Coefficient Functional Bounds for Certain Classes of Sakaguchi Type Functions

S. Balaji, B. Srutha Keerthi

Abstract: The purpose of this paper is to obtain bounds on coefficient functional for certain classes of Sakaguchi type functions involving generalized multiplier transformation operator, which are defined using Quasi-subordination. Some consequences of the main result are mentioned and relevance with some of the known results are also pointed out.

Index Terms: About four key words or phrases in a natural order, separated by commas.

I. INTRODUCTION

Let $A$ denotes the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$  \hspace{1cm} (1.1)

which are analytic in the open unit disk $D = \{z \in \mathbb{C} : |z| < 1\}$. By combining the definitions of the operators $\text{Im}(\alpha,1)$ and $\text{Dm}(\alpha,1)$ recently author [17] (see also [16]) defined a new generalized multiplier differintegral operator $\text{Im}(\alpha,1) : A \to A$, where $\alpha \in \mathbb{C}^*$, $1 \leq \alpha < \infty$. Then $\text{Dm}(\alpha,1) : A \to A$ and

$$J^{\alpha}(\alpha,\lambda)(f(z)) = z + \sum_{n=2}^{\infty} \left(1 + \frac{a_n}{\lambda} \right) \tilde{a_n} z^n,$$  \hspace{1cm} (1.2)

where $\alpha \in \mathbb{C}^*$, $m \in \mathbb{Z}$, $1 \leq \alpha < \infty$. It is easily seen from (1.5) that for a function $f \in A$ of the form (1.1), we have

$$J^{\alpha}(\alpha,\lambda)(f(z)) = z + \sum_{n=2}^{\infty} \left(1 + \frac{\lambda}{\alpha} \frac{n(n-1)}{\lambda} \right) \tilde{a_n} z^n,$$  \hspace{1cm} (1.3)

where $\alpha \in \mathbb{C}^*$, $m \in \mathbb{Z}$, $1 \leq \alpha < \infty$ such that $2 - \frac{1}{\alpha} \notin \mathbb{N}$ for $-m \in \mathbb{N}$.

We remark that the operator $\text{Im}(\alpha,1)$ generalizes several previously studied operators due to Cho and Srivastava[5], Al-Oboudi[3], Bernardi[4], Flett[8], Jung et al. [9], Salagean[20] etc. Using the generalised multiplier transformation operator $\text{Dm}(\alpha,1)$, we now define the following two subclasses of $A$.

Definition 1.1. Let $\phi(z)$ be analytic in $D$ of the form

$$\phi(z) = B_0 z + B_1 z^2 + \lambda A \quad \text{if and only if} \quad \phi(z) = B_0 z + B_1 z^2 + \lambda A,$$  \hspace{1cm} (1.4)

A function $f \in A$ is said to be in the class $L_{\Omega}^{\alpha}$ if and only if

$$f(z) \in L_{\Omega}^{\alpha} \Leftrightarrow (\alpha, \lambda, \alpha, t, \lambda; \phi) \in L_{\Omega}^{\alpha}$$  \hspace{1cm} (1.5)

where $\lambda \geq 0$, $t \in \mathbb{C}$, with $s \neq t$, $|t| < 1$, $\alpha \in \mathbb{C}^*$, $m \in \mathbb{Z}$, $1 \leq \alpha < \infty$ and

$$m \notin \mathbb{N}.$$  \hspace{1cm} (1.6)

Remark 1.2. Setting $\lambda = 0$, $s = 1$, $t = 0$, $m = 0$ and $\phi(z) = \psi(z) - 1$.

Where $\psi(z)$ is analytic in $D$ with $\psi(0) = 1$, then (1.5) reduces to

$$zf(z) = f(z) \lambda + \frac{1}{\lambda} \frac{\psi(z)}{\lambda} - 1 \quad \hspace{1cm} (1.7)$$

The class of all functions $f \in A$ satisfying (1.7) is denoted by $S^{\alpha}_1(\psi)$, which is studied recently by Mohd and Darus [14]. Further, setting

$$\lambda = 0, s = 1, t = 0, m = 1, \alpha = 1, \lambda = 0 \quad \text{and} \quad \phi(z) = \psi(z) - 1$$  \hspace{1cm} (1.8)

where $\psi(z)$ is analytic in $D$ with $\psi(0) = 1$, then $L_{\Omega}^{\alpha}(1,0,1,0; \psi) \equiv C^2(\psi)$, where $C^2(\psi)$ studied in Mohd and Darus [14].

In the following lemma, $\Omega$ is the class of analytic functions, normalized by $\phi(0) = 0$, and satisfying the condition $|\phi(z)| < 1$. We shall use this lemma to prove our main results.

Lemma 1.3. [11] If $\alpha \in \Omega$, then for any complex number $t$

$$|a_2 - a_1| \leq \max \{1, |t|\} \quad \hspace{1cm} (1.9)$$

The result (1.6) is sharp for the function $\phi(z) = z2$ or $\phi(z) = z$.

II. MAIN RESULTS

Theorem 2.1.

Let $A_1 = \left\{1 + \frac{i \alpha}{1 + i \alpha} \right\}_{i = 1, 2, \ldots}$, and $f \in L_{\Omega}^{\alpha}(\alpha, \lambda, s, t, \lambda; \phi)$, then for any complex number $\mu$ we have

$$|h_1| \leq \frac{B_1}{|s + t(\lambda - 1)| + 2|A_1|} \quad \hspace{1cm} (1.10)$$

and

$$|h_1| \leq \frac{B_1}{|s + t(\lambda - 1)| + 2|A_1|} \quad \hspace{1cm} (1.11)$$

$$1 + \max \left\{1, \frac{|s + t(\lambda - 1)|}{|s + t(\lambda - 1)| + 2|A_1|} \right\} B_1 + |B_2| \quad \hspace{1cm} (1.12)$$

S. Balaji, Mathematics Division, School of Advanced Sciences
VIT Chennai, Chennai - 600 127, India.

B. Srutha Keerthi, Mathematics Division, School of Advanced Sciences
VIT Chennai, Chennai - 600 127, India.

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\[ (s-\Omega)^{1+\nu}[\lambda^\nu(s)\psi(z)]^{-1} = \psi(z) \phi(z) \]

(2.1)

Let \( \phi(z) \) is given by (1.4), \( \psi(z) \) and \( \omega(z) \) are given by,

\[ \psi(z) = c_0 + c_1z + c_2z^2 + \Lambda \]

(2.2)

\[ \omega(z) = \omega_1 + \omega_2z^2 + \Lambda \]

(2.3)

By using (2.4), (2.5), in (2.1)and equating co-efficients both sides, we get

\[ a_2 = \frac{B_c c_0 \omega_1}{\lambda (s+1)(\lambda -1)+2} A_1 \]

and

\[ a_3 = \frac{\lambda (s+1)(\lambda -1)+2}{2(s+\lambda)(\lambda -1)+2} [B_c c_0 \omega_1 + c_1 (B_c c_1 + B_c \omega_1)] \]

(2.4)

(2.5)

Since \( \psi(z) \) is analytic and bounded in D, we have \( 0 < n \leq 1 \) \(-\infty < k \leq 0 \), [15, page 172]. Using this fact and well-knowledge inequality \( |a_1| \leq 1 \), we get \( |a_1| = \frac{\lambda}{\lambda -1} \) and using Lemma 1.3, we get,

\[ \frac{1+\nu(B_c \omega_1)}{(\lambda -1)(s+1)+2} \]

\[ \left[ \frac{\lambda (s+1)(\lambda -1)+2}{2(s+\lambda)(\lambda -1)+2} \right] A_1 \]

\[ \left[ \frac{\lambda (s+1)(\lambda -1)+2}{2(s+\lambda)(\lambda -1)+2} \right] A_1 \]

(2.6)

Next following the same procedure as the Theorem 2.1 we get the desired result. □

Remark 2.2.

In the special case when we substitute the values of m and \( \phi \) as given in (1.6), then Theorem 2.1 corresponds to the result given recently by [14, Theorem 2.1]. Furthermore, if we set the values of parameters and \( \phi \) as given in (1.8). Then Theorem 2.1 yields a recently established result due to [14, Theorem 2.4].

Theorem 2.3.

Let \( A_i = \left[ \frac{s-\Omega}{s+1+\lambda} \right] \), \( i = 1, 2, ..., \) If \( f \in A \), such that the function \( \frac{(s-\Omega)^{1+\nu}[\lambda^\nu(s)\psi(z)]^{-1}}{[\lambda^\nu(s)\psi(z)]^{-1}} \) majorized by \( \psi(z) \) then

\[ \left[ \frac{a_2}{(s+\lambda)(\lambda -1)+2} A_1 \right] \]

(2.7)

and

\[ \left[ \frac{a_3}{(s+\lambda)(\lambda -1)+2} A_1 \right] \]

Proof.

Taking \( \omega(z) = z \) in the proof of Theorem 2.1 we get

\[ a_2 = \frac{c_0 B_c}{(\lambda -1)(s+1)+2} A_1 \]

\[ a_3 = \frac{1}{(\lambda -1)(s+1)+2} \]

where

\[ c_0 B_c \]

Now following the same procedure as the Theorem 2.1 we get the desired result. □

CONCLUSION

This paper deals to obtain bounds on coefficient functional for certain classes of Sakaguchi type functions involving generalized multiplier transformation operator, which are defined as quasi-subordination. Some consequences of the main result are mentioned and relevance with some of the known results are also pointed out.

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