result we have

$$T_0\left(TB(n)\right)=T_0\left(TB(2)\right)+\left(n-2\right)T_0\left(F\right)$$

$T_1\left(TB(n)\right)=T_1\left(TB(2)\right)+\left(n-2\right)T_1\left(F\right)$. Further we know from the definition of one part of $TB(n)(\downarrow^n)$ denoted by $T(F)$ consists of 3 labels (both vertices and edges) labelled with 0's and 3 labels(both vertices and edges) labelled with 1's by removing 1 part successively from the result. We find that the result on continuation on n times reduces to the basic graph $TB(2)(\downarrow^2)$.

Theorem 3.6 : If G is a Triangular belt graph $TB(n)(\downarrow^n)$ then the following are equivalent

(a) $TB(n)(\downarrow^n)$ is homo cordial graph

(b) $T_0\left(TB(n)\right)=T_0\left(TB(2)\right)+\left(n-2\right)T_0\left(F\right)$ and $T_1\left(TB(n)\right)=T_1\left(TB(2)\right)+\left(n-2\right)T_1\left(F\right)$

(c) Each part of $TB(n)(\downarrow^n)$ has 3 0's and 3 1's

Proof: In order to prove that they are equivalent . Let us prove (a) implies (b) , (b) implies (c) and (c) implies (a)

To Prove (a) Implies (b)

Consider triangular belt graph $TB(n)(\downarrow^n)$ as being proved in Theorem.3.1 by labelling of vertices $u_1,u_2,...u_n$ and for $v_1,v_2,...v_n$ as follows

$$f\left(u_i\right)=1 \text{ for } 1\leq i\leq n$$

$$f\left(v_i\right)=0 \text{ for } 1\leq i\leq n$$

Then the induced edge labelling for the triangular belt $TB(n)(\downarrow^n)$ are

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$f^*(u_i u_{i+1}) = 1$ for $1 \le i \le n-1$

$f^*(v_i v_{i+1}) = 1$ for $1 \le i \le n-1$

$f^*(u_i v_i) = 0$ for $1 \le i \le n$

$f^*(u_i v_{i+1}) = 0$ for $1 \le i \le n-1$

Noticing the induced edge labelling we find that the number of vertices labelled with 0 is n and the number of vertices labelled with 1 is n and that the number of edges labelled with 0 is n+1 and the number of edges labelled with 1 is n. Hence $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. Therefore the triangular belt $TB(n)(\downarrow^n)$ graph is a homo cordial labelling graph

From the definition of one part of $TB(n)(\downarrow^n)$ we can claim that $T_0(TB(n)) = T_0(TB(2)) + (n-2)T_0(F)$ and $T_1(TB(n)) = T_1(TB(2)) + (n-2)T_1(F)$.

Hence (a) implies (b)

To prove (b) implies (c)

Consider $T_0(TB(n)) = T_0(TB(2)) + (n-2)T_0(F)$ and $T_1(TB(n)) = T_1(TB(2)) + (n-2)T_1(F)$. As the triangular belt graph is homo cordial labelling from the labelling procedure adopted we can ascertain the number of vertices and edges labelled with 0 and 1's for the basic triangular belt graph $TB(2)(\downarrow^2)$ and by substituting in the given result we can find that e ach part of $TB(n)(\downarrow^n)$ has 3 0's and 3 1's .

To prove (c) implies (a)

Since each part of $TB(n)(\downarrow^n)$ consists of 3 0's and 3 1's continuing in this pattern of calculating we can obtain the labelling procedure defined for the triangular belt graph $TB(n)(\downarrow^n)$ resulting in proving that $TB(n)(\downarrow^n)$ is homo cordial labelling graph.

Hence the above statements are equivalent. Hence the proof.

IV RESULTS

In this paper we have considered triangular belt graph $TB(n)(\downarrow^n)$ and proved that it is homo cordial labelling graph and have identified a generalisation method for triangular belt graph to label the vertices and edges.

V CONCLUSION

We wish to identify some more graphs which can be labelled and proved to be homo cordial graphs and identify the generalisation condition.

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AUTHORS PROFILE

S.Sriram, Assistant Professor, Department of Mathematics, Patrician College of Arts and Science, Adyar, Chennai-20

Dr. R.Govindarajan, Associate Professor and Head(Retd.) ,U.G and P.G Department of Mathematics D.G.Vaishnav Collegege, Arumbakamma, Chennai.