Heterogeneous Server Queue with A Threshold on Slow Server

R. Kalyanaraman, S. Kalaiselvi

Abstract: In this paper we consider a two heterogeneous server Markovian queue. In which, the second server has a threshold for service. For this model, the steady state results have been obtained. Some particular models, performance measure and numerical models are presented.

Keywords: Heterogeneous server - slow server - threshold - steady state -performance measures. 2000 Mathematics Subject Classification: 90B22, 60K25 and 60K30.

I. INTRODUCTION

In the literature, most of the work on multi-server queuing systems, it has been generally assumed that the servers are homogeneous. This is valid only when the service process is mechanically or electrically controlled. In 1981, Neuts and Takahashi have pointed out that for the queuing system with two heterogeneous servers, analytical results are intractable. Even though, some researchers focused their studies on queue with two heterogeneous servers. The equilibrium analysis for the general input and exponential service time and with n severs was given in Kendall(1958). A non-constructive existence theorem for the stationary distribution of general input and general service time was presented in Kiefer and Wolfowitz(1955). Karlin and Mc.Gregor(1958) obtained the busy period distribution for the M/M/S queue. Krishnamoorthi(1963) consider a Poisson queue with two heterogeneous servers and with violation of the First-in-First-out principle.

Heffer(1969) has analyzed waiting time distribution of $M/E_k/S$ queue. A Markovian queuing system with balking and two heterogeneous servers has been considered in Singh(1970). In this paper, the author determines the capacity of the slower server and obtains the optimal service rates. Singh(1973) discussed a Markovian queue with the number of servers depending upon the queue length. Desmit(1983 a,b) presented an approach to identify the distribution of waiting times and queue lengths for the queue $GI/H_2/S$. He reduced the problem to the solution of the Wiener-Hopf-type equations and then used a factorization method to solve the system.

Lin and Kumar (1984) has analyzed the optimal control of a queuing system with two heterogeneous servers. Rubinovitch(1985a,b) studied the problem of a heterogeneous two channels queuing systems. In his first paper he discussed three simple models and gave the condition when to discard the slower server depending on the expected number of customer in the system. In the second paper he studied a queuing model with a stalling concept. In 1999,Abou-El-Ata and Shawky introduced a simpler approach to find the condition when to discard the slower server in a heterogeneous two channels queue.

Barcelo(2003) has obtained an approximation for the mean waiting time of $M/H_{2b}/S$ queue. Shin and Moon(2009) has carried out an approximate analysis for $M/G\ /C$ queue. Arkat and Farahani(2014), has used a partial-fraction decomposition approach to the $M/H_2/2$ queue. Zhernovyi(2011)analyzed queue with switching of service modes and threshold blocking of input flow. Kopytko and Zhernovyi(2011) investigated Markovian queue with switching of service mode.
Kalyanaraman and Senthilkumar(2018) analyzed a two heterogeneous server Markovian queue with switching service models. In the same year the authors discussed heterogeneous server Markovian queue with restricted admissibility and with reneging. Kalyanaraman and Senthil kumar(2018) analyzed a two heterogeneous server queue with restricted admissibility. In real life, Models of queuing system with different intensity of service are used for the study of telecommunication process.

In this paper, we consider an $M/M/2$ queue with heterogeneous servers. In addition, there is a threshold policy on the slow servers. In section 2, the queueing system has been discussed and analyzed in steady state. In section 3, we derive some performance measures related to the model discussed in section 2. In section 4, by taking some particular models. In section 5, we obtain same numerical results related to the model discussed in this article.

II. THE MODEL AND ANALYSIS

We consider an $M/M/2$ queueing system with a single waiting line of infinite waiting capacity. Customers arrive at the system according to a Poisson process with rate λ. The service times of customers follows two different exponential distributions with rate μ_1, μ_2 respectively corresponding to the two servers. Also $\mu_1 > \mu_2$.

If there are less than K customers in the system the first server works and the second server stays in ideal state. Once the system size reaches K, the second server also starts work.

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Each customer is served only one server at a time and the queue discipline is first come first served. For the $p_n(t)$be the probability that there are n customers in the system at time t .,$n \geq 0, t \geq 0$. In study state $\lim_{t\to\infty} p_n(t) = p_n$
Using general birth death arguments the following difference equation has been derived

$$\lambda p_0 = \mu_1 p_1 \quad (1)$$

$$(\lambda + \mu_1)p_n = \lambda p_{n-1} + \mu_1 p_{n+1} \quad n = 1,2, \ldots \ldots K-1 \quad (2)$$

$$(\lambda + \mu_1)p_K = \lambda p_{K-1} + \mu p_{K+1} \quad (3)$$

$$(\lambda + \mu)p_n = \lambda p_{n-1} + \mu p_{n+1} \quad n = K+1, K+2 \ldots \ldots \quad (4)$$

For the analysis, the following probability generating function has been defined:

$$P(z) = \sum_{n=0}^{\infty} p_n z^n : |z| \leq 1 \quad (5)$$

Multiplying equations (1)-(4), by suitable power of z an summing over n=0 to ∞, and then using equation (5), we get

$$P(z) = \frac{(\mu_1 - \mu)\sum_{n=1}^{K} p_n z^n - \mu p_0}{\lambda z - \mu} \quad (6)$$

Equation (6) can be written as

$$P(z) = \frac{(\mu_1 - \mu)}{\mu}\sum_{n=1}^{K} p_n z^n - \left(1 - \frac{\lambda z}{\mu}\right)^{-1} + p_0\left(1 - \frac{\lambda z}{\mu}\right)^{-1} \quad (7)$$

Equating the coefficient of z^n on both sides of equations (7), we get

$$p_n = \left(\frac{\lambda}{\mu_1}\right)^{-1} p_0, \quad n = 1,2, \ldots K,$$

$$= \left(\frac{\lambda}{\mu_1}\right)^{K}\left(\frac{\lambda}{\mu}\right)^{n-K} p_0, \quad n = K+1, K+2, \ldots \quad (8)$$

Using the normalization condition$\sum_{n=0}^{\infty} p_n = 1$, we get

$$p_0 = \frac{\mu_1^K(\mu - \lambda)(\mu_1 - \lambda)}{\mu_1^{K+1}(\mu - \lambda) - \lambda^{K+1}(\mu - \mu_1)} \quad (9)$$

Equation (8) and (9)together represents the probability distribution in steady state of the model designed in this article and it is easy to see that the stability condition is $\frac{\lambda}{\mu} < 1$.

III. SOME PERFORMANCE MEASURES

In this section, some performance measure such as probability that both the servers are ideal. Probability that both the servers are busy, expected number of customers in the system, second moment of the number of customers in the system and expected waiting time of a customers in the system have been obtained by straight forward calculations:

(i) Probability that both the servers are idle(p_0)

$$= \frac{\mu_1^K(\mu - \lambda)(\mu_1 - \lambda)}{\mu_1^{K+1}(\mu - \lambda) - \lambda^{K+1}(\mu - \mu_1)}$$

(ii) Probability that both the servers are busy

$$p_B = 1 - p_0\left(\frac{\mu_1 - \lambda}{\mu_1}\right)$$

(iii) Expected number of customers in the system

$$L = P'(1) = \frac{\lambda p_0}{\mu_1^K}\left[\frac{\mu_1(\mu_1^K - \lambda^K) - K\lambda^K(\mu_1 - \lambda)}{(\mu_1 - \lambda)^2}\right]$$

mathematical frame work of the above defined model, the following probabilities have been defined:

(iv) Second moment of number of customers in the system

$$L_1 = \frac{\lambda^2 p_0}{(\mu_1 - \lambda)^3}\left[\frac{\lambda(\mu^K - \lambda^K) + 2\mu(\mu^K - (K+1)\lambda^K) + K(K+1)\lambda(K-1)(\mu - \lambda)^2}{\mu^K(\lambda - \mu)}\right] - \frac{2\lambda L}{(\lambda - \mu)}$$

(v) Expected waiting time of customers in the system (using Little's law)

$$W = \frac{L}{\lambda}$$

IV. NUMERICAL STUDY

In this section, we present some numerical illustrations corresponding to the model discussed in this paper. For the numerical illustration, let us take the value of λ from 0.1 to 1, $K = 5, \mu_1 = 2.9, \mu_2 = 2.5$, (Table1, Graph1), $\mu_1 = 2.7, \mu_2 = 2.3$, (Table2, Graph2), $\mu_1 = 2.5, \mu_2 = 2.1$, (Table3, Graph3). The values of p_0, p_B and L_1 are presented in the takes and the values of L and W are presented in the graphs.

arrival rate	probability of down time	prob. of all the server are busy	Second moment of number of customers in the system
0.1	0.965517	0.067776	0.001624
0.2	0.931036	0.133173	0.006768
0.3	0.896562	0.196185	0.015841
0.4	0.862112	0.256800	0.029219
0.5	0.827712	0.314997	0.047163
0.6	0.793404	0.370748	0.069692
0.7	0.759243	0.424023	0.096384
0.8	0.725297	0.474785	0.126090
0.9	0.691646	0.523003	0.156528
1.0	0.658382	0.568646	0.183732

Table 1: $K = 5, \mu_1 = 2.9, \mu_2 = 2.5$ (λ Various from 0.1 to 1)

arrival rate	probability of down time	prob. of all the server are busy	Second moment of number of customers in the system
0.1	0.962963	0.072702	0.001800
0.2	0.925928	0.142659	0.007488
0.3	0.888904	0.209863	0.017481
0.4	0.851913	0.274297	0.032096
0.5	0.814992	0.335932	0.051410
0.6	0.778199	0.394734	0.075027
0.7	0.741609	0.450660	0.101714
0.8	0.705314	0.503668	0.128880
0.9	0.669421	0.553719	0.151821
1.0	0.634048	0.600784	0.162697

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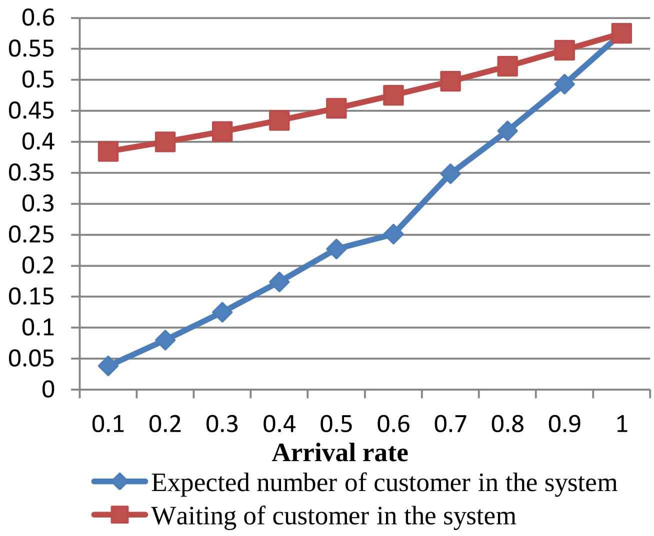
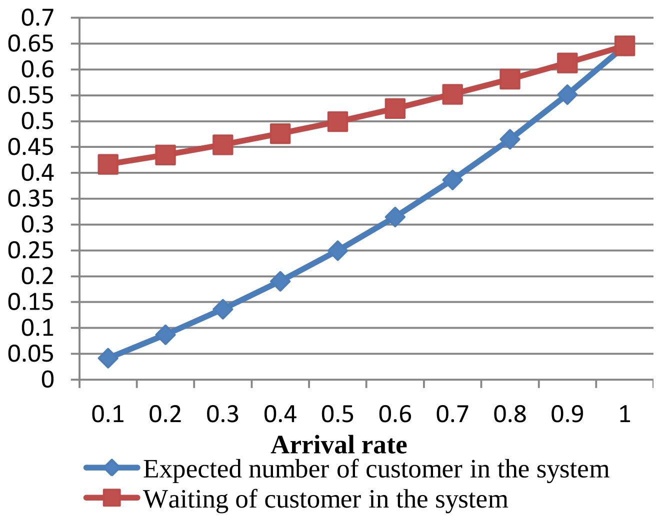
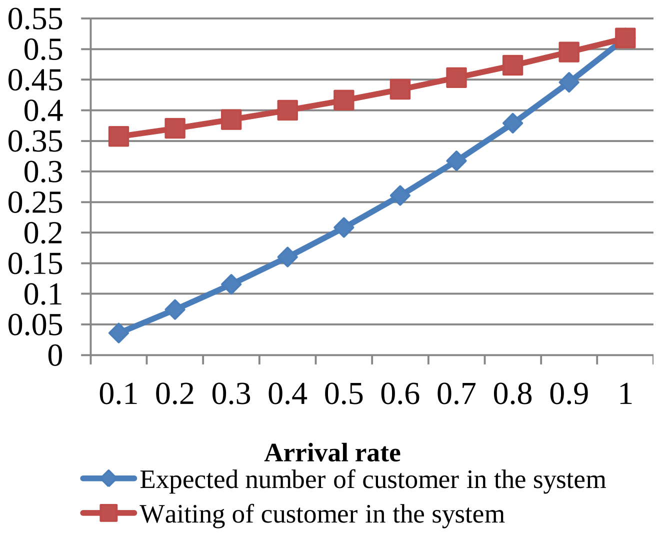
Table2: $\mu_1 = 2.7, \mu_2 = 2.3$(λ Various from 0.1 to 1)

arrival rate	probability of down time	prob. of all the server are busy	Second moment of number of customers in the system
0.1	0.960000	0.078400	0.001996
0.2	0.920003	0.153597	0.008281
0.3	0.880022	0.225581	0.019229
0.4	0.840088	0.294326	0.034991
0.5	0.800256	0.359795	0.055230
0.6	0.760606	0.421940	0.078672
0.7	0.721241	0.480706	0.102431
0.8	0.682289	0.536043	0.120992
0.9	0.643893	0.587908	0.124762
1.0	0.606208	0.636275	0.098076

Table 3:$\mu_1 = 2.7, \mu_2 = 2.3$(λ Various from 0.1 to 1.0)







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