Lh Labeling of Some Graphs

Farisa M, Parvathy K.S

Abstract: ***A graph G with n vertices is said to have an LH Labeling if there exists a bijective function f : V(G) → {1,2,3,…,\|V(G)\|} such that the induced map f* : E(G) → N, the set of natural numbers defined by f* (uv) =*** $\frac{LCM(f(u),f(v))}{HCF(f(u),f(v))}$ ***is injective. (LCM and HCF denotes the least common multiple and highest common factor respectively). A graph that admits an LH labeling is called an LH graph. In this paper we prove that the splitting graphs of a path, comb and theta, sparkler graph and H graph are LH graphs.***

Keywords:Graph labeling, H graph, Splitting graph, Theta graph.

I. Introduction

Let G = G(V,E) be a simple, finite, connected and undirected graph.We provide a brief description of the basic ideas needed for the present work.

If for every vertex v of V(G) assign a non- negative integer f(v), then V(G) is said to be labeled or numbered. The graph G is called a labeled graph if each edge e = uv is given the value

f(uv) = f(u) * f(v) where * is either addition, multiplication, modulo addition or absolute difference. Without any additional constraints labeling can be done in infinitely many ways. Applications of labeled graphs can be seen in different types of networks and mathematical models. [4][7]. We refer to Gallian for a detailed survey of graph labeling techniques.[7].

Study on graph labeling concentrate more on the classes of graphs for which a specific labeling can be done. Infinitely many types of labeling can be done to a graph. Motivated from this idea, we are defining the concept of LH labeling of graphs. Here, we intend to apply the elementary class concepts least common multiple (LCM) and highest common factor (HCF) in labeling [4]. In this paper LH labeling of some class of graphs are investigated.

Splitting graph of a graph

For a graph G, the splitting graph S' of G is obtained by adding to each vertex v a new vertex v' so that v' is adjacent to every vertex that is adjacent to v in G, that is N(v) = N(v') [9].Degree splitting graph of a graph :Let G = (V,E) be a graph with V= S1 U S2 U…….U St U T where each Si is a set of vertices having at least two vertices and having the same degree and T = V \ U Si. The degree splitting graph of G denoted by DS(G) is obtained from G by adding w1, w2, ..., wtand joining wi to each vertex of Si ($1 \leq i \leq t$)[14].

H graph : The H- graph of a path Pn is the graph obtained from two copies of Pn with vertices

v1,v2,……, vnand u1,u2,…,un by joining the vertices v(n+1)/2 and u(n+1)/2 by means of an edge if n is odd and the vertices u(n/2)+1 and v n/2 if n is even [12].

Theta graph :A Theta graph is a block with two non-adjacent vertices of degree 3 and all other vertices of degree 2 and is denoted by Tα [1].

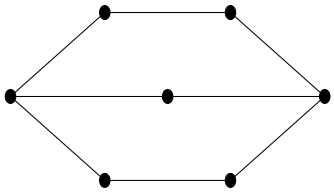

Fig 1: Theta graph

A comb is a caterpillar in which each vertex in the path is joined to exactly one pendant vertex.

A sparkler, denoted as Pm +n, is a graph obtained from the path Pmand appending n edges to an endpoint. This is a special case of a caterpillar. That is the graph obtained by joining an end vertex of a path to the centre of a star [2][8][11].A vertex switching of a graph G is a graph Gv obtained by taking a vertex of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

LH Labeling of a graph : A graph G with n vertices is said to have an LH Labeling if there exists a bijective function f : V(G) → {1,2,3,…,\|V(G)\|} such that the induced map f* : E(G) → N, the set of natural numbers defined by f* (uv) = $\frac{\text{LCM}(f(u),f(v))}{\text{HCF}(f(u),f(v))}$ is injective. A graph that admits an LH labeling is called an LH graph. The path Pn, Odd Cycles, the star K1,n, and are LH graphs[4].

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II. LH labeling of some graphs

In this section we prove that the Theta graph, H graph, Sparkler graph,Heawoodgraph and the splitting graphs of a path, comb and Theta are LH graphs.

Theorem 2.1

Splitting graph of a path Pn is an LH graph.

Proof :

Let {vi, 1≤ i ≤ n} be the vertices of a path Pn and Let {ui, 1≤ i ≤ n} be the new set of vertices added to Pnto form the split graph of Pn. Then

$|V(S'(Pn))| = 2n.$

Define f: V(G) →{1,2,....2n } by

$f(v1) = 1$

$f(v2) = 2n-1$

$$f(vi) = \begin{cases} f(vi-2) + 2 \text{ if } i \text{ is odd} \\ f(vi-2) - 2 \text{ if } i \text{ is even.} \end{cases}$$

$f(u1) = 2n.$

$f(u2) = 2.$

$$f(ui) = \begin{cases} f(u\,i-2) - 2 \text{ if } i \text{ is odd.} \\ f(ui-2) + 2 \text{ if } i \text{ is even.} \end{cases}$$

It is very easy to comprehend that the labeling function meets the conditions of an LH labeling and the graph is an LH graph.

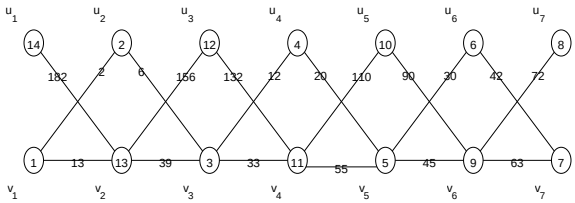


Fig 2 :LH labeling of S'(P7)

Theorem 2.2

The Splitting graph of of a comb S'(PnΘ K1) is an LH graph.

Proof :

Let {vi, 1≤ i ≤ n} and { v'i, 1 ≤ i ≤ n } be the vertices of comb in which {v'i, 1 ≤ i ≤ n } are the pendant vertices.

Let {ui, 1≤ i ≤ n } and { u'i, 1≤ i ≤ n } be the newly added vertices to obtain the splitting graph and let G denotes the splitting graph of a comb.

Then |V(G)| = 4n.

Define f : V(G) →{1,2,3,..., 4n } as follows:

$f(v1) = 4n - 1$

$f(vi) = f(vi-1) - 2, 2 \le i \le n$

f(v'1) = 4n.

$f(v'i) = f(v'i-1) - 2, \ 2 \le i \le n.$

$f(ui) = 2i - 1, 1 \le i \le n.$

$f(u'i) = f(v'n) - 2i, \ 1 \le i \le n$

As per the labeling pattern f defined, the splitting graph of a comb is an LH graph.

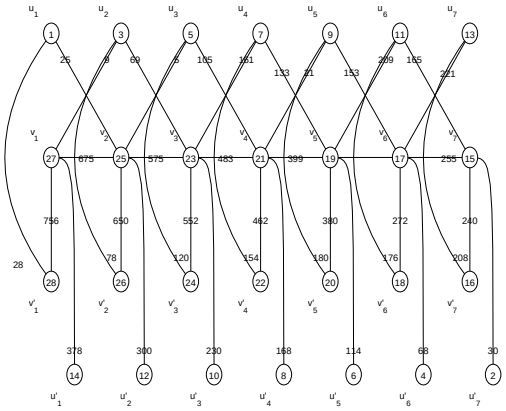


Fig 3 : LH labeling of S'(P7 Θ K1)

Theorem 2.3

The Sparkler graph Pm +n is an LH graph.

Proof :

Let {vi, i = 1 to m} be the vertices of the path Pm and {ui, i = 1 to n} be the vertices joined to the vertex vm to form the sparkler graph Pm +n .

|V (Pm +n)| = m + n.

Define f : V(Pm +n) → {1,2,3,..., m + n } as follows

$f(vm) = 1.$

$f(ui) = i + 1, 1 \le i \le n.$

$f(v1) = m + n$

$f(vi) = f(\ vi-1) - 1, 2 \le i \le m-1$

The vertex function defined above induces an injective function f* : E(G) →N. Thus f is an LH labeling of Pm +n. Hence Pm +n is an LH graph.

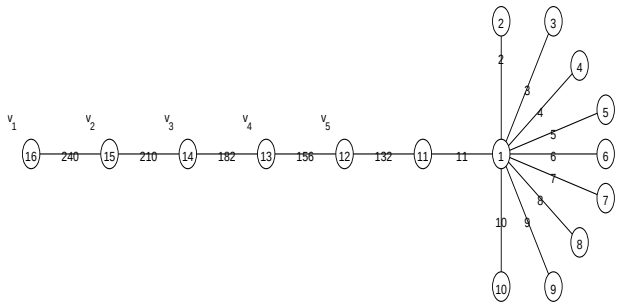


Fig 4 : LH labeling of P7+9

Theorem 2.4

Theta graph Tα is an LH graph.

Proof :

Let v0, v1,v2,v3,v4, v5, v6 are the vertices of the Theta graph Tα with centre v0 and |V Tα)| = 7. We define the vertex labeling f : V(Tα) →{1,2,3,4,5,6,7}as follows:

f(v0) = 6, f(v1) = 7, f(v2) = 1, f(v3) = 2, f(v4)= 5, f(v5) =4 and f(v6) = 3.

In view of the labelling pattern, Theta graph is an LH graph.

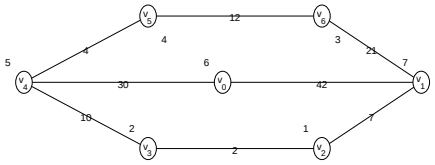


Fig 5 : LH labeling of Tα.

Theorem 2.5

The splitting graph of a Theta graph S' (Tα) is an LH graph.

Proof :

Let v0, v1,v2,v3,v4, v5, v6 are the vertices of the Theta graph Tα with centrev0 and u0, u1, u2, u3, u4, u5, u6 be the newly added vertices corresponding to vi, $0 \le i \le 6$ to obtain the splitting graph of a Theta graph.

|V(S'(Tα))| = 14.

We define the vertex labeling f : V(S'(Tα)) → {1,2,3,4,....,14} as follows:

f(v0) = 11, f(v1)= 7, f(v2) = 1, f(v3)= 3, f(v4)= 13,f(v5)= 9, f(v6) = 5

f(u0)= 6, f(u1) = 12, f(u2)= 2, f(u3)= 4, f(u4)= 14, f(u5)= 8, f(u6)=10.

In view of the labelling pattern, splitting graph of a Theta graph S'(Tα) is an LH graph.

Theorem 2.6

The degree splitting graph of a Theta graph DS(Tα)is an LH graph.

Proof :Let v0, v1,v2,v3,v4, v5, v6 are the vertices of the Theta graph Tα with centre v0 and |V (Tα)| = 7.

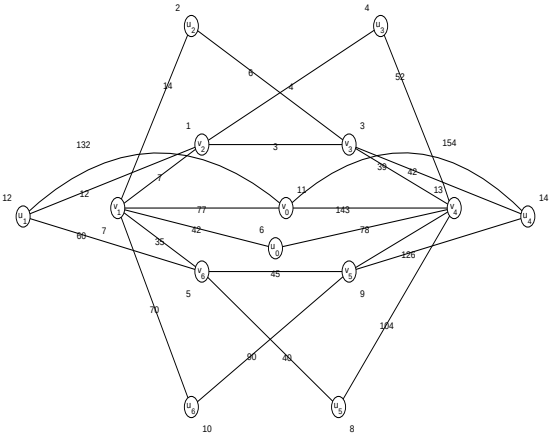


Fig 6 : LH Labeling of S'(Tα)

It has two vertices of degree 3 and all other vertices are of degree 2. Therefore, V (Tα) = S1 U S2 where S1 = {v1,v4} and S2 = {v0, v2, v3, v5, v6}. Add two vertices w1 and w2 corresponding to the sets S1and S2 and make w1 adjacent to v1and v4 and w2 adjacent tov2, v3, v5, v6 and v0, to obtain the degree splitting graph DS(Tα).

|V(DS(Tα))| = 9.

We define the vertex labeling f : V(S'(Tα)) → {1,2,3,4,...., 9} as follows:

f(v0) = 6, f(v2)= 1, f(v3)= 2, f(v4)= 5, f(v5)= 8, f(v6)= 3, f(w1)= 9, f(w2)= 7.

In view of the labeling pattern, the degree splitting graph of a Theta graph DS(Tα) is an LH graph.

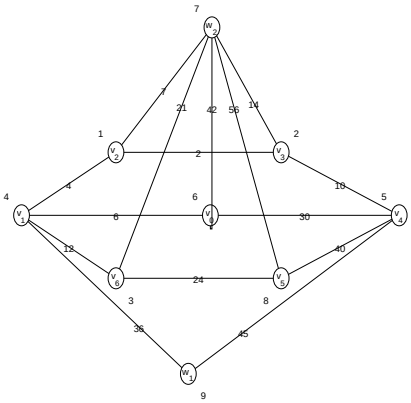


Fig 7: LH labeling of DS(Tα)

Theorem 2.7

The switching of any vertex in the Theta graph is an LH graph.

Proof :

Let v0, v1,v2,v3,v4, v5, v6 are the vertices of the Theta

graph Tα with centre v0 and |V(Tα)| = 7. Let Gs be the graph obtained from Tα after switching the vertex vi,

$0 \leq i \leq 6$. Clearly |V(Gs)| = 7.

We define the vertex labeling f : |V(GS) | →{1,2,3....,7} as follows :

Case 1 : switching of the center vertex v0

f(v0) = 1, f(v1)= 6, f(v2)= 5, f(v3)= 4, f(v4)= 7, f(v5)= 3 and f(v6) = 2

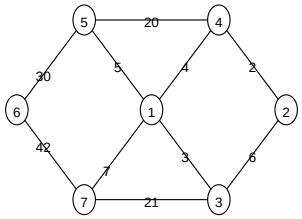


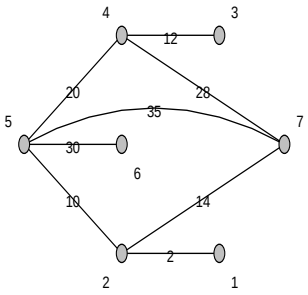
Fig 8.

Case 2 : switching of any vertex of degree 3

In Tα, only two vertices are of degree 3. The vertex in which switching is done is labeled with 5, centre vertex with 6 and the vertices adjacent to 5 with 7,2 and 4. The vertex adjacent to 4 is labeled with 3 and the vertex which is adjacent to 2 with 1.

Case 3 : switching of any vertex of degree 2 other than the centre

The vertex in which switching is done is labeled with 7, centre vertex with 2 and the pendant vertex with 6. Label the adjacent vertices of centre with 5 and 1. The vertex adjacent to 5 is labeled with 3 and the vertex adjacent to 4 is labeled with 1.



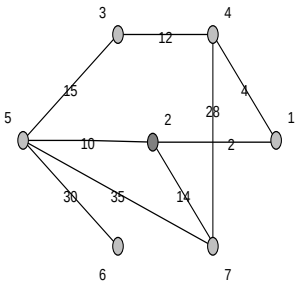


Fig 9.

In all cases the graph is an LH graph.

Theorem 2.8

The H-graph is an LH graph.

Proof :

Let G be an H-graph. The vertex and the edge set of G are given by V (G) = {ui, vi / $1 \leq i \leq n$}

and E (G) = {u iui+1, vivi+1 / $1 \leq i \leq n-1$} U {u (n+1)/2 v(n+1)/2 if n is odd or u(n/2) +1v_n/2 if n is even}.

Then |V(G)| = 2nand |E(G)| = 2n-1.

Define f : V(G) →{1,2,3,..., 2n } as follows

f(vi) = 2i - 1, $1 \leq i \leq n$

f(u1) = 2n

f(ui) = f(ui-1) - 2, $2 \leq i \leq n$

The labeling function satisfies the conditions of an LH labeling and the graph is an LH graph.

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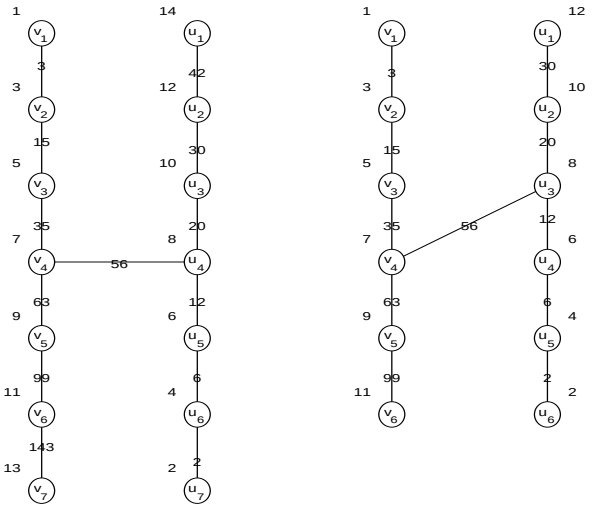

Fig. 10 : LH labeling of H7 and H6

Remark: The Heawood Graph [5][17] is an LH graph and the labeling pattern is shown below :

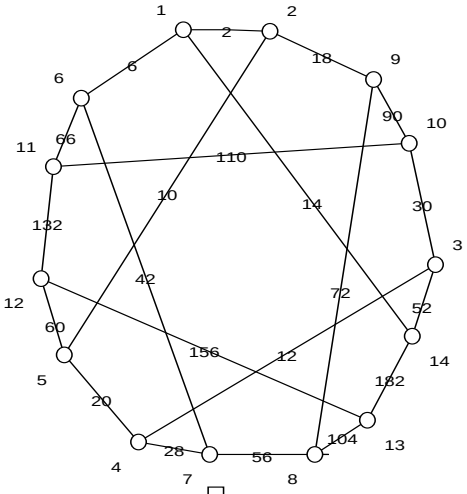

Fig.11 : LH Labeling of Heawood graph.

III. CONCLUSION

Throughout this paper we conclude that Theta graph and types of Theta graphs and its structure of theta graphs. In modern world how to use graph in network theory and how to optimized and save energy and time etc..

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