Hamiltonian Laceability in the Interleaver graph of the Brick Product Graph $C(2n,1,5)$

R. A. Daisy Singh, R. Murali

Abstract:A good interleaver for turbo codes can be constructed from 3-regular hamiltonian graphs having large girth. The girth of a graph is the shortest cycle contained in a graph. In topological point of view it is important for any interconnection network to have various graph theoretic properties which includes girth. Turbo codes are used in 3G/4G mobile communications and in satellite communications. Also, interleavers have been used in communication systems for a long time. The classical use of interleaver is to randomise the location of errors. In this paper we present a construction of interleaver graphs IG_N ***from the brick product graph*** $C(2n,1,5)$ ***and explore its Hamiltonian laceability properties.***

Keywords : Brick product graphs, fault tolerance, Hamiltonian laceability, interleaver graph.

I. INTRODUCTION

Interleaver plays a very important part in the construction of good turbo codes which are the class of error correcting codes introduced by Berrou and Glavieux in 1993. AryaMazumdar et.al. [4] have proposed a new construction of interleavers for turbo codes from 3-regular hamiltonian graphs. Alspach et.al.[3] showed that most $C(2n,1,r)$ are hamiltonianlaceable for $3 \le r \le 9$. Inspired by their work theauthors R.A. Daisy Singh and Murali. R [6] have discussed the hamiltonianlaceability properties of interleaver graph IG_N, $N \ge 8$ constructed from the brick product graph $C(2n,1,3)$. Extending this work we explore the construction of interleaver graphs IG_N, $N \ge 12$ from the brick product graph $C(2n,1,5)$ and discuss laceability properties.

A. Definition

A connected graph G is Hamiltonian laceableif between every pair of distinct vertices in G at an odd distance there exists a Hamiltonian path. G is Hamiltonian-t- laceable (Hamiltonian-t^*- laceable) if there exists in it a hamiltonian path between every pair (atleast one pair) of distinct vertices u and v with the property $d(u,v)=t$ such that $1 \le t \le diam(G)$ where $diam(G)$ is the diameter of G.

B. Definition

Let P be a path between the vertices a_i and a_j and P' be a path between the vertices a_j and a_k in a graph G then the path $P \cup P'$ is obtained by extending P from a_i to a_j and a_j to a_K through the common vertex a_j. That is if $P: a_i \to a_j$ and $P': a_j \to a_k$ then $P \cup P': a_i \to a_j \to a_k$.

C. Definition

Let m, n and r be the positive integers. Let $C_n = a_{0,} a_{1,} a_{2,} a_{3,} \ldots\ldots a_{(2n-1),} a_0$ denote a cycle of order $2n$. The (m,r)-brick product of C_{2n} denoted by $C(2n,m,r)$ is defined for $m=1,$ werequire r to be odd and greater than 1. Then $C(2n,m,r)$ is obtained from C_{2n} by adding chords $C_n = a_{2k}\, a_{(2k+r),}$ $k = 1,2,\ldots.n,$ where the computation is performed under modulo $2n.$

D. Definition

A graph G^* is k - edge fault tolerant with respect to a graph G if every graphobtained by removing any k edges from G^* contains G. A graph G is k - edge fault tolerant Hamiltonian laceableif $G-F$ remains Hamiltonianlaceable for every $F \subseteq E(G)$ with $|F| \le k$.

II. PRELIMINARIES: CONSTRUCTION OF INTERLEAVER FROM BRICK PRODUCT GRAPH $C(2n,1,5)$

Let G be the graph of $C(2n,1,5)$. Let $2n = N$ be the number of vertices labelled from 1 to N placed on a cycle of the graph. Initially the fixed cycle becomes the Hamiltonian cycle of the graph. To construct IG_N from G we first draw the upper and lower chain with N vertices. Note that the upper and lower chains are the two cycles of length N. If i is the vertex

Revised Manuscript Received on March 26, 2019.

R. A. Daisy Singh, Assistant Professor, Department of Mathematics B.N.M. Institute of Technology, Bengaluru – 560070.
R. Murali, Professor, Department of Mathematics, Dr.Ambedkar Institute of Technology, Bengaluru – 560056.

from the upper chain and j is the vertex from the lower chain then i and j should be connected in the IG_N if there is an edge between i and j in G which is not a part of the Hamiltonian cycle of G.

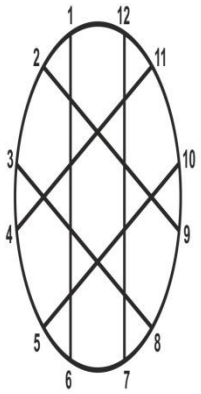

Figure-1(a)

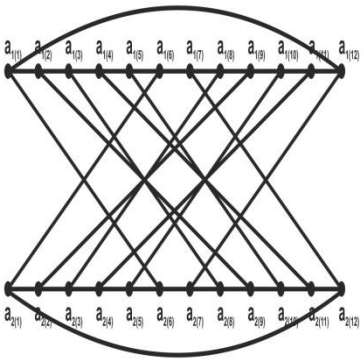

Figure-1(b)

Figure-1(b) shows the IG_{12} constructed from the graph $C(12,1,5)$ of figure 1(a).

III. TERMINOLOGIES

We need to first introduce the following terminologies to establish our results.

For each vertex a_{1i} and a_{2i} of IG_N, we shall write:

$a_{1i}\,P[N]= a_{1i}\,(a_{1(i+1)})(a_{1(i+2)})(a_{1(i+3)})(a_{1(i+4)})\ldots\ldots(a_{1(i+N-1)})$,

$a_{1i}\,P^{-1}[N]= a_{1i}\,(a_{1(i-1)})(a_{1(i-2)})(a_{1(i-3)})(a_{1(i-4)})\ldots\ldots(a_{1(i-N+1)})$.

$a_{2i}\,Q[N]= a_{2i}\,(a_{2(i+1)})(a_{2(i+2)})(a_{2(i+3)})(a_{2(i+4)})\ldots\ldots(a_{2(i+N-1)})$.

$a_{2i}\,Q^{-1}[N]= a_{2i}\,(a_{2(i-1)})(a_{2(i-2)})(a_{2(i-3)})(a_{2(i-4)})\ldots\ldots(a_{2(i-N+1)})$

for $1 \le i < N$.

Note that the symbols $P[N], P^{-1}[N], Q[N], Q^{-1}[N]$ are the paths of order N where as the symbols

$R[m], R^{-1}[m], S[m], S^{-1}[m], T[m], T^{-1}[m], I[m], I^{-1}[m], J[m]$

$W^{-1}[m], F[m], F^{-1}[m], L[m]$ areself explanatory.

$R[m]=a_{11} \to a_{1N}$, $R^{-1}[m]=a_{1N} \to a_{11}$,

$S[m]=a_{21} \to a_{2N}$, $S^{-1}[m]=a_{2N} \to a_{21}$

$L[m]=a_{1(i+1)} \to a_{2(i+1)}$ for $i=m=1$.

$T[m]=a_{1i} \to a_{2(i+5)}$, $T^{-1}[m]=a_{2(i+5)} \to a_{1i}$,

$I[m]=a_{2i} \to a_{1(i+5)}$, $I^{-1}[m]=a_{1(i+5)} \to a_{2i}$ for all odd i and $1 \le i \le N-5$.

$J[m]=a_{1j} \to a_{2(N-3)}$, $J^{-1}[m]=a_{2(N-3)} \to a_{1j}$,

$K[m]=a_{2j} \to a_{1(N-3)}$, $K^{-1}[m]=a_{1(N-3)} \to a_{2j}$

$W[m]=a_{1(N-1)} \to a_{2j}$, $W^{-1}[m]=a_{2j} \to a_{1(N-1)}$,

$F[m]=a_{2(N-1)} \to a_{1j}$, $F^{-1}[m]=a_{1j} \to a_{2(N-1)}$

for $i=m=1,\ j=i+1$.

IV. RESULTS

A. *Theorem*

The graph IG_N, $N \ge 12$ is Hamiltonian-t^*- laceable for all N and odd t such that $1 \le t < 6$.

Proof: Let IG_N be the interleaver graph of order N. Let $V = V_1 \cup V_2$ be the set of vertices such that $V_1 = \{a_{i(j)} : i = 1, 1 \le j \le N\}$, $V_2 = \{a_{i(j)} : i = 2, 1 \le j \le N\}$. Let E be the set of edges of IG_N such that $E = \{b_k : 1 \le k \le 3N\}$.

To establish the result we discuss the following cases.

Case:1 In IG_N, $d(a_{1(1)}, a_{2(6)}) = 1$. Then the Hamiltonian path P between the vertices $a_{1(1)}$ and $a_{2(6)}$ is as follows.

Subcase : (i) For $N = 10r + 2$, $r = 1, 2, 3\ldots\ldots$

$P : a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}\,Q^{-1}[2]T^{-1}[m]P[2]F^{-1}[m]\{Q^{-1}[2]T^{-1}[m]$

$P[2]I^{-1}[m]\}_r\,S[m]T^{-1}[m]P[2]I^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}Q^{-1}[2]K[m]P[2]I^{-1}[m]$

$\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}Q^{-1}[2]W^{-1}[m]P[2]I^{-1}[m]\{Q^{-1}[2]\,T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}\,Q^{-1}[2]\,a_{2(6)}$.

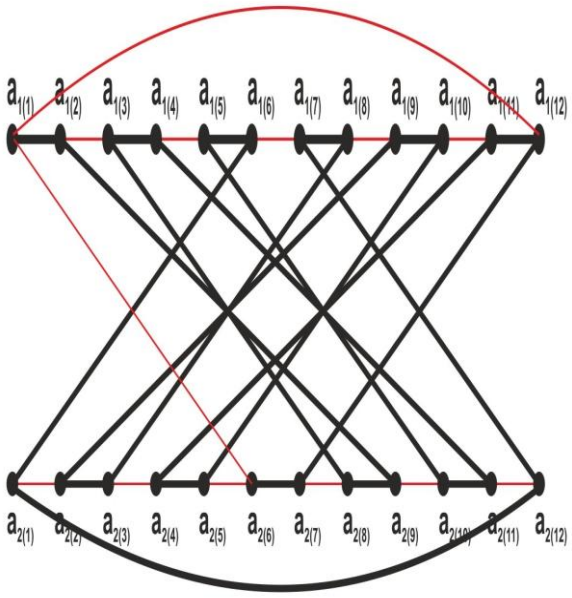

Figure-2. Hamiltonian path between $a_{1(1)}$ and $a_{2(6)}$ in IG_{12}

Subcase : (ii) For $N = 10r + 4,\ r = 1, 2, 3.....$

$P{:}a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_r\, S[m]T^{-1}[m]P[2]I^{-1}[m]\,\{Q^{-1}[2]T^{-1}[m]P[2]$
$I^{-1}[m]\}_{r-1}\, Q^{-1}[2]W^{-1}[m]P[2]I^{-1}[m]\,\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}\ \ Q^{-1}[2]T^{-1}[m]P[2]$
$F^{-1}[m]\,\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_r\ \ Q^{-1}[2]K[m]P[2]I^{-1}[m]\,\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}$
$Q^{-1}[2]a_{2(6)}.$

Subcase : (iii) For $N = 10r + 6,\ r = 1, 2, 3.....$

$P{:}a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_r\, Q^{-1}[2]K[m]\,\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_r$
$P[2]F^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_r Q^{-1}[2]W^{-1}[m]\,\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_r\ P[2]$
$I^{-1}[m]S[m]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_r a_{2(6)}.$

Subcase : (iv) For $N = 10r + 8,\ r = 1, 2, 3.....$

$P{:}a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_r Q^{-1}[2]W^{-1}[m]\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_r P[2]$
$I^{-1}[m]Q^{-1}[2]K[m]\,\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_r\ P[2]I^{-1}[m]S[m]\,\{T^{-1}[m]P[2]I^{-1}[m]\,Q^{-1}[2]\}_r$
$T^{-1}[m]P[2]F^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_r\ Q^{-1}[2]a_{2(6)}.$

Subcase : (v) For $N = 10r + 10,\ r = 1, 2, 3.....$

$P{:}a_{1(1)}P[2]J[m]Q[2]\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[2]\}_{r-1}T^{-1}[m]P^{-1}[3]\,\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}$
$\{T[m]Q^{-1}[2]I[m]P^{-1}[3]\}\,\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}I^{-1}[m]Q[2]T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[3]$
$\{I[m]P^{-1}[2]T[m]Q^{-1}[2]\}\,\{I[m]P[2]T[m]Q^{-1}[2]\}_{r-1}I[m]P[2]K^{-1}[m]\}\,Q^{-1}[2]I[m]P^{-1}[2]\ T[m]$
$\{Q^{-1}[2]I[m]P[2]T[m]\}_r Q^{-1}[2]F[m]P^{-1}[2]T[m]Q^{-1}[3]a_{2(6)}.$

Case :2 In IG_N, $d(a_{1(1)}, a_{2(8)}) = 3.$ Then the Hamiltonian path P between the vertices $a_{1(1)}$ and $a_{2(8)}$ is as follows.

Subcase : (i) For $N = 10r + 2,\ r = 1, 2, 3.....$

$P: a_{1(1)}\, T[m]Q[2]\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}\, I[m]P^{-1}[2]W[m]\,\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}$
$Q[2]I[m]P^{-1}[2]K^{-1}[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_{r}\, S^{-1}[m]\,\{I[m]P^{-1}[2]T[m]Q[2]\}_{r}\, F[m]$
$P^{-1}[3]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1} Q^{-1}[2]a_{2(8)}.$

Subcase : (ii) For $N = 10r+4,\ r = 1,2,3.....$

$P: a_{1(1)}\, T[m]Q[2]\,\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1} Q[2]I[m]P^{-1}[2]K^{-1}[m]\,\{Q[2]I[m]P^{-1}[2]$
$T[m]\}_{r} Q[2]F[m]P^{-1}[3]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r}\, S[m]\{T^{-1}[m]P[2]I^{-1}[m]$
$Q^{-1}[2]\}_{r}\, W^{-1}[m]P[2]I^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1} Q^{-1}[2]a_{2(8)}.$

Subcase : (iii) For $N = 10r+6,\ r = 1,2,3.....$

$P: a_{1(1)}\, T[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_{r}\, S^{-1}[m]I[m]P^{-1}[2]\,\{T[m]Q[2]I[m]P^{-1}[2]\}_{r}\, W[m]$
$\{Q[2]I[m]P^{-1}[2]T[m]\}_{r}\, Q[2]F[m]P^{-1}[3]J[m]\,\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r}\, Q^{-1}[2]K[m]$
$P[2]I^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1} Q^{-1}[2]a_{2(8)}.$

Subcase : (iv) For $N = 10r+8,\ r = 1,2,3.....$

$P: a_{1(1)}\, T[m]Q[2]\,\{I[m]P^{-1}[2]T[m]Q[2]\}_{r}\, F[m]P^{-1}[3]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r}$
$Q^{-1}[2]W^{-1}[m]\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_{r}\, P[2]I^{-1}[m]Q^{-1}[2]K[m]\{P[2]I^{-1}[m]Q^{-1}[2]\,T^{-1}[m]\}_{r}\, P[2]$
$I^{-1}[m]S[m]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r}\, a_{2(8)}.$

Subcase : (v) For $N = 10r+10,\ r = 1,2,3.....$

$P: a_{1(1)}\, P[2]J[m]\ \{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r}\, Q^{-1}[3]I[m]P^{-1}[3]I^{-1}[m]Q[2]W^{-1}[m]$
$\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_{r-1} P[2]I^{-1}[m]Q^{-1}[3]\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1} I[m]P^{-1}[2]$
$K^{-1}[m]\{Q^{-1}[2]I[m]P[2]T[m]\}_{1}\,\{Q^{-1}[2]I[m]P^{-1}[2]T[m]\}_{1}\,\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}$
$Q^{-1}[3]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1} T^{-1}[m]P[2]I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[3]T[m]a_{2(8)}.$

Case :3 In IG_N, $d(a_{1(1)}, a_{2(2)}) = 5.$ Then the Hamiltonian path P between the vertices $a_{1(1)}$ and $a_{2(2)}$ is as follows.

Subcase : (i) For $N = 10r+2,\ r = 1,2,3.....$

$P: a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r-1}\, Q^{-1}[2]T^{-1}[m]P[2]F^{-1}[m]\{Q^{-1}[2]T^{-1}[m]$
$P[2]I^{-1}[m]\}_{r}\, S[m]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1} T^{-1}[m]P[2]I^{-1}[m]Q[2]W^{-1}[m]\{P[2]$
$I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_{r-1} P[2]I^{-1}[m]Q^{-1}[3]\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1} I[m]P^{-1}[2]K^{-1}[m]\, a_{2(2).}$

Subcase : (ii) For $N = 10r+4,\ r = 1,2,3.....$

$P: a_{1(1)}\,\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1} T[m]Q^{-1}[2]I[m]P^{-1}[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}$
$I^{-1}[m]Q[2]W^{-1}[m]P[2]I^{-1}[m]\,\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r-1} Q[2]T^{-1}[m]P^{-1}[2]F^{-1}[m]$
$Q[2]S^{-1}[m]\,\{I[m]P[2]T[m]Q^{-1}[2]\}_{r-1} I[m]P[2]T[m]Q^{-1}[2]J^{-1}[m]\{P[2]T[m]Q^{-1}[2]\ I[m]\}_{r}$
$P^{-1}[2]K^{-1}[m]\}\, a_{2(2)}.$

Subcase : (iii) For $N = 10r+6,\ r = 1,2,3.....$

$P: a_{1(1)}P[2]J[m]\,\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r}\, Q[2]W^{-1}[m]P[2]I^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P[2]$
$I^{-1}[m]\}_{r}\, S[m]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1} T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[3]\{I[m]P^{-1}[2]T[m]$
$Q[2]\}_{r}\, F[m]P^{-1}[2]\{T[m]Q[2]I[m]P^{-1}[2]\}_{r}\, K^{-1}[m]a_{2(2)}.$

Subcase : (iv) For $N = 10r+8,\ r = 1,2,3.....$

$P: a_{1(1)}P[2]J[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r-1}\, Q[2]T^{-1}[m]P[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}$

$T[m]Q^{-1}[2]F[m]P^{-1}[2]T[m]Q^{-1}[4]\{I[m]P^{-1}[2]T[m]Q^{-1}[2]\}_1\{I[m]P[2]T[m]Q^{-1}[2]\}_{r-1}$
$I[m]P^{-1}[2]W[m]\{Q^{-1}[2]I[m]P^{-1}[2]T[m]\}_1\{Q^{-1}[2]I[m]P[2]T[m]\}_{r-1}Q^{-1}[2]I[m]P^{-1}[3]$
$\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}\{I^{-1}[m]Q[2]T^{-1}[m]P[2]\}_1 I^{-1}[m]Q[2]a_{2(2)}.$

Subcase : (v) For $N=10r+10$, $r=1,2,3.....$

$P:a_{1(1)}P[2]J[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r-1}Q[2]T^{-1}[m]P^{-1}[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}$
$T[m]Q^{-1}[2]I[m]P^{-1}[2]W[m]Q^{-1}[2]I[m]P^{-1}[2]T[m]Q^{-1}[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_{r-1}$
$P^{-1}[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}I^{-1}[m]Q[2]T^{-1}[m]P[2]I^{-1}[m]Q[4]T^{-1}[m]P[2]F^{-1}[m]$
$\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_r Q[2]T^{-1}[m]P[2]I^{-1}[m]Q[2]a_{2(2)}.$ Hence the proof.

B. *Theorem*

The graph IG_N, $N\geq 12$ is 1-edge fault tolerant Hamiltonian-t^*- laceable for all N and even t such that $2\leq t\leq 6.$

Proof: Let IG_N be the interleaver graph of order N. Let $V=V_1\cup V_2$ be the set of vertices such that $V_1=\{a_{i(j)}: i=1, 1\leq j\leq N\}$, $V_2=\{a_{i(j)}: i=2, 1\leq j\leq N\}$. Let E be the set of edges of IG_N such that $E=\{b_k: 1\leq k\leq 3N\}\cup E'$ where $E'=\{a_{1(2)}-a_{2(2)}\}.$

To establish the result we discuss the following cases.

Case : 1 In IG_N, $d(a_{1(1)},a_{2(5)})=2.$ Then the Hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(5)}$ is $P:P_1\cup P_2\cup P_3.$

Subcase : (i) For $N=10r+2$, $r=1,2,3.....$

$P: P_1\rightarrow a_{1(1)}\{T[m]Q[2]I[m]P^{-1}[2]\}_r W[m]Q^{-1}[2]\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}I[m]P^{-1}[2]$
$T[m]S^{-1}[m]\{I[m]P^{-1}[2]T[m]Q[2]\}_r F[m]\{P^{-1}[2]T[m]Q[2]I[m]\}_{r-1}P^{-1}[2]T[m]Q[2]J^{-1}[m]$
$a_{1(2)}\cup P_2\rightarrow a_{2(5)}\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}I[m]P^{-1}[2]K^{-1}[m]a_{2(2)}\cup P_3\rightarrow a_{1(2)}L[m]\ a_{2(2)}.$

Subcase : (ii) For $N=10r+4$, $r=1,2,3.....$

$P: P_1\rightarrow a_{1(1)}P[2]a_{1(2)}\cup P_2\rightarrow a_{2(5)}Q[3]I[m]P^{-1}[4]\{T[m]Q[4]I[m]P^{-1}[4]\}_{r-1}T[m]Q^{-1}[2]$
$F[m]\{P^{-1}[2]T[m]Q[2]I[m]\}_r P^{-1}[2]W[m]Q^{-1}[2]\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}I[m]P^{-1}[2]$
$T[m]Q^{-1}[3]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1}T^{-1}[m]P[2]I^{-1}[m]Q[2]a_{2(2)}\cup P_3\rightarrow a_{1(2)}L[m]\ \ a_{2(2)}.$

Subcase : (iii) For $N=10r+6$, $r=1,2,3.....$

$P: P_1\rightarrow a_{1(1)}P[2]a_{1(2)}\cup P_2\rightarrow a_{2(5)}Q[3]I[m]P[3]\{I^{-1}[m]Q^{-1}[2]T^{-1}[m]P[2]\}_1 F^{-1}[m]Q[2]$
$\{T^{-1}[m]P[4]I^{-1}[m]Q^{-1}[4]\}_{r-1}T^{-1}[m]P^{-1}[3]\{T[m]Q[2]I[m]P^{-1}[2]\}_{r-1}T[m]Q^{-1}[3]\ \{T^{-1}[m]$
$P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1}\{T^{-1}[m]P[2]I^{-1}[m]Q[2]\}_1 W^{-1}[m]P[2]\{I^{-1}[m]Q^{-1}[2]T^{-1}[m]P[2]\}_r$
$I^{-1}[m]Q[2]a_{2(2)}\cup P_3\rightarrow a_{1(2)}L[m]\ a_{2(2)}.$

Subcase : (iv) For $N=10r+8, r=1,2,3,.....$

$P: P_1\rightarrow a_{1(1)}T[m]Q[2]\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}I[m]P^{-1}[2]T[m]Q^{-1}[2]J^{-1}[m]a_{1(2)}\cup$
$P_2\rightarrow a_{2(5)}I[m]P^{-1}[2]\{T[m]Q[2]I[m]P^{-1}[2]\}_{r-1}\{T[m]Q^{-1}[2]I[m]P^{-1}[2]\}_1 W[m]$
$\{Q^{-1}[2]I[m]P^{-1}[2]T[m]\}_1\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}Q^{-1}[2]I[m]P^{-1}[4]T[m]Q^{-1}[2]\ F[m]P^{-1}[2]T[m]$
$\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}Q[3]T^{-1}[m]\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_{r-1}P[2]I^{-1}[m]Q[2]a_{2(2)}$
$\cup P_3\rightarrow a_{1(2)}L[m]\ a_{2(2)}.$

Subcase : (v) For $N=10r+10$, $r=1,2,3,.....$

$P: P_1\rightarrow a_{1(1)}\{T[m]Q[2]I[m]P^{-1}[2]\}_r\ \{T[m]Q^{-1}[2]I[m]P^{-1}[2]\}_1 W[m]Q^{-1}[2]I[m]P^{-1}[2]$

$T[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}Q^{-1}[3]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1}T^{-1}[m]P[2]I^{-1}[m]$
$Q[2]a_{2(2)} \cup P_2 \to a_{2(5)}\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}I[m]P^{-1}[2]T[m]Q^{-1}[2]I[m]P^{-1}[4]T[m]\ Q^{-1}[2]$
$F[m]\{P^{-1}[2]T[m]Q[2]I[m]\}_r\ P^{-1}[2]T[m]Q^{-1}[2]J^{-1}[m]a_{1(2)} \cup P_3 \to a_{1(2)}\ L[m]\ a_{2(2)}.$

Case :2 In IG_N, $d(a_{1(1)}, a_{2(3)}) = 4.$ Then the Hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is $P: P_1 \cup P_2 \cup P_3$.

Subcase :(i) For $N = 10r + 2$, $r = 1, 2, 3.....$

$P: P_1 \to a_{1(1)}\{T[m]Q[2]I[m]P^{-1}[2]\}_r\ W[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}\ Q[2]I[m]P^{-1}[2]$
$K^{-1}[m]a_{2(2)} \cup P_2 \to a_{2(3)}\{I[m]P^{-1}[2]T[m]Q[2]\}_{r-1}I[m]P^{-1}[2]T[m]S^{-1}[m]\{I[m]P^{-1}[2]$
$T[m]Q[2]\}_r\ F[m]\{P^{-1}[2]T[m]Q[2]I[m]\}_{r-1}\ P^{-1}[2]T[m]Q[2]J^{-1}[m]a_{1(2)} \cup P_3 \to a_{1(2)}\ L[m]\ a_{2(2)}.$

Subcase :(ii) For $N = 10r + 4$, $r = 1, 2, 3.....$

$P: P_1 \to a_{1(1)}\{T[m]Q[2]I[m]P^{-1}[2]\}_r\ K^{-1}[m]a_{2(2)} \cup P_2 \to a_{2(3)}\{I[m]P^{-1}[2]T[m]Q[2]\}_r$
$F[m]P^{-1}[2]\{T[m]Q[2]I[m]P^{-1}[2]\}_r\ W[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_r\ S^{-1}[m]\{I[m]P^{-1}[2]\ T[m]Q[2]\}_r$
$J^{-1}[m]a_{1(2)} \cup\ P_3 \to a_{1(2)}\ L[m]\ a_{2(2)}.$

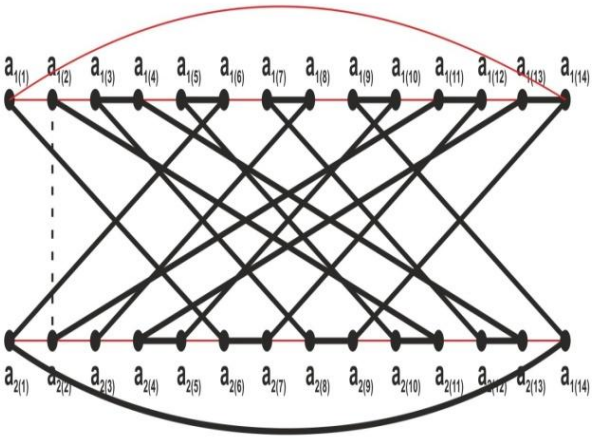

Figure-3. Hamiltonian path between $a_{1(1)}$ to $a_{2(3)}$ in IG_{14}

Subcase :(iii) For $N = 10r + 6$, $r = 1, 2, 3.....$

$P: P_1 \to a_{1(1)}T[m]Q[4]I[m]\{P^{-1}[4]T[m]Q[4]I[m]\}_{r-1}\ P^{-1}[4]T[m]Q^{-1}[2]F[m]P^{-1}[3]a_{1(2)}$
$\cup P_2 \to a_{2(3)}\ I[m]P^{-1}[2]T[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1}\ Q[3]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_r$
$W^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\ \{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r-1}\ \{T^{-1}[m]P[2]I^{-1}[m]Q[2]\}_1 a_{2(2)}$
$\cup P_3 \to a_{1(2)}\ L[m]\ a_{2(2)}.$

Subcase :(iv) For $N = 10r + 8$, $r = 1, 2, 3.....$

$P: P_1 \to a_{1(1)}T[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_r\ Q^{-1}[3]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_r\ W^{-1}[m]$
$\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_{r-1}\ P[2]I^{-1}[m]Q^{-1}[3]I[m]\ \{P^{-1}[4]T[m]Q[4]I[m]\}_{r-1}P^{-1}[4]$
$T[m]Q^{-1}[2]F[m]P[2]T[m]Q^{-1}[3]T^{-1}[m]P^{-1}[2]a_{1(2)} \cup P_2 \to a_{2(3)}I[m]P^{-1}[3]I^{-1}[m]Q[2]$
$\cup P_3 \to a_{1(2)}\ L[m]\ a_{2(2)}.$

Subcase :(v) For $N = 10r + 10$, $r = 1, 2, 3.....$

$P: P_1 \to a_{1(1)}T[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_r\ Q[3]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_r$
$T^{-1}[m]$

$P^{-1}[2]a_{1(2)} \cup P_2 \rightarrow a_{2(3)}\, I[m]P[3]I^{-1}[m]Q^{-1}[2]W^{-1}[m]P[2]I^{-1}[m]Q^{-1}[3]I[m]P^{-1}[4]T[m]$
$Q^{-1}[2]F[m]P[2]T[m]\{Q[2]I[m]P^{-1}[2]T[m]\}_{r-1} Q[3]\{T^{-1}[m]P[4]I^{-1}[m]Q^{-1}[4]\}_{r-1}T^{-1}[m]$
$P^{-1}[2]I^{-1}[m]Q[2]a_{2(2)} \cup P_3 \rightarrow a_{1(2)}\, L[m]\, a_{2(2)}.$

Case :3 In IG_N, $d(a_{1(1)}, a_{2(1)}) = 6.$ Then the Hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(1)}$ is $P: P_1 \cup P_2 \cup P_3$.

Subcase :(i) For $N = 10r + 2$, $r = 1, 2, 3.....$

$P: P_1 \rightarrow a_{1(1)}R[m]P^{-1}[2]W[m]Q^{-1}[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_{r-1} P[3]\{I^{-1}[m]Q[4]T^{-1}[m]$
$P^{-1}[4]\}_{r-1} I^{-1}[m]Q[4]T^{-1}[m]P[2]F^{-1}[m]Q[2]T^{-1}[m]\{P^{-1}[2]I^{-1}[m]Q[2]T^{-1}[m]\}_{r-1}\ P^{-1}[3]$
$\{T[m]Q^{-1}[2]\ I[m]P[2]\}_{r-1} T[m]Q^{-1}[2]J^{-1}[m]a_{1(2)} \cup P_2 \rightarrow a_{2(1)}P[2]a_{2(2)} \cup P_3 \rightarrow a_{1(2)}\ L[m]a_{2(2)}.$

Subcase :(ii) For $N = 10r + 4$, $r = 1, 2, 3.....$

$P: P_1 \rightarrow a_{1(1)}T[m]Q^{-1}[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_{r-1}P[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}$
$I^{-1}[m]Q[2]T^{-1}[m]P[2]F^{-1}[m]Q[2]\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[2]\}_{r} W^{-1}[m]P[2]I^{-1}[m]Q[2]T^{-1}[m]$
$\{P^{-1}[2]I^{-1}[m]Q[2]T^{-1}[m]\}_{r-1} P[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}T[m]Q^{-1}[2]J^{-1}[m]a_{1(2)} \cup P_2 \rightarrow$
$a_{2(1)}P[2]a_{2(2)} \cup P_3 \rightarrow a_{1(2)}\ L[m]a_{2(2)}.$

Subcase :(iii) For $N = 10r + 6$, $r = 1, 2, 3.....$

$P: P_1 \rightarrow a_{1(1)}T[m]Q^{-1}[2]\ \{I[m]P[2]T[m]Q^{-1}[2]\}_{r-1}F[m]P^{-1}[2]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}T[m]$
$Q^{-1}[2]I[m]P[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}I^{-1}[m]Q[2]T^{-1}[m]P[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}$
$T[m]Q^{-1}[2]I[m]P^{-1}[2]W[m]\{Q^{-1}[2]I[m]P[2]T[m]\}_{r}Q^{-1}[2]J^{-1}[m]a_{1(2)} \cup P_2 \rightarrow a_{2(1)}P[2]\ a_{2(2)}$
$\cup P_3 \rightarrow a_{1(2)}\ L[m]a_{2(2).}$

Subcase :(iv) For $N = 10r + 8$, $r = 1, 2, 3.....$

$P: P_1 \rightarrow a_{1(1)}\ \{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}T[m]Q^{-1}[2]I[m]P^{-1}[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}$
$I^{-1}[m]Q[2]W^{-1}[m]P[2]I^{-1}[m]Q[4]\{T^{-1}[m]P[2]I^{-1}[m]Q[4]\}_{1}\{T^{-1}[m]P^{-1}[4]I^{-1}[m]Q[4]\}_{r-1}$
$T^{-1}[m]P[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}T[m]Q^{-1}[2]I[m]P^{-1}[4]T[m]Q^{-1}[2]F[m]P^{-1}[3]a_{1(2)}$
$\cup P_2 \rightarrow a_{2(1)}P[2]\ a_{2(2)} \cup P_3 \rightarrow a_{1(2)}\ L[m]a_{2(2).}$

Subcase :(v) For $N = 10r + 10$, $r = 1, 2, 3.....$

$P: P_1 \rightarrow a_{1(1)}R[m]P^{-1}[2]W[m]Q^{-1}[2]I[m]P[3]I^{-1}[m]Q[4]T^{-1}[m]P[2]F^{-1}[m]\{Q[2]T^{-1}[m]$
$P^{-1}[2]I^{-1}[m]\}_{r}\ Q[2]T^{-1}[m]P[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}T[m]Q^{-1}[2]I[m]P[3]I^{-1}[m]Q[4]$
$\{T^{-1}[m]P^{-1}[4]I^{-1}[m]Q[4]\}_{r-1}T^{-1}[m]P[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r-1}T[m]Q^{-1}[2]J^{-1}[m]a_{1(2)}$
$\cup P_2 \rightarrow a_{2(1)}P[2]\ a_{2(2)} \cup P_3 \rightarrow a_{1(2)}\ L[m]a_{2(2).}$

Hence the proof.

IV. CONCLUSION

The study of hamiltonicityand Hamiltonian laceabilityhaslotofsigni ficance incomputernetworks.Inthispaperwe have establishedthehamiltonian-t^*-laceabilityofaninterleavergraphconstructedfrombrickproduct graph $C(2n,1,5)$. Precisely, wehaveshownthat theinterleavergraphs IG_N foreven N, $N \geq 12$ ishamiltonian-t^*-laceableforallodd t suchthat $1 \leq t < 6$ and1-edgefaulttoleranthamiltonian-t^*-laceableforalleven t suchthat $2 \leq t \leq 6$.Our results shows that theexistence of Hamiltonian pathsinsuchnetworkssufficeto solvedata communication problems.

ACKNOWLEDGEMENT

The first author thankfully acknowledges the support and encouragement providedbytheManagement,Director,Principal,Dean,HOD andStaffofBNM Institute of Technology, Bengaluru. The authorsarealsothankful to the Management, HOD, Researchanddevelopmentcentre, DepartmentofMathematics, Dr.AmbedkarInstituteofTechnology, Bengaluru.

REFERENCES

1. B.Alspach,C.Q.Zhang.“ Hamiltoncyclesincubiccayleygraphson dihedralgroups” ArsCombinatoria,vol.28 , 1989,PP :101-108.
2. S.Y.Hsieh,C.H.Chen,C.W.Ho.“Fault free Hamiltoniancyclesinfaultyarrangementgraphs”. IEEETrans.ParallelDistributedsystemsvol. 10(32), 1999,PP:223-237.
3. B.Alspach,,C.C.Chen,KevinMcAvaney.“ OnaclassofHamiltonianLaceable3-regular
4. graphs”. Elsevier, vol.151,1996,PP:19-38.
5. AryaMazumdar,A.K.Chaturvedi,AdrishBanerjee, “ Constructionof turbo code interleaversfrom3-regularHamiltonian graphs”,IEEE, vol. 10(4),2006,PP:284-286.
6. Pascal O.Vontobel.“OntheconstructionofTurbocodeinterleaversbasedongraph swithlargegirth”.IEEE, 2002,PP:1408-1412.
7. R.A.Daisy Singh, Murali R. “Interleaver graph of brick product graph C(2n,1,3) and Hamiltonian laceability”, International journal of pure and applied mathematics, vol.119, No.14 , 2018, PP : 213-219.
8. AnnapoornaM,MuraliR.“ Hamiltonian Laceabilityinthelinegraphofthe(wl,n,k) graph”. InternationalJournalofComputerApplication,vol.5, No.5,2015, PP :8-15.
9. Frank Harary, John.P.Hayes. “Edgefaulttoleranceingraphs”., Networks:AninternationalJournal, Issue-2,vol.23,1993,PP:135-142.
10. R.Murali, K.S.Harinath. “ Hamiltonian -n*-laceablegraphs”, Fareast journalofappliedmathematics. vol. 391,1999,PP:59-79.
11. LeenaNShenoy,R.Murali, Laceabilityonaclassofregulargraphs,Internationaljournalofcomputationalscienceand Mathematics,vol. 20, No.3,2010,PP:397-406.

AUTHORS PROFILE

R. A. Daisy Singh
Assistant Professor,
Department of Mathematics
B.N.M. Institute of Technology,
Bengaluru – 560070.

R. Murali
Professor,
Department of Mathematics
Dr.Ambedkar Institute of Technology,
Bengaluru – 560056.