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Coefficient Functional Bounds for Certain Classes of Sakaguchi Type Functions

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Abstract: ***The purpose of this paper is to obtain bounds on coefficient functional for certain classes of Sakaguchi type functions involving generalized multiplier transformation operator, which are defined using Quasi-subordination. Some consequences of the main result are mentioned and relevance with some of the known results are also pointed out.***

Index Terms: ***About four key words or phrases in alphabetical order, separated by commas.***

I. INTRODUCTION

Let A denotes the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1.1)$$

which are analytic in the open unit disk D = {z ∈C : |z| <1}. By combining the definitions of the operators Im(α,l) andDm(α,l) recently author [17] (see also [16])defined a new generalized multiplier differintegral operatorJm(α,l) : A→A, where

α∈C*, l∈ $\tilde{\mathbb{C}}$ and m ∈Z = {0, ±1, ±2, ...} by

$$J^m(\alpha,\lambda)f(z) = \begin{cases} \frac{1+\lambda}{\alpha} z^{1-\frac{1+\lambda}{\alpha}} \int_0^z t^{\frac{1+\lambda}{\alpha}-2} I^{m-1}(\alpha,\lambda)f(t)dt, & \text{if } -m \in \mathbb{N} \text{ and } \\ & 2-\frac{1+\lambda}{\alpha} \notin \mathbb{N} \\ f(z) & \text{if } m=0 \\ \frac{\alpha}{1+\lambda} z^{2-\frac{1+\lambda}{\alpha}} \frac{d}{dz}\left[z^{\frac{1+\lambda}{\alpha}-1} D^{m-1}(\alpha,\lambda)f(z)\right], & \text{if } m \in \mathbb{N} \end{cases} \qquad (1.2)$$

It is easily seen from (1.5) that for a function f∈Aof the form (1.1), we have

$$J^m(\alpha,\lambda)f(z) = z + \sum_{n=2}^{\infty}\left(\frac{1+\lambda+\alpha(n-1)}{1+\lambda}\right)^m a_n z^n, \quad z \in D \qquad (1.3)$$

whereα∈C*, m ∈Z, l∈ $\tilde{\mathbb{C}}$ suchthat $2-\frac{1+\lambda}{\alpha}$ ∉N for −m ∈N.

We remark that the operator Jm(α, l) generalizes severalpreviously studied operators due to, Cho and Srivastava[5],Al-Oboudi[3], Bernardi[4], Flett[8], Jung etal. [9], Salagean[20] etc. Using the generalizedmultiplier transformation operator Jm(α, l), we now definethe following two subclasses of A.

Definition 1.1. Let ϕ(z) be analytic in D of the form

$$\phi(z) = B_1 z + B_2 z^2 + \Lambda, \quad B_1 \in \mathbb{R}, B_1 > 0 \qquad (1.4)$$

A function f∈A is said to be in the class $L_q^m(\alpha,\lambda,s,t,\lambda;\phi)$ if and only if

$$\frac{[(s-t)z]^{1-\lambda}[J^m(\alpha,\lambda)f(z)]'}{[J^m(\alpha,\lambda)f(sz)-J^m(\alpha,\lambda)f(tz)]^{1-\lambda}} - 1 \,\pi_q\, \phi(z) \qquad (1.5)$$

whereλ≥ 0, s, t ∈C, with s ≠ t, |t| ≤ 1,α∈C*, m ∈Z, l∈ $\tilde{\mathbb{C}}$ and $2-\frac{1+\lambda}{\alpha}$ ∉N (m ∈N).

Remark 1.2. Setting λ = 0, s = 1, t = 0, m = 0 and

$$\phi(z) = \psi(z) - 1 \qquad (1.6)$$

Where ψ(z) is analytic in D with ψ(0) = 1, then(1.5) reduces to

$$\frac{zf'(z)}{f(z)} - 1 \,\pi_q\, \phi(z) \qquad (1.7)$$

The class of all functions f ∈A satisfying (1.7) isdenoted by $S_q^*(\psi)$, which is studied recently by Mohd and Darus [14]. Further, setting

$$\lambda = 0, S = 1, t = 0, m = 1, \alpha = 1, \lambda = 0 \text{ and } \phi(z) = \psi(z) - 1 \qquad (1.8)$$

whereψ(z) is analytic in D with ψ(0) = 1, then $L_q^m(1,0,1,0,0;\psi-1) := C_q(\psi)$, where Cq(ψ) studiedin Mohd and Darus [14].

In the following lemma, Ω is the class of analytic functionsω, normalized by ω(0) = 0, and satisfying the condition|ω(z)| < 1. We shall use this lemma to prove our main results.

Lemma 1.3. [11] If ω∈Ω, then for any complex number t

$$\left|\omega_2 - t\omega_1^2\right| \le \max\{1; |t|\} \qquad (1.9)$$

The result (1.6) is sharp for the function ω(z) = z2or ω(z) = z.

II. MAIN RESULTS

Theorem2.1.

Let Ai= $\left(1+\frac{i\alpha}{1+\lambda}\right)^m$, i = 1, 2,.... If f ∈ $L_q^m(\alpha,\lambda,s,t,\lambda;\phi)$, then for any complex number μ we have

$$|a_2| \le \frac{B_1}{[(s+t)(\lambda-1)+2]|A_1|}$$

$$|a_3| \le \frac{B_1}{[(\lambda-1)(s^2+st+t^2)+3]|A_2|}$$

$$\left\{1+\max\left(1, \left|\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}\right|B_1 + \frac{|B_2|}{B_1}\right)\right\}$$

and

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$$\left|a_3-\mu a_2^2\right|\le\frac{B_1}{[(\lambda-1)(s^2+st+t^2)+3]\left|A_2\right|}\left\{1+\max\left(1,\left|\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}+\frac{\mu[(\lambda-1)(s^2+st+t^2)+3]A_2}{[(s+t)(\lambda-1)+2]^2A_1^2}\right|B_1+\frac{\left|B_2\right|}{B_1}\right)\right\}$$

Proof.

If f $\in L_q^m(\alpha,\lambda,s,t,\lambda;\phi)$ then there existsanalytic functions ψ and ω with |ψ(z)| ≤ 1, ω(0) = 1, and |ω(z)| ≤ 1 such that

$$\frac{[(s-t)z]^{1-\lambda}[J^m(\alpha,\lambda)f(z)]'}{[J^m(\alpha,\lambda)f(sz)-J^m(\alpha,\lambda)f(tz)]^{1-\lambda}}-1=\psi(z)\phi(\omega(z)) \quad (2.1)$$

Let ϕ(z) is given by(1.4),ψ(z) and ω(z) are given by,

$$\psi(z)=c_0+c_1z+c_2z^2+\Lambda \quad (2.2)$$

and

$$\omega(z)=\omega_1z+\omega_2z^2+\Lambda \quad (2.3)$$

A simple calculation provides us,

$$\frac{[(s-t)z]^{1-\lambda}[J^m(\alpha,\lambda)f(z)]'}{[J^m(\alpha,\lambda)f(sz)-J^m(\alpha,\lambda)f(tz)]^{1-\lambda}}-1$$
$$=[(s+t)(\lambda-1)+2]A_1a_2z+\Big[[(\lambda-1)(s^2+st+t^2)+3]A_2a_3+\frac{1}{2}(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]A_1^2a_2^2\Big]z^2+\Lambda \quad (2.4)$$

$$\psi(z)\phi(\omega(z))=B_1C_0\omega_1z+[B_1C_1\omega_1+c_0(B_1\omega_2+B_2\omega_1^2)]z^2+\Lambda \quad (2.5)$$

By using (2.4), (2.5), in (2.1)and equating co-efficients both sides, we get

$$a_2=\frac{B_1c_0\omega_1}{[(s+t)(\lambda-1)+2]A_1}$$

and

$$a_3=\frac{1}{[(\lambda-1)(s^2+st+t^2)+3]A_2}\Big[B_1c_1\omega_1+c_0(B_1\omega_2+B_2\omega_1^2)-\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}B_1^2c_1c_0^2\omega_1^2\Big]$$

Since ψ(z) is analytic and bounded in D, we have |cn|≤ 1−|c0|2≤ 1 n > 0, [15, page 172].Using this fact and well-knowninequality |ω1| ≤ 1, we get $|a_2|\le\frac{B_1}{|A_1|}$ and using Lemma 1.3, we get,

$$\left|a_3-\mu a_2^2\right|\le\frac{B_1}{[(\lambda-1)(s^2+st+t^2)+3]\left|A_2\right|}\left\{\left|c_1\omega_1\right|+\left|c_0\right|\right|\omega_2-\left[\left(\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}+\frac{\mu[(\lambda-1)(s^2+st+t^2)+3]A_2}{[(s+t)(\lambda-1)+2]^2A_1^2}\right)B_1c_0-\frac{B_2}{B_1}\right]\omega_1^2\left|\right\} \quad (2.6)$$

$$\le\frac{B_1}{[(\lambda-1)(s^2+st+t^2)+3]\left|A_2\right|}\left\{1+\max\left(1,\left|\left(\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}+\frac{\mu[(\lambda-1)(s^2+st+t^2)+3]A_2}{[(s+t)(\lambda-1)+2]^2A_1^2}\right)B_1c_0-\frac{B_2}{B_1}\right|\right)\right\}$$

$$\le\frac{B_1}{[(\lambda-1)(s^2+st+t^2)+3]\left|A_2\right|}\left\{1+\max\left(1,\left|\left(\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}+\frac{\mu[(\lambda-1)(s^2+st+t^2)+3]A_2}{[(s+t)(\lambda-1)+2]^2A_1^2}\right)\right|B_1+\frac{\left|B_2\right|}{B_1}\right)\right\}$$

which is required result. Further setting μ = 0 in (2.6)we get the bound on |a3|. This completes the proof of Theorem2.1. □

Remark 2.2.

In the special case when we substitute the values of m and ϕ asgiven in (1.6), then Theorem 2.1 corresponds to theresult given recently by [14, Theorem 2.1]. Furthermore, if weset the values of parameters and ϕ as given in (1.8). Then Theorem 2.1 yields a recentlyestablished result due to [14, Theorem 2.4].

Theorem 2.3.

Let Ai= $\left(1+\frac{i\alpha}{1+\lambda}\right)^m$, i = 1, 2,.... If f ∈A, such that the function $\frac{[(s-t)z]^{1-\lambda}[J^m(\alpha,\lambda)f(z)]'}{[J^m(\alpha,\lambda)f(sz)-J^m(\alpha,\lambda)f(tz)]^{1-\lambda}}-1$ majorized byϕ(z) then

$$\left|a_2\right|\le\frac{B_1}{[(s+t)(\lambda-1)+2]\left|A_1\right|}$$

and

$$\left|a_3\right|\le\frac{1}{[(\lambda-1)(s^2+st+t^2)+3]\left|A_2\right|}\left\{\left|B_2\right|+B_1+\left|\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]}\right|B_1^2\right\}$$

and for any complex number μ

$$\left|a_3-\mu a_2^2\right|\le\frac{1}{[(\lambda-1)(s^2+st+t^2)+3]\left|A_2\right|}\left\{\left|B_2\right|+B_1+\left|\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]}{2[(s+t)(\lambda-1)+2]^2}+\frac{\mu[(\lambda-1)(s^2+st+t^2)+3]A_2}{[(s+t)(\lambda-1)+2]^2A_1^2}\right|B_1^2\right\}$$

Proof.

Taking ω(z) = z in the proof of Theorem 2.1 we get

$$a_2=\frac{c_0B_1}{[(\lambda-1)(s+t)+2]A_1}$$

$$a_3=\frac{1}{[(\lambda-1)(s^2+st+t^2)+3]A_2}\left[c_0B_2+c_1B_1-\frac{(\lambda-1)(s+t)[(\lambda-2)(s+t)+4]c_0^2B_1^2}{2[(s+t)(\lambda-1)+2]^2}\right]$$

Now following the same procedure as the Theorem 2.1 we getthe desired result. □

CONCLUSION

This paper deals to obtain bounds on coefficientfunctional for certain classes of Sakaguchi type functions involvinggeneralized multiplier transformation operator, which are definedusing Quasi-subordination. Some consequences of the main result arementioned and relevance with some of the known results are also pointedout.

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