Properties of Multiplication, Determinant, Rank and Eigenvalues of Fuzzy Soft Matrices

K.L Swarna, Magie Jose

Abstract: In this paper some properties of multiplication of fuzzy soft matrices are studied. Then, some properties of determinants are presented. Besides ,rank and eigen values of fuzzy soft matrices are also discussed.

Key words: Soft set , Fuzzy soft set , Fuzzy soft matrix.

I. INTRODUCTION

Soft set theory was first introduced by Molodtsov[9] in 1999.Maji,Biswas and Roy[8]introduced several basic ideas of soft sets in 2001.Cagman and Enginoglu defined soft matrices in 2010 and applied soft matrices in decision making problems.AgainCagman and Enginoglu[6]introduced fuzzy soft matrices and constructed decision making problem in 2012.In 2012 TanusreeMitraBasu et al [7]introduced different types of fuzzy soft matices.SanijibMondal and Madhumangal Pal [5]studied Inversibility and eigenvalues of Intuitoinistic fuzzy matrix in 2013. N Sarala and M Prabhavathi[2]studied Determination of fuzzy soft matrices in 2016.Some new operations of fuzzy soft matrices were introduced by SeyyedHosseinJafariPetroudi et al [1]in 2017.

In this paper we study some properties of multiplication of fuzzy soft matrices and determinants.Thenwe discuss the rank and eigenvalues of fuzzy soft matrices.

II. PRELIMINARIES

Definition 2.1:[9]Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Let $A \subseteq E$.A pair (F_A,E) is called a soft set (over U) where $F_A$is a mapping given by F_A:$E \to P(U)$such that $F(e) = \emptyset$ if $e \notin A$.

Definition 2.2:[8]Let U be an initial universe set and E be a set of parameters.Let$P(U)$ denote the collection of all fuzzy subsets of U,then (F_A,E) is called a fuzzy soft set (FSS) over U, where $F_A : E \to P(U)$ such that $F_A(e) = \Phi$ if $e \notin A$, Φ is a null fuzzy set.

Definition 2.3:[7]Let (F_A,E) be a fuzzy soft set over U.Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u,e): e \in A\ ,\ u \in F_A(e)\}$ which is called relation form of(F_A, E). The characteristic function of R_A is written by $\mu_{R_A}: U \times E \to [0,1]$ where $\mu_{R_A}(u,e) \in [0,1]$ is the membership value of $u \in E$ for each $e \in U$. If $U = \{u_1, u_2, \dots u_m\} and E = \{e_1, e_2, \dots e_n\} and \mu_{ij} = \mu_{R_A}(u_i, e_j),\ \ i = 1,2, \dots, m and j = 1,2, \dots, n$we can define a matrix

$[\mu_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \dots & a_{mn} \end{bmatrix}$, which is called a fuzzy soft matrix(FSM) of order mxn corresponding to the fuzzy soft set (F_A,E) over U.

III. PROPERTIES OF MULTIPLICATION

Definition 3.1:[1]Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two fuzzy soft matrices .Then A and B are compatible for addition and the sum of A and B is defined as $[a_{ij}]_{m\times n} + [b_{ij}]_{m\times n} = [c_{ij}]_{m\times n}$ where$[c_{ij}]_{m\times n}$is a fuzzy soft matrix such that $c_{ij} = \min(1\ ,\ a_{ij} + b_{ij})$.

Definition 3.2:[1]Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$ be two fuzzy soft matrices .Then A and B are compatible for multiplication and the product of A and B is defined as $[a_{ij}]_{m\times n} \times [b_{ij}]_{n\times p} = [c_{ij}]_{m\times p}$ where$[c_{ij}]_{m\times p}$is a fuzzy soft matrix such that $c_{ik} = \max(a_{ij} b_{jk})$ for $j = 1,2, \dots, n$.

Proposition 3.1: Let A , B and C be fuzzy soft matrices .Then

1. Fuzzy soft matrix multiplication does not satisfy Commutative. Law$AB \neq BA$

2.Fuzzy soft matrix multiplication satisfiesAssociative Law . $A(BC) = (AB)C$

3 .. Fuzzy soft matrix multiplication does not satisfy Distributive Laws

$$(i)\ A\ (B + C) \neq AB + AC$$

$$(ii)(A + B)C \neq AC + BC$$

4. Fuzzy soft matrix multiplication satisfies Identity Law

$(i)AI_n = A(ii)I_mA = A$, where A is a FSM of order $m \times n$.

5. Fuzzy soft matrix multiplication satisfies Law for Transposition$(AB)^T = B^T A^T$

Proof:1. $(AB)_{ik} = \max(a_{ij} b_{jk}) \neq \max(b_{ij} a_{jk}) = (BA)_{ik}$

2. $[A(BC)]_{ik} = \max(a_{ij} (bc)_{jk}) = \max(a_{ij} \max(b_{jp} c_{pk})) = \max((ab)_{ip} c_{pk}) = [(AB)C]_{ik}$

3.$(i)[A(B + C)]_{ik} =$
$\max\ (a_{ij} (b + c)_{jk}) =$
$\max(a_{ij} \min(1, b_{jk} + c_{jk}))$

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$\neq \min(1, \max(a_{ij}b_{jk}) + \max(a_{ij}c_{jk})) = (AB + AC)_{ik}$

(ii)The proof is similar as above.

4. $(i)(AI)_{ik} = \max(a_{ij}I_{jk}) = A_{ik}$,because$I_{jk} = 0\ if j \neq k and I_{jk} = 1\ if j = k$

(ii)The proof is similar as above.

5.$(AB)^T{}_{ik} = (AB)_{ki} = \max(a_{kj}b_{ji}) = \max(b_{ij}^T a_{jk}^T) = (BTAT)ik$

IV. DETERMINANT OF A FUZZY SOFT SQUARE MATRIX

Let S_n be a symmetric group on {1 , 2 , 3 , … , n}

Determinant of a fuzzy soft square matrix of order 3

If$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Then $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= min\ [\max\ (a_{11}, min(\max(a_{22}, a_{33}), \max(a_{32}a_{23}))),$
$\max(a_{12}, min(\max(a_{21}, a_{33}), \max(a_{31}a_{23}))),$
$\max(a_{13}, min(\max(a_{21}, a_{32}), \max(a_{31}, a_{22})))]$

Example 4.2:

Let$A = \begin{bmatrix} 0.2 & 0 & 1 \\ 1 & 0.4 & 0.7 \\ 0.3 & 0.5 & 0.9 \end{bmatrix}$

Then, $|A| = 0.7$

Properties of determinants

Proposition 4.1:Let$A$ be a fuzzy soft square matrix.

(i)If two rows of A are interchanged to produce amatrix B,then$\det(A) = \det(B)$

(ii) If two columns of A are interchanged to produce amatrix$B$,then$\det(A) = \det(B)$

Proof:(i)If

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$,then$|A| = \min(\max(a_{11}, a_{22}), \max(a_{21}, a_{12}))$

Interchanging rows , we get

$B = \begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix}$,then$|B| = \min(\max(a_{21}, a_{12}), \max(a_{11}, a_{22}))$

Therefore $\det(A) = \det(B)$

(ii)The proof is similar as above.

Proposition 4.2: Let A and B be fuzzy soft square matricesand $a \in [0,1]$. Then,

$(i)\det(A) = \det(A^T)$

$(ii)\ \det(aA) = a\det(A)$

$(iii)\det(a + A) = a + \det(A)$

$(iv)\ \det(AB) \neq \det(A)\det(B)$

$(v)\det(A + B) \neq \det(A) + \det(B)$

Proof: (i) If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then$|A| = \min(\max(a_{11}, a_{22}), \max(a_{21}, a_{12}))$

We have,$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$ and

$|A^T| = \min(\max(a_{11}, a_{22}), \max(a_{12}, a_{21}))$

Therefore,$\det(A) = \det(A^T)$

$(ii)\ \det(aA) = \begin{vmatrix} aa_{11} & aa_{12} \\ aa_{21} & aa_{22} \end{vmatrix} = a\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = adet(A)$

Definition 4.1: Let $A = [\mu_{ij}]_{n\times n}$ be an $n \times n$FSM .Then the determinant of A,denoted by $det\ (A)$ or $|A|$, is defined as $|A| = \prod_{\pi\in s_n} (a_{1\pi(1)} + a_{2\pi(2)} + a_{3\pi(3)} + \cdots + a_{n\pi(n)})$ where the product is taken over all of S_n.

Determinant of a fuzzy soft square matrix of order 2

If$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Then$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \min(\max(a_{11}, a_{22}), \max(a_{21}, a_{12}))$

Example 4.1:

Let $A = \begin{bmatrix} 0.7 & 0.3 \\ 0.8 & 0.6 \end{bmatrix}$

Then $|A| = 0.7$

$(iii)\det(a + A) = \begin{vmatrix} \max(a, a_{11}) & \max(a, a_{12}) \\ \max(a, a_{21}) & \max(a, a_{22}) \end{vmatrix}$,

$= \min(\max(\max(a, a_{11}), \max(a, a_{22})), (\max(\max(a, a_{21}), \max(a, a_{12}))$

$= \min(\max(a, \max(a_{11}, a_{22})), \max(a, \max(a_{21}, a_{12}))$

$= \min(a, \max(a_{11}, a_{22}), \max(a_{21}, a_{12}))$

$= \max(a, \min(\max(a_{11}, a_{22}), \max(a_{21}, a_{12})))$

$= a + \det(A)$

(iv)Let $A = \begin{bmatrix} 0.3 & 1 \\ 0.4 & 0.5 \end{bmatrix} and B = \begin{bmatrix} 0.2 & 0.6 \\ 0 & 0.7 \end{bmatrix}$, then$AB = \begin{bmatrix} 0.06 & 0.7 \\ 0.08 & 0.35 \end{bmatrix}$.

$\det(A) = 0.5$, $\det(B) = 0.6$and$\det(AB) = 0.35$

Therefore $\det(AB) \neq \det(A)\det(B)$

$(v) Let A = \begin{bmatrix} 0.5 & 0.4 \\ 0.7 & 0.6 \end{bmatrix} and B = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$then$A + B = \begin{bmatrix} 0.8 & 0.6 \\ 0.8 & 1 \end{bmatrix}$

$\det(A) = 0.6\ and\ \det(B) = 0.2$ and$\det(A + B) = 0.8$.

Therefore $\det(A + B) \neq \det(A) + \det(B)$

V. RANK OF A FUZZY SOFT MATRIX

Definition 5.1:AFSM is said to be singular if its determinant is equal to zero.It is said to be non-singular if its determinant is not equal to zero.

Example 5.1:

The fuzzy soft matrix$A = \begin{bmatrix} 0 & 0.3 & 0.2 \\ 0.6 & 0 & 0.6 \\ 0.1 & 0.7 & 0 \end{bmatrix}$is singular , because$\det(A) = 0$

Example 5.2:

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The fuzzy soft matrix $A = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.0 & 0.6 \\ 0.1 & 0.7 & 0.0 \end{bmatrix}$ is non singular ,because$\det(A) = 0.2 \neq 0$

Proposition 5.1:AFSM is singular if all of its principal diagonal[4] elements are zero .

Proof:Suppose$A$ is an $n \times n$fuzzy soft matrix .Then

$\det(A)$

$= \min(\max(a_{11}, \min(\max(a_{22}, \min(\max(a_{33}, \min(\max(a_{44}, \cdots$

$\min(\max(a_{n-1n-1}, a_{nn}), \max(a_{nn-1}, a_{n-1n}))$ ….and this will be zero only when $a_{ii} = 0$ for $i = 1,2, \dots, n$.Hence all the principal diagonal elements are zero.

Example 5.3:The fuzzy soft matrix in example 5.1

Definition 5.2:Rank of a fuzzy soft matrix A is defined to be the order of the largest square sub matrix of A whose determinant is not equal to zero.

Example 5.4:

Consider the FSM in example 5.1.

Since $\det(A) = 0$, the rank of the FSM is not equal to 3.

But $\begin{vmatrix} 0.3 & 0.2 \\ 0.0 & 0.6 \end{vmatrix} = 0.2 \neq 0$

Therefore the rank is 2.

Proposition 5.2:The rank of a non-singular FSMis same as the order of the fuzzy soft matrix.

Proof:Let$A$ be an $n \times n$ non-singular FSM .Then , $\det(A) \neq 0$ .So rank of A = order of$A = n$

VI. EIGEN VALUE OF A FUZZY SOFT MATRIX

Definition 6.1:[7] Let Abe a fuzzy soft matrix of order $n \times n$ andλ be a scalar such that $\lambda \in [0,1]$ .Then λ is called an eigenvalue of A and a non-zero vector X be called a row (column) eigenvector associated with the eigenvalue λ of A if $XA = \lambda X$ $(AX = \lambda X)$.

Proposotion 6.1: If $A = [a_{ij}]$ is aFSM of order n×n, such that $a_{1i} = a_{2i} = a_{3i} = \cdots = a_{i-1i} = a_{i+1i} = \cdots = a_{ni} = c$where $i \in \{1,2,3,\cdots, n\}$, $c \in [0,1]$ and$c < a_{ii}$, then a_{ii} is an eigenvalue corresponding to the column eigenvector $[c, c, , \cdots, c, 1, c, \cdots, c]^T$, where 1 is theith entry.

Proof: Let$X = [c, c, , \cdots, c, 1, c, \cdots, c]^T$.Then,

$$AX = \begin{bmatrix} \max(a_{1k}x_{k1}) \\ \max(a_{2k}x_{k1}) \\ \max(a_{3k}x_{k1}) \\ \vdots \\ \max(a_{ik}x_{k1}) \\ \vdots \\ \max(a_{nk}x_{k1}) \end{bmatrix} = \begin{bmatrix} c \\ c \\ c \\ \vdots \\ a_{ii} \\ \vdots \\ c \end{bmatrix} = a_{ii} \begin{bmatrix} c \\ c \\ c \\ \vdots \\ 1 \\ \vdots \\ c \end{bmatrix} = a_{ii}X$$, where

$k = 1,2,3, \dots, n$.

Therefore a_{ii} is the eigen value corresponding to X

Example 6.1:

Let $A = \begin{bmatrix} 0.6 & 0.0 & 0.7 \\ 0.7 & 0.5 & 0.8 \\ 0.8 & 0.0 & 0.9 \end{bmatrix}$ and $X = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Then0.5 is the eigenvalue of A corresponding to the eigen vector X.

Example 6.2:

Let $A = \begin{bmatrix} 0.6 & 0.3 & 0.7 \\ 0.7 & 0.5 & 0.8 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$ and $X = \begin{bmatrix} 0.3 \\ 1 \\ 0.3 \end{bmatrix}$

Then 0.5 is the eigenvalue of A corresponding to the eigen vector X.

Proposition 6.2:If$A = [a_{ij}]$ is a fuzzy soft matrix of order n×n, such that $a_{i1} = a_{i2} = a_{i3} = \cdots = a_{ij-1} = a_{ij+1} = \cdots = a_{in} = c$ where $i \in \{1,2,3,\cdots, n\}$, $c \in [0,1]$ and$c < a_{ii}$,then a_{ii} is an eigenvalue corresponding to the row eigenvector$[c, c, \cdots, c, 1, , c \cdots, c]$, where 1 is the$ith$ entry.

Proof:The proof is similar as the proof of Proposition 6.1

Example 6.3:

Let $A = \begin{bmatrix} 0.7 & 0.9 & 0.8 \\ 0.0 & 0.7 & 0.0 \\ 0.9 & 0.6 & 0.1 \end{bmatrix}$ and $X = [0 \quad 1 \quad 0]$

Then 0.7 is the eigenvalue of A corresponding to the eigen vector X.

Example 6.4:

Let $A = \begin{bmatrix} 0.7 & 0.9 & 0.8 \\ 0.5 & 0.7 & 0.5 \\ 0.9 & 0.6 & 0.1 \end{bmatrix}$ and $X = [0.5 \quad 1 \quad 0.5]$

Then0.7 is the eigenvalue of A corresponding to the eigen vector X.

Proposition 6.3:[7]If $A = [a_{ij}]$ is a fuzzy soft matrix of order n×n, such that $a_{1i} = a_{2i} = \cdots = a_{ni} = \lambda \geq a_{ij}$ where $i, j \in \{1,2,3,\cdots, n\}$, then λ is an eigenvalue corresponding to the column eigenvector $[1,1, , \cdots, 1, \cdots, 1]^T$

Proof: Let$X = [1,1, , \cdots, 1, \cdots, 1]^T$.Then,

$$AX = \begin{bmatrix} \max(a_{1k}1) \\ \max(a_{2k}1) \\ \max(a_{3k}1) \\ \vdots \\ \max(a_{ik}1) \\ \vdots \\ \max(a_{nk}1) \end{bmatrix} = \begin{bmatrix} \max(a_{1k}) \\ \max(a_{2k}) \\ \max(a_{3k}) \\ \vdots \\ \max(a_{ik}) \\ \vdots \\ \max(a_{nk}) \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \\ \lambda \\ \vdots \\ \lambda \\ \vdots \\ \lambda \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} =$$

λX , where$k = 1,2,3, \dots, n$

Therefore λ is the eigen value corresponding to X

Example 6.5:

Let $A = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 0.5 & 0.6 \end{bmatrix}$ and $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Then 0.7 is the eigenvalue of A corresponding to the eigen vector X.

Proposition 6.4:[7]If $A = [a_{ij}]$ is a fuzzy soft matrix of order n×n, such that $a_{i1} = a_{i2} = a_{i3} = \cdots = a_{in} = \lambda \geq a_{ij}$ where $i, j \in \{1,2,3,\cdots, n\}$,then λ is an eigenvalue corresponding to the row eigenvector$[1,1, \dots, 1]$.

Proof:The proof is similar as the proof of Proposition 6.3

Example 6.6:

Let $A = \begin{bmatrix} 0.4 & 0.2 & 0.5 \\ 0.6 & 0.3 & 0.4 \\ 0.8 & 0.8 & 0.8 \end{bmatrix}$ and $X = [1 \quad 1 \quad 1]$

Then 0.8 is the eigenvalue of A corresponding to the eigen vector X.

VII. CONCLUSION

In this paper we have considered the concept of fuzzy soft matrices and discussed some properties of product of fuzzy soft matrices. We also defined the determinant and studied some of the properties. Then we discussed the rank and eigenvalues of fuzzy soft matrices.

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